Solution

**ENGR 6913 Advanced Engineering Mathematics**

## Assignment No.1

The following problems from the textbook (Advanced Engineering Mathematics by Kreyszig, $8^{th}$ Edition):

1. Problem **1.3**: 9
2. Problem **1.3**: 17
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# ENGR 6913
## Assignment No. 1

**1.3** 9 $\quad y' = \dfrac{x^2+y^2}{xy}$

$$y' = \frac{x}{y} + \frac{y}{x} = \frac{1}{u} + u$$

$y/x = u$

$y = ux$

$\dfrac{dy}{dx} = u + x\dfrac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{1}{u} + u$$

$$u\,du = \frac{dx}{x}$$

$$\frac{u^2}{2} = \ln|x| + C^*$$

$$x = Ce^{u^2/2} \Rightarrow \underline{x = Ce^{y^2/2x^2}}$$

or $\quad y^2/x^2 = 2\ln|x| + C_2 \Rightarrow \underline{y = x\sqrt{2\ln|x| + C_2}}$

**1.3** 17 $\quad y'\cosh^2 x - \sin^2 y = 0 \qquad y(0) = \pi/2$

$$\frac{dy}{\sin^2 y} = \frac{dx}{\cosh^2 x}$$

$$\operatorname{cosec}^2 y \,.\, dy = \operatorname{sech}^2 x\, dx$$

$$-\cot y = \tanh x + C$$

$$y(0) = \pi/2 \Rightarrow C = 0$$

$\left\{ \text{Since } \dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \text{ and } \dfrac{d}{dy}(\cot y) = -\operatorname{cosec}^2 y \right.$

$$\cot y = -\tanh x$$

1.3 25 $\quad y' = \dfrac{1-2y-4x}{1+y+2x}$ $\qquad$ let $v = y+2x$

$$\frac{dv}{dx} = \frac{dy}{dx} + 2$$

$$y' = \frac{1-2v}{1+v}$$

$$\frac{dv}{dx} - 2 = \frac{1-2v}{1+v} \qquad \frac{dv}{dx} = \frac{2+2v+1-2v}{1+v}$$

$$\frac{dv}{dx} = \frac{3}{1+v} \Rightarrow (1+v)\,dv = 3\,dx$$

$$v + \frac{v^2}{2} = 3x + C$$

$$y + 2x + \frac{1}{2}\left(y^2 + 4x + 4x^2\right) = 3x + C^*$$

$$2y + 4x + (y+2x)^2 = 6x + C$$

$$2y - 2x + (y+2x)^2 = C$$

1.4 4 $\quad$ Acceleration $y'' = 7t$

$$y' = \frac{7t^2}{2} + C_1 \qquad (\text{No air resistance } C_1 = 0)$$

$$y = \frac{7}{6}t^3 + C_2 \qquad (y(0) = 0 \Rightarrow C_2 = 0)$$

After 10 sec. : initial speed = final speed from previous 10 seconds!

$$y'(10) = \frac{7}{2}(10)^2 = \frac{700}{2} = 350$$

Distance reached after first 10 sec,

$$y(10) = \frac{7}{6}(10)^3 = \frac{7000}{6} = 1167$$

At the peak $v = 0$, $S = 0$

For further flight after the peak $s = \frac{1}{2} g t^2$

$$v = \frac{ds}{dt} = gt = 350 \, m/s \Rightarrow t = \frac{350}{9.81} = 35.7 \text{ sec.}$$

Further distance $s = \frac{9.81}{2} (35.7)^2 = 6244$ m

So total distance $= 1167 + 6244 = 7411$ m.

1.4 8 For the motion of a particle at constant acceleration, we have

$$s = \frac{1}{2} a t^2 + v_i t$$

$s$ : distance

$t$ : time

$$v = \frac{ds}{dt} = at + v_i$$

$v_i$ : initial speed

$a$ : acceleration.

Since $v(0) = 10^3 \Rightarrow v_i = 10^3$

$$v(10^{-3}) = a(10^{-3}) + 10^3 = 10^4$$

$$a = 9 \times 10^6$$

So, $s = \frac{1}{2} \times 9 \times 10^6 (10^{-3})^2 + 10^3 (10^{-3}) = 5.5$ m.

1.4 13 According to Newton's law of cooling

$$\frac{dT}{dt} = k(T - T_A) = -k(T_A - T)$$

$$\frac{dT}{T_A - T_A} = -k \, dt$$

$$\ln(T_A - T_A) = -kt + C$$

$$T(1) = 12 \Rightarrow \ln(22 - 12) = -k + C$$

$$T(0) = 5 \Rightarrow \ln(22 - 5) = 0 + C$$

$C = 2.83 \qquad k = 0.53$

$$t = \frac{C - \ln(T_A - T)}{k} = \frac{2.83 - \ln(22 - 21.9)}{0.53}$$

$t = 9.69$ min.

1.4 (14) $y$ = amount of salt in the tank at time $t$

Each gallon of brine has $y/400$ lb of salt.

Rate of change of salt in the tank = Salt inflow rate − Salt outflow rate

$$\frac{dy}{dt} = 0 - 2 \times \frac{y}{400} = -0.005y$$

$$y = C e^{-0.005t} \qquad y(0) = 100 \Rightarrow C = 100$$

$$y = 100 e^{-0.005t}$$

$$y = 100 e^{-0.005 \times 60} = 74.1 \text{ lb.}$$

1.4 (18) Newton's second law $\Sigma F = m s''$

$\Sigma F = W \sin\alpha - \mu N = mg \sin\alpha - \mu . mg \cos\alpha$

$\cancel{m} g(\sin\alpha - \mu \cos\alpha) = \cancel{m} s''$

$s'' = 9.81(\sin 30 - 0.2 \cos 30) = 3.206$

$s' = 3.206t + C_1 \qquad s'(0) = 0 \Rightarrow C_1 = 0$

$s' = 3.206t$

$s = 3.206 \frac{t^2}{2} + C_2 \qquad s(0) = 0 \Rightarrow C_2 = 0$

$s = 3.206\, t^2/2$

For $s = 10\,m$ $\quad t = \sqrt{\dfrac{2 \times 10}{3.206}} = 2.5$ sec.

$s'(2.5) = 3.206 \times 2.5 = 8$ m/s

1.4 (20)

(a) For free-fall condition $a = g$, $v = gt$

and $h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\dfrac{2h}{g}}$ put in the relationship for $v = gt = g\sqrt{\dfrac{2h}{g}} = \sqrt{2gh}$

(b) Rate of change of mass in the tank

= Mass inflow rate − Mass outflow rate

$$\frac{d}{dt}(\rho B(h) h) = 0 - \rho A_j v_j$$

$B(h)$: Area of cross-section of tank

$A_j$ = Area of cross-section of jet of water (hole).

$v_j$ = Velocity of jet of water.

$\rho$ = Mass density of fluid in the tank.

$$\frac{dh}{dt} = -\frac{A \times 0.6\sqrt{2 \times 980}}{B(h)} \cdot \sqrt{h}$$

$$h' = -26.56 \frac{A}{B(h)} \sqrt{h}$$

(c)

$$\frac{dh}{\sqrt{h}} = -26.56 \frac{A}{B} dt$$

$$2\sqrt{h} = -26.56 \frac{A}{B} t + C \Rightarrow h^{1/2} = -13.28 \frac{A}{B} t + C$$

(d) $d = 1$ cm $\qquad D = 100$ cm

$A = \frac{\pi d^2}{4} \qquad B = \frac{\pi D^2}{4}$

$$A/B = \frac{\pi d^2/4}{\pi D^2/4} = \left(\frac{d}{D}\right)^2 = \left(1/100\right)^2 = 1\times10^{-4}$$

$$h^{1/2} = -0.001328t + C$$

$h(0) = 150$ cm $\qquad C = \sqrt{150} = 12.25$

$$h^{1/2} = -0.001328t + 12.25$$

When the tank will be empty $h=0$

$$t = \frac{12.25}{0.001328} = \underline{9222 \text{ sec}} = 154 \text{ min.}$$

(e) If A is increased, the rate of change of h ($h'$) will also increase. In other words the tank will be emptied faster. If the tank has larger Area (B), $h'$ will decrease, which means it will take longer to empty the tank.

<u>1.5</u> 39 Let us select $u = xy^2 + y^3 = C$

$$du = 2xy\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0$$

$$(2xy + 3y^2)\frac{dy}{dx} + y^2 = 0$$

$$y^2\,dx + (2xy + 3y^2)\,dy = 0 \qquad \text{(Exact equation)}$$

Considering $F = y$

$$y\,dx + (2x + 3y)\,dy = 0 \qquad \text{(Non-exact)}$$

Considering $F = xy$

$$\frac{y}{x}\,dx + \left(2 + 3\,\frac{y}{x}\right)dy = 0 \qquad \text{(Non-exact)}$$

Considering $F = xy^2$

$$\frac{1}{x}\,dx + \left(\frac{2}{y} + \frac{3}{x}\right)dy = 0 \qquad \text{(Non-exact)}$$