Groundwater and well hydraulics

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Steady unidirectional flow

Confined aquifer

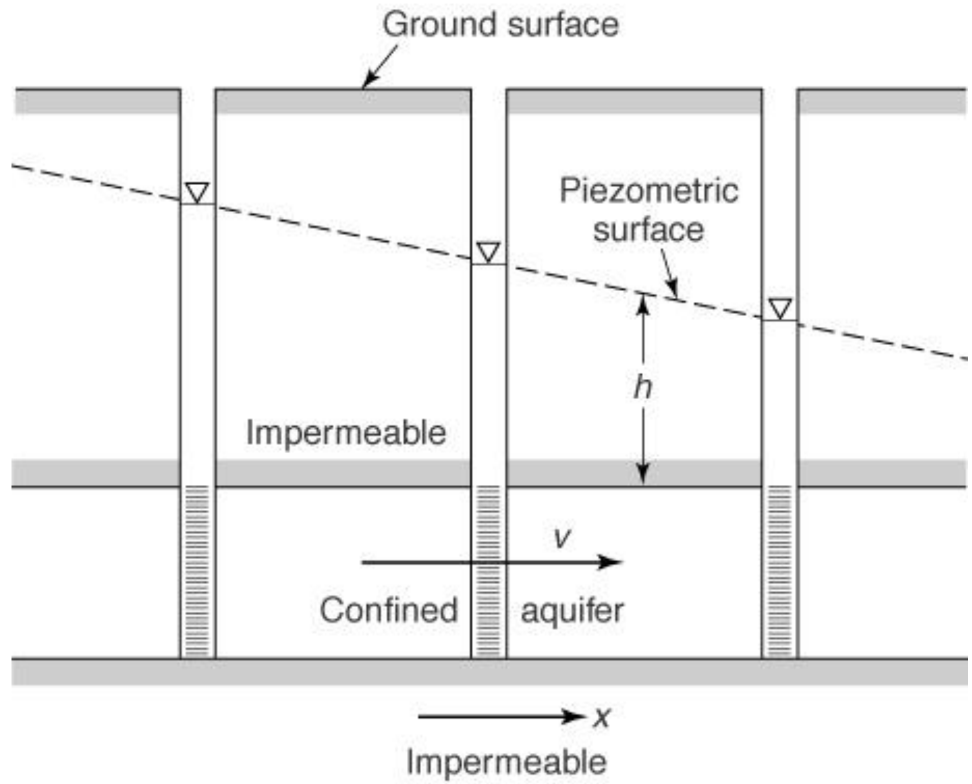
$$\frac{\partial^2 h}{\partial x^2} = 0$$

$$h = C_1 x + C_2$$

$$h = 0 \quad at \quad x = 0 \quad and$$

$$\partial h / \partial x = -v / K \ (Darcy)$$

$$h = -\frac{vx}{K}$$

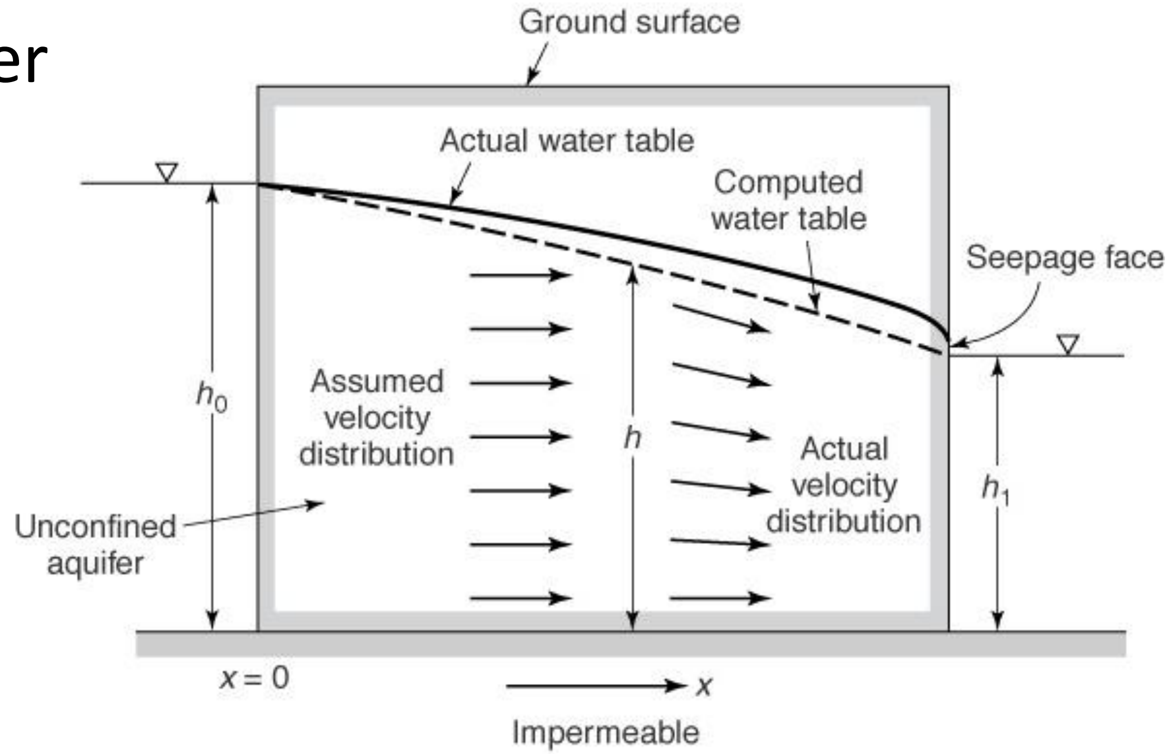


Example 4.1.1

Unconfined aquifer

$$q = -Kh\frac{dh}{dx}$$

$$qx = -\frac{K}{2}h^2 + C$$



$$h = h_0 \quad at \quad x = 0$$

$$q = \frac{K}{2x}\left(h_0^2 - h^2\right)$$

Example 4.1.2

A stratum of clean sand and gravel between two channels (see Figure 4.1.2) has a hydraulic conductivity $K = 10^{-1}$ cm/sec, and is supplied with water from a ditch ($h_0 = 6.5$ m deep) that penetrates to the bottom of the stratum. If the water surface in the second channel is 4 m above the bottom of the stratum and its distance to the ditch is $x = 150$ m (which is also the thickness of the stratum), estimate the unit flow rate into the gallery.

Solution

The flow is computed using the Dupuit equation (4.1.6) for unit flow, where

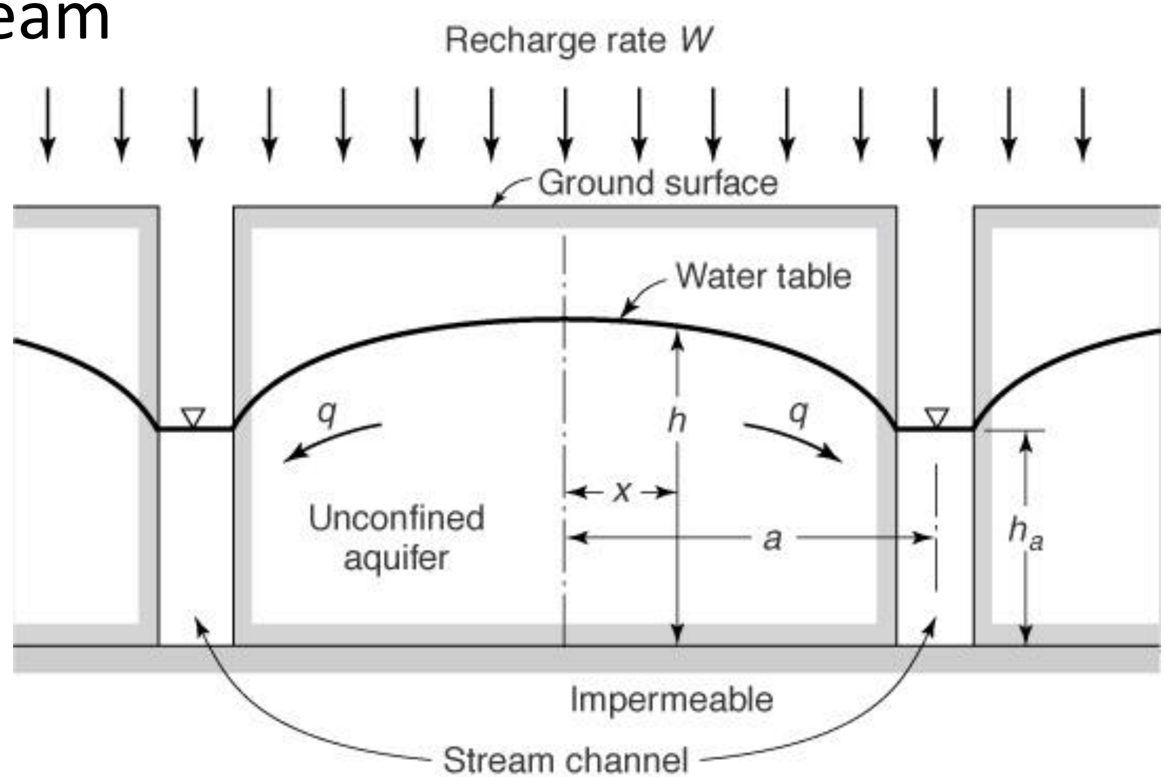
$$K = 10^{-1} \text{ cm/sec} = 86.4 \text{ m/day}$$

$$q = \frac{K}{2x}\left(h_0^2 - h^2\right) = \frac{86.4 \text{ m/day}}{2(150 \text{ m})}\left(6.5^2 - 4^2\right)\text{m}^2 = 7.56 \text{ m}^2\text{/day}$$

Base flow to a stream

$$q = -Kh\frac{dh}{dx}$$

$$q = Wx$$



$$h = h_a \quad at \quad x = a$$

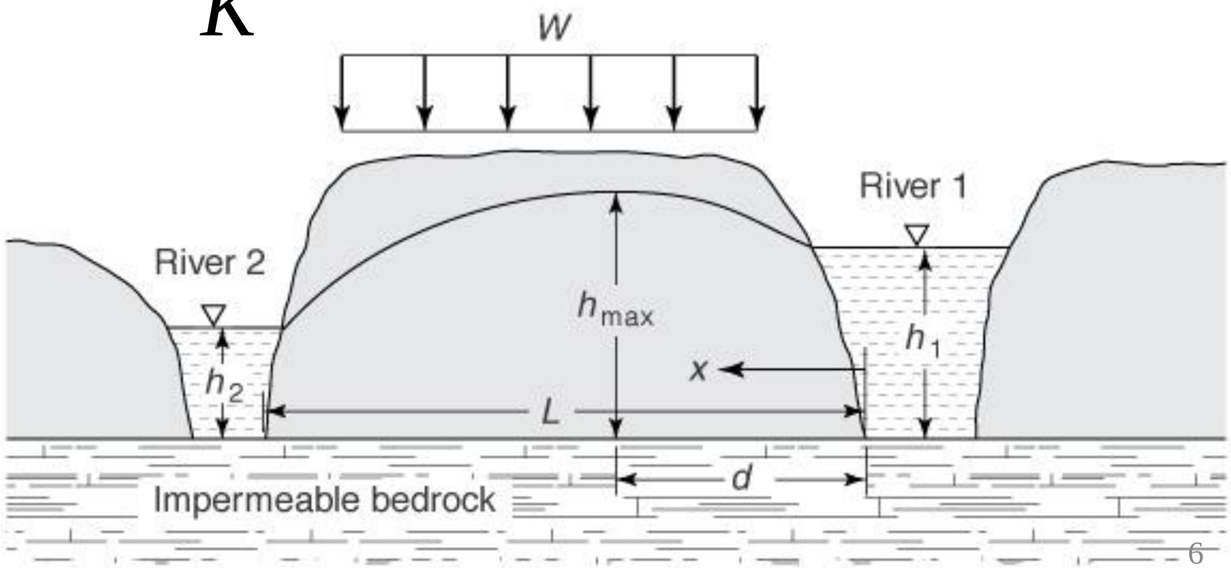
$$h^2 = h_a^2 + \frac{W}{K}\left(a^2 - x^2\right)$$

$$\frac{d}{dx}\left(Kh\frac{dh}{dx}\right)=-W \Rightarrow \frac{d^2}{dx^2}\left(h^2\right)=-\frac{2W}{K}$$

$$h^2=\frac{Wx^2}{K}+c_1x+c_2$$

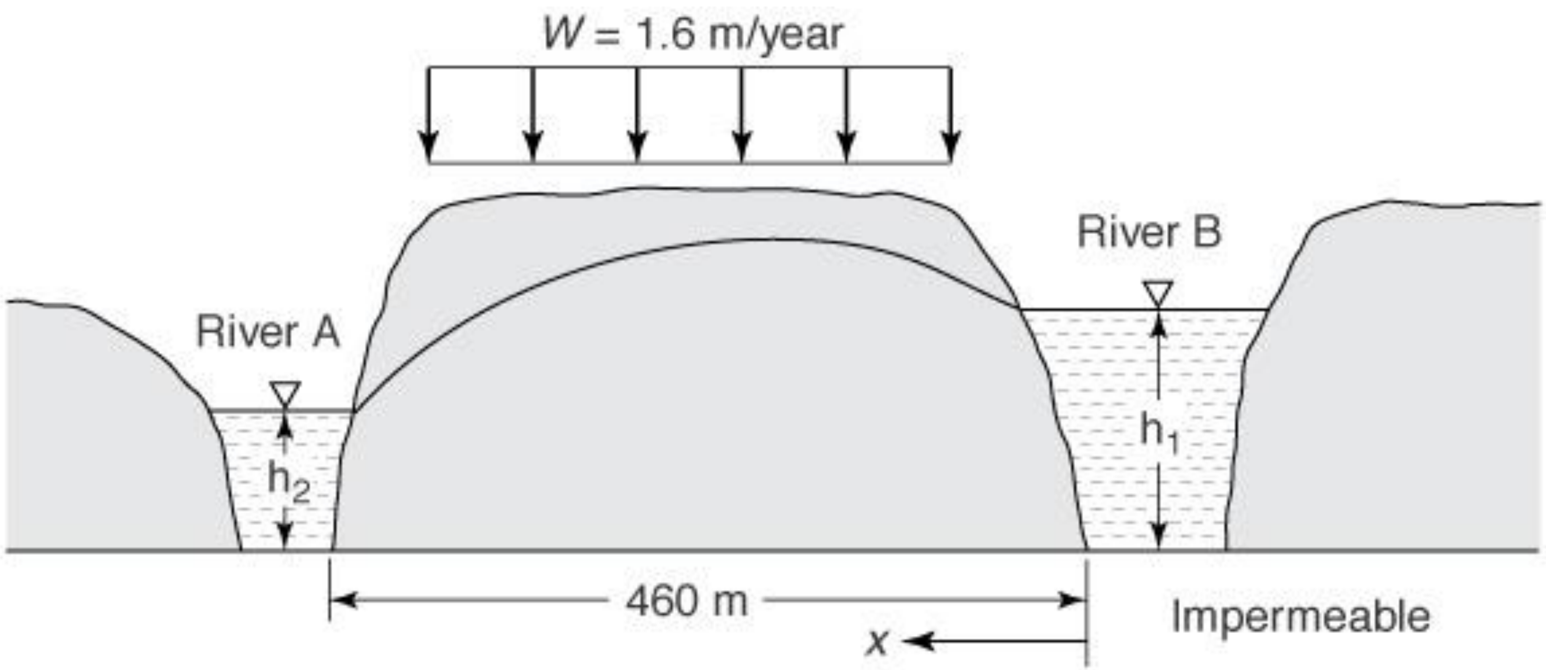
$$h=h_1 \ at \ x=0 \quad and \quad h=h_2 \ at \ x=L$$

$$h^2=h_1^2-\frac{\left(h_1^2-h_2^2\right)x}{L}+\frac{W}{K}\left(L-x\right)x$$



Example 4.1.3

An unconfined aquifer of clean sand and gravel is located between two fully penetrating rivers (see Figure 4.1.5) and has a hydraulic conductivity of $K = 10^{-2}$ cm/sec. The aquifer is subject to a uniform recharge of 1.6 m/year. The water surface elevations in rivers A and B are 8.5 m and 10 m, respectively, above the bottom. Estimate (a) the maximum elevation of the water table and the location of groundwater divide, (b) the travel times from groundwater divide to both rivers ($n_e = 0.35$), and (c) the daily discharge per kilometer from the aquifer into both rivers.



(a) The maximum elevation of the water table occurs at the location of the groundwater divide computed using Equation 4.1.17 with $W = 1.6$ m/year $= 0.0044$ m/day and $K = 10^{-2}$ cm/s $= 8.64$ m/day:

$$d = \frac{L}{2} - \frac{K}{W}\frac{\left(h_1^2 - h_2^2\right)}{2L} = \frac{460\text{ m}}{2} - \frac{8.64\text{ m/d}}{0.0044\text{ m/d}}\frac{\left(10^2 - 8.5^2\right)\text{m}^2}{2(460\text{ m})} = 171\text{ m from river B}$$

The maximum head at the divide is computed using Equation 4.1.18:

$$h_{\max} = \sqrt{h_1^2 - \frac{\left(h_1^2 - h_2^2\right)d}{L} + \frac{W}{K}(L-d)d}$$

$$= \sqrt{10^2 - \frac{\left(10^2 - 8.5^2\right)(171)}{460} + \frac{0.0044}{8.64}(460-171)(171)} = 10.7\text{m}$$

(b) The average pore velocity is computed using Darcy's law with the Dupuit assumptions:

$$v_A = \left(\frac{K}{n_e}\right)\left(\frac{\Delta h}{\Delta x}\right) = \left(\frac{8.64\text{ m/d}}{0.35}\right)\left(\frac{10.7-8.5}{460-171}\right)\frac{\text{m}}{\text{m}} = 0.190\text{ m/day}$$

So the travel time from the groundwater divide to river A is

$$t = \frac{L_A}{v_A} = \frac{460\text{ m} - 171\text{ m}}{0.190\text{ m/day}} = 1524\text{ days} = 4.18\text{ years}$$

Similarly, the travel time from the groundwater divide to river B is computed as

$$v_B = \left(\frac{K}{n}\right)\left(\frac{\Delta h}{\Delta x}\right) = \left(\frac{8.64\text{ m/d}}{0.35}\right)\left(\frac{10.7-10}{171}\right) = 0.101\text{ m/day}$$

$$t = \frac{L_A}{v_A} = \frac{171\text{m}}{0.104\text{ m/day}} = 1692\text{ days} = 4.64\text{ years}$$

(c) From Equation 4.1.16, for $x = 0$:

$$q_x = \frac{K\left(h_1^{\,2} - h_2^{\,2}\right)}{2L} - W\left(\frac{L}{2} - x\right)$$

$$= \frac{(8.64\text{ m/d})\left(10^2 - 8.5^2\right)\text{m}^2}{2(460)\text{m}} - (0.0044\text{ m/d})\left(\frac{460}{2} - 0\right)\text{m} = -0.751\left(\text{m}^3/\text{day}\right)/\text{m}$$

The minus sign occurs due to opposite flow direction to the x-axis (see Figure 4.1.5). So, (0.751 × 1000 m) = 751 m^3/day is the daily discharge from the aquifer per kilometer into river B.

Similarly, for $x = 460$ m:

$$q_x = \frac{K\left(h_1^{\,2} - h_2^{\,2}\right)}{2L} - W\left(\frac{L}{2} - x\right)$$

$$= \frac{(8.64\text{ m/d})\left(10^2 - 8.5^2\right)}{2(460)} - (0.0044\text{ m/d})\left(\frac{460}{2} - 460\right) = 1.27\left(\text{m}^3/\text{day}\right)/\text{m}$$

The daily discharge from the aquifer per kilometer into river A is (1.27 × 1000 m) = 1270 m^3/day.

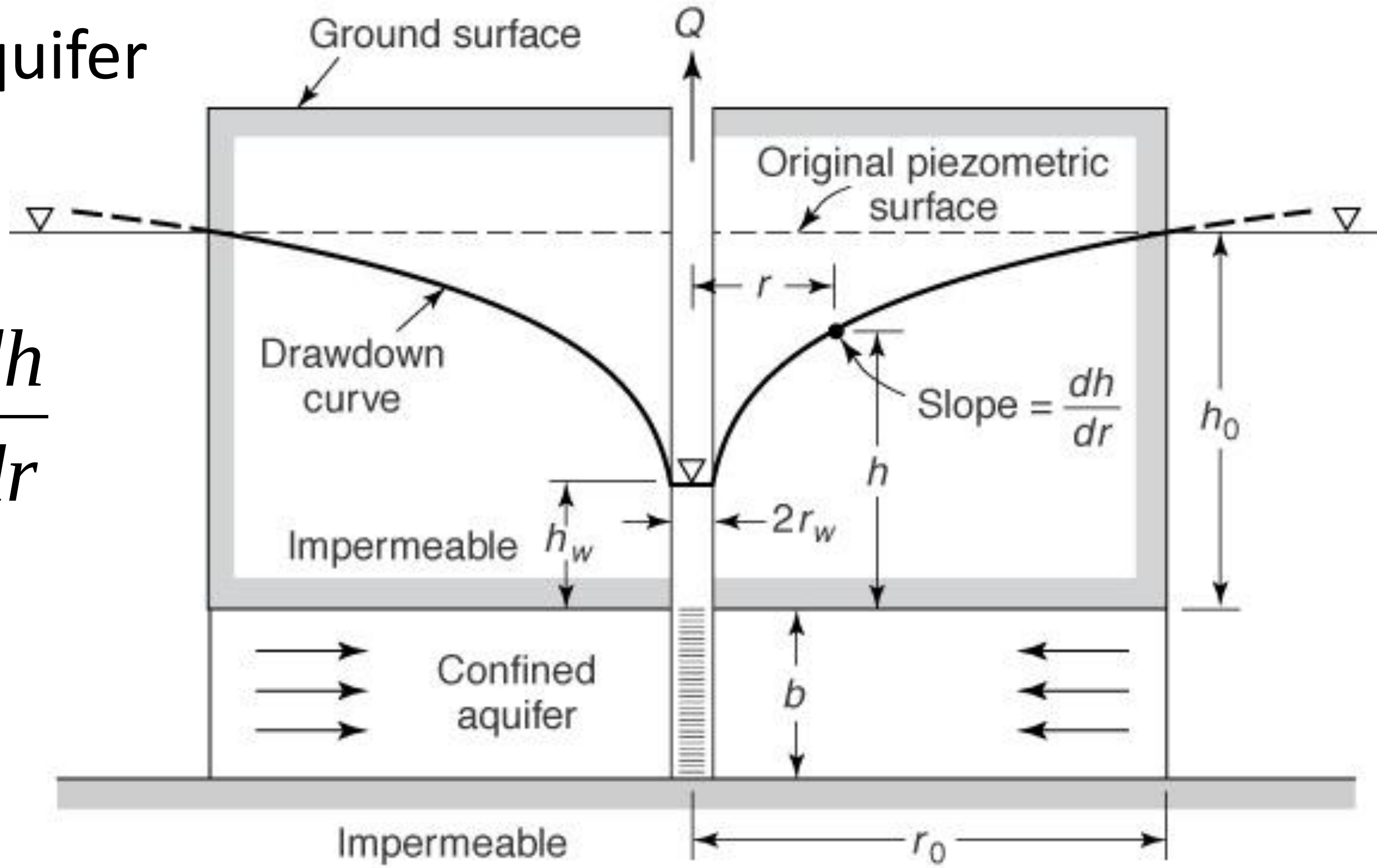
Steady radial flow to a well

Confined aquifer

$$Q = Av$$

$$= -2\pi rbK \frac{dh}{dr}$$

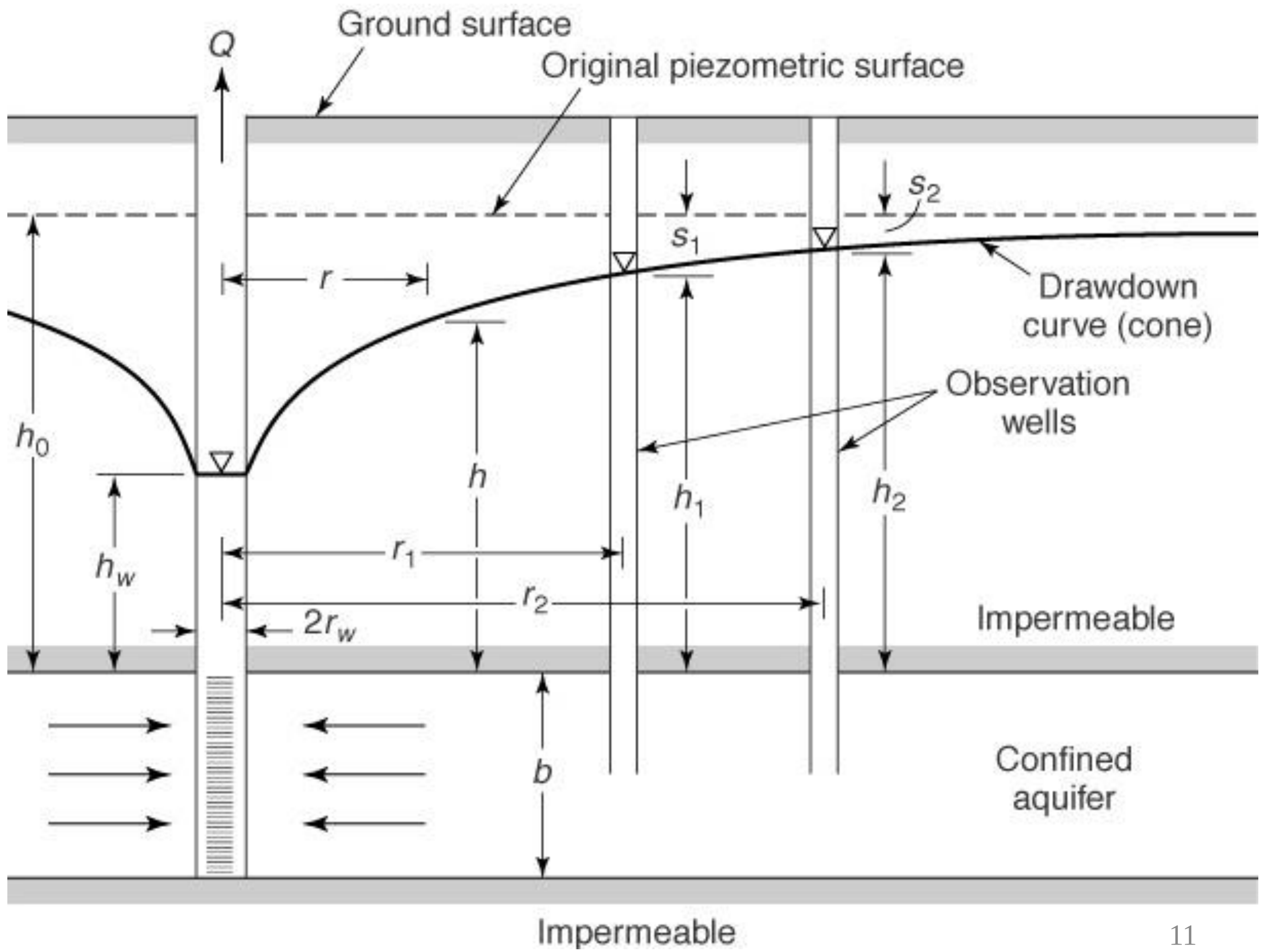
$$dh = \frac{Q}{2\pi Kb} \frac{dr}{r}$$



$$Q = 2\pi bK \frac{h_0 - h_w}{\ln(r_0 / r_w)}$$

Thiem equation

$$T = Kb = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right) = \frac{Q}{2\pi(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right)$$



Example 4.2.1

A well fully penetrates a 25-m thick confined aquifer. After a long period of pumping at a constant rate of $0.05\ m^3/s$, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Determine the hydraulic conductivity and the transmissivity. What type of unconsolidated deposit would you expect this to be?

Use Equation 4.2.5 to compute the hydraulic conductivity with $Q = 0.05\ m^3/s$, $r_1 = 50$ m, $r_2 = 150$ m, $s_1 = h_0 - h_1$, and $s_2 = h_0 - h_2$, so $s_1 - s_2 = h_2 - h_1 = 3 - 1.2 = 1.8$ m. $Q = 0.05\ m^3/s = 4320\ m^3/day$, and

$$K = \frac{Q}{2\pi b(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ m^3/day}{2\pi(25\ m)(1.8\ m)} \ln\left(\frac{150}{50}\right) = 16.8\ m/day$$

The transimissivity is $T = Kb = (16.8\ m/day)(25\ m) = 420\ m^2/day$. Referring to Figure 3.2.2 and Table 3.2.1 with $K = 1.94 \times 10^{-4}$ m/s shows that this aquifer is probably a medium clean sand. ■

Example 4.2.2

A 1-m diameter well penetrates vertically through a confined aquifer 30 m thick. When the well is pumped at 113 m^3/hr, the drawdown in a well 15 m away is 1.8 m; in another well 50 m away, it is 0.5 m. What is the approximate head in the pumped well for steady-state conditions and what is the approximate drawdown in the well? Also compute the transmissivity of the aquifer and the radius of influence of the pumping well. Take the initial piezometric level as 40 m above the datum.

First determine the hydraulic conductivity using Equation 4.2.5: $Q = 113\ \text{m}^3/\text{hr} = 2712\ \text{m}^3/\text{day}$. Then

$$K = \frac{Q}{2\pi b(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right) = \frac{2712\ \text{m}^3/\text{day}}{2\pi(30\ \text{m})(1.8\ \text{m} - 0.5\ \text{m})} \ln\left(\frac{50}{15}\right) = 13.3\ \text{m/day}$$

The transmissivity is $T = Kb = 13.3\ \text{m/day} \times 30\ \text{m} = 400\ \text{m}^2/\text{day}$.

To compute the approximate head, h_w, in the pumped well, rearrange Equation 4.2.5 and use $h_2 = h_0 - s_2 = 40 - 0.5 = 39.5$ m

$$h_w = h_2 - \frac{Q}{2\pi Kb} \ln\left(\frac{r_2}{r_w}\right) = 39.5\ \text{m} - \frac{2712\ \text{m}^3/\text{day}}{2\pi(13.3\ \text{m/day})(30\ \text{m})} \ln\left(\frac{50\ \text{m}}{0.5\ \text{m}}\right) = 34.5\ \text{m}$$

Drawdown is then

$$s_w = h_0 - h_w = 40\ \text{m} - 34.5\ \text{m} = 5.5\ \text{m}$$

The radius of influence (R) of pumping well can be found by rearranging Equation 4.2.5 and solving for r_0 which is R:

$$R = (r_1)\exp\left[\frac{2\pi Kb(h_0 - h_1)}{Q}\right] = (15\ \text{m})\exp\left[\frac{2\pi(13.3\ \text{m/day})(30\ \text{m})(40\ \text{m} - 38.2\ \text{m})}{2712\ \text{m}^3/\text{day}}\right] = 79\ \text{m}$$

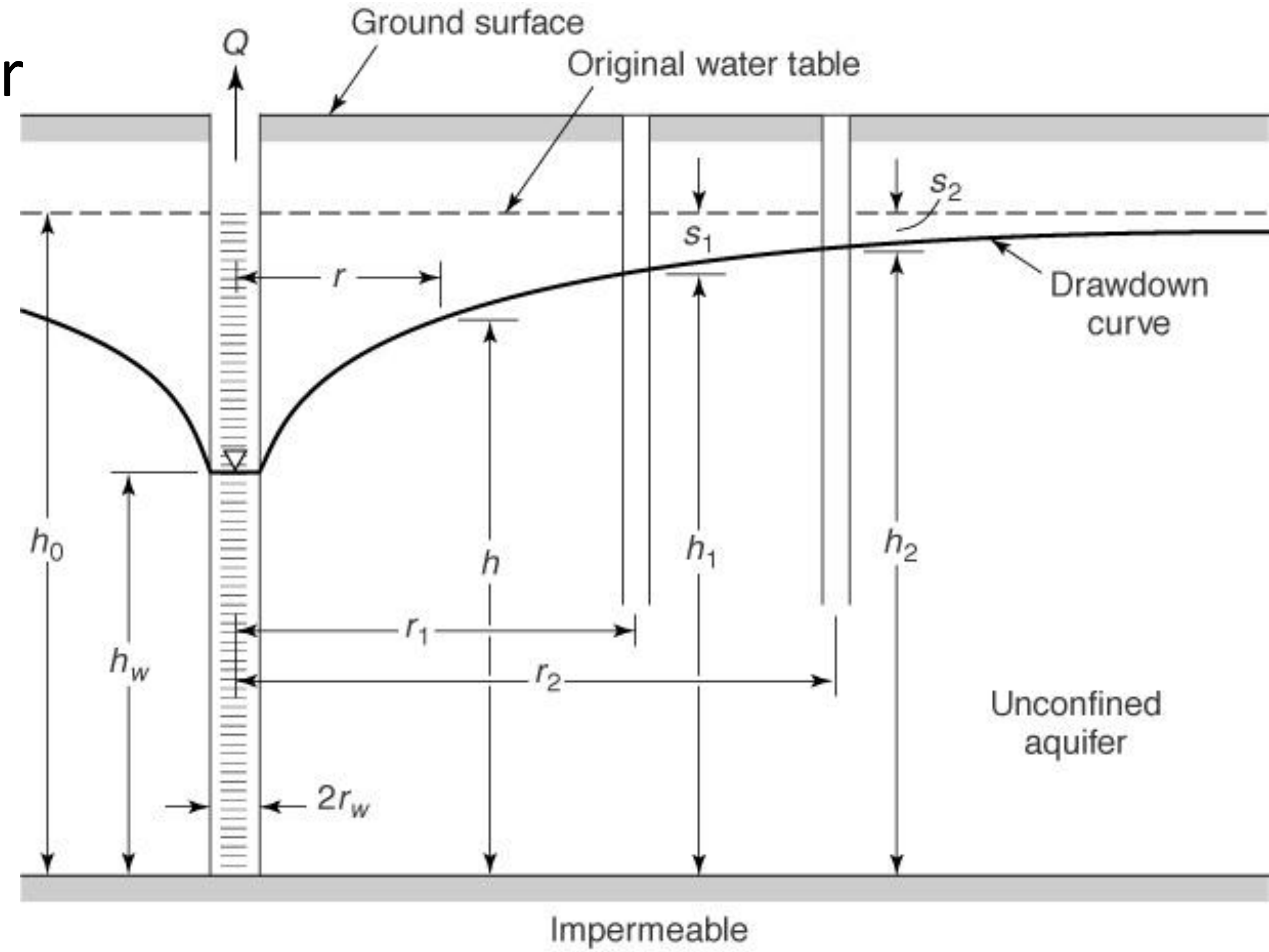
Unconfined aquifer

$$Q = Av$$

$$= -2\pi r K h \frac{dh}{dr}$$

$$h\,dh = -\frac{Q}{2\pi K b}\frac{dr}{r}$$

$$Q = \pi K \frac{h_0^2 - h_w^2}{\ln\left(r_0 / r_w\right)}$$



$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)}\ln\left(\frac{r_2}{r_1}\right)$$

Example 4.2.3

A well penetrates an unconfined aquifer. Prior to pumping the water level (head) is $h_0 = 25$ m. After a long period of pumping at a constant rate of 0.05 m^3/s, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Compute the hydraulic conductivity of the aquifer and the radius of influence of pumping well. What type of deposit is the aquifer material?

Use Equation 4.2.10 to compute K with $Q = 0.05\ m^3/s = 4320\ m^3/day$, $r_1 = 50$ m, $r_2 = 150$ m, $h_1 = 25 - 3 = 22$ m, and $h_2 = 25 - 1.2 = 23.8$ m.

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ \text{m}^3/\text{day}}{\pi\left(23.8^2 - 22^2\right)} \ln\left(\frac{150\ \text{m}}{50\ \text{m}}\right) = 18.3\ \text{m/day}$$

The deposit is probably a medium clean sand. Equation 4.2.10 is used to compute the radius of influence:

$$R = (r_1)\exp\left[\frac{K\pi\left(h_0^2 - h_1^2\right)}{Q}\right] = (50\ \text{m})\exp\left[\frac{(18.3\ \text{m/day})\pi\left(25^2 - 22^2\right)}{4320\ \text{m}^3/\text{day}}\right] = 327\ \text{m}$$

Example 4.2.4

A well 0.5 m in diameter penetrates 33 m below the static water table. After a long period of pumping at a rate of 80 m^3/hr, the drawdowns in wells 18 and 45 m from the pumped well were found to be 1.8 and 1.1 m respectively. (a) What is the transmissivity of the aquifer? (b) What is the approximate drawdown in the pumped well? (c) Determine the radius of influence of the pumping well.

(a) Use Equation 4.2.10 for steady-state radial flow to a well in an unconfined aquifer to compute the hydraulic conductivity, where $Q = 80\ \text{m}^3/\text{hr} = 1920\ \text{m}^3/\text{day}$; $h_1 = 33 - 1.8 = 31.2$ m.; $h_2 = 33 - 1.1 = 31.9$ m; $r_2 = 45$ m and $r_1 = 18$ m:

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{1920\ \text{m}^3/\text{day}}{\pi\left(31.9^2 - 31.2^2\right)} \ln\left(\frac{45}{18}\right) = 12.7\ \text{m/day}$$

The transmissivity is computed as $T = Kb = 12.7\ \text{m/day} \times 33\ \text{m} = 418\ \text{m}^2/\text{day}$.

(b) Next compute the head and drawdown at the well. First rearrange Equation 4.2.10 to solve for the head at the well:

$$h_w = \sqrt{h_2^2 - \frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right)} = \sqrt{31.2^2 - \frac{1920\ \text{m}^3/\text{day}}{\pi(12.68\ \text{m/day})} \ln\left(\frac{18\ \text{m}}{0.25\ \text{m}}\right)} = 27.7\ \text{m}$$

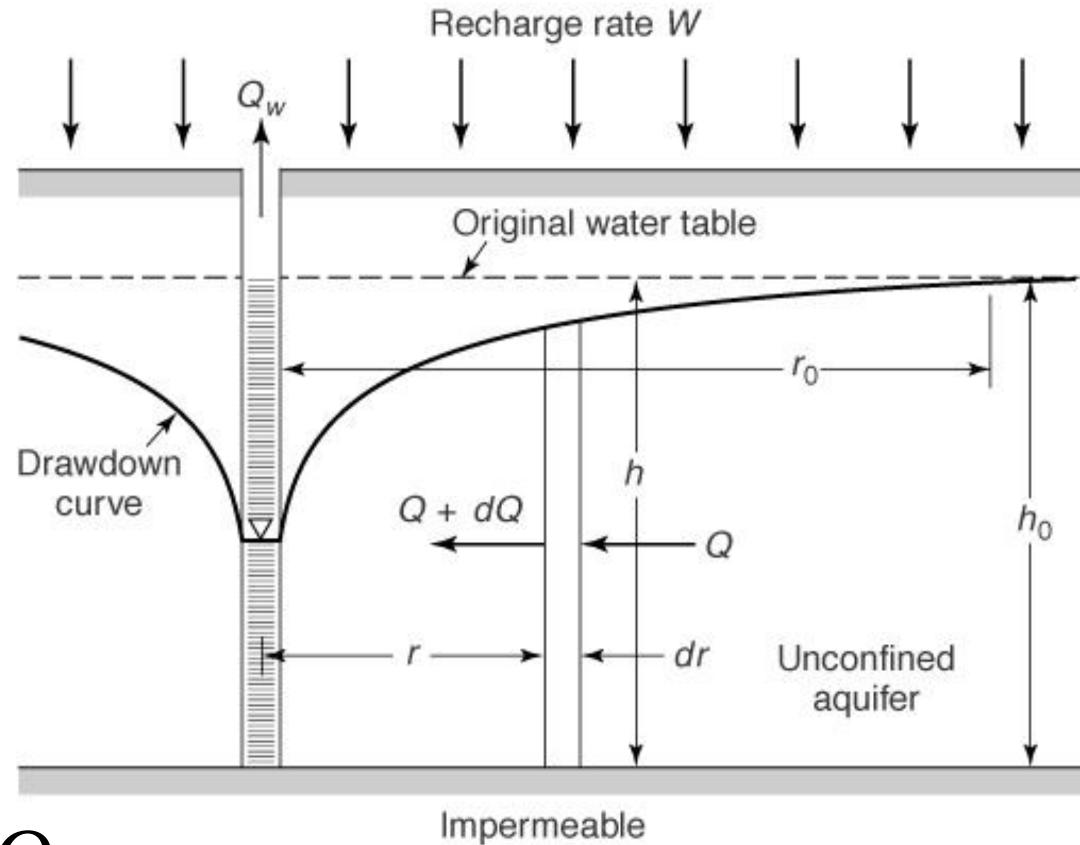
The drawdown is computed as $s_w = 33\ \text{m} - 27.7\ \text{m} = 5.3$ m.

(c) The radius of influence of the pumping well is computed by rearranging Equation 4.2.5:

$$R = (r_1)\exp\left(\frac{\pi K\left(h_0^2 - h_1^2\right)}{Q}\right) = (45\ \text{m})\exp\left(\frac{\pi(12.68\ \text{m/day})\left(33^2 - 31.9^2\right)}{1920\ \text{m}^3/\text{day}}\right) = 198\ \text{m}$$

■

Unconfined aquifer
With uniform recharge



$$dQ = -2\pi r\,dr\,W$$

$$Q = -\pi r^2 W + C$$

$$Q = Q_w \quad at \quad r = 0$$

$$Q = -\pi r^2 W + Q_w$$

$$-2\pi r K h \frac{dh}{dr} = -\pi r^2 W + Q_w$$

$$Q = 0 \quad at \quad r = r_0$$

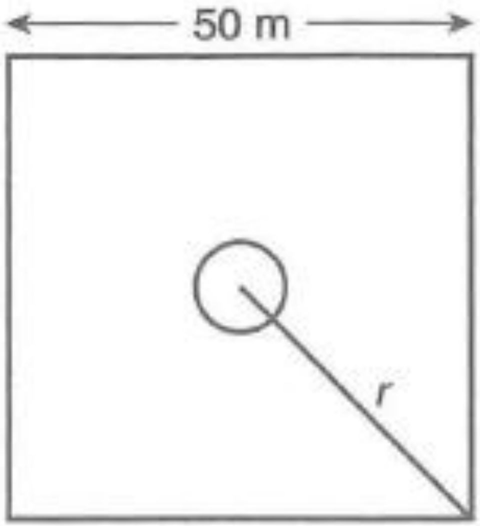
$$h_0^2 - h^2 = \frac{W}{2K}\left(r^2 - r_0^2\right) + \frac{Q_w}{\pi K}\ln\left(\frac{r_0}{r}\right)$$

Example 4.2.5

A pumping well is to be used to maintain a lowered water table at a construction site. The site is square, 50 m on a side, and the 25-cm diameter well is located at the center of the square, as shown in the figure. The hydraulic conductivity of the unconfined aquifer is estimated to be about 1×10^{-5} m/s or 0.864 m/day. The bottom of the aquifer is approximately horizontal at a depth of 20 m below the ground surface. Under natural conditions, the water table is nearly horizontal at a depth of 1 m below the ground surface and the unconfined aquifer is uniformly recharged at a rate $W = 0.06$ m/day. During the construction period, the water table must be lowered a minimum of 3 m over the site. Assuming steady-state conditions, compute the minimum pumping rate required.

Solution

The given condition is satisfied if the drawdown at any of the corner points is 3 m.



The well discharge is expressed in terms of the radius of influence using Equation 4.2.17 as $Q_w = \pi r_0^2 W = (0.06)\,\pi r_0^2$. Substitute this relationship along with $h_0 = 19$ m, $h = 16$ m, $W = 0.06$ m/day, $K = 0.864$ m/day, and $r = \sqrt{25^2 + 25^2} = 35.35$ m into Equation 4.2.17 to obtain

$$19^2 - 16^2 = \frac{0.06}{2 \times 0.864}\left(35.35^2 - r_0^2\right) + \frac{\left(0.06\pi r_0^2\right)}{\pi \times 0.864}\ln\left(\frac{r_0}{35.35}\right)$$

Solving the above equation using an iterative procedure yields $r_0 \approx 70$ m. The minimum pumping rate is $Q_w = \pi(70^2)\,(0.06) \approx 924\ \mathrm{m^3/day}$ or $0.01069\ \mathrm{m^3/s}$. ■

Unsteady radial flow in a confined aquifer (Theis equation)

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u} du = \frac{Q}{4\pi T} W(u)$$

$$= \frac{Q}{4\pi T}\left(-0.5772 - \ln(u) + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \cdots \right)$$

$s =$ Drawdown in an observation well at a distance r from the pumping well.

$t =$ Time since pumping began

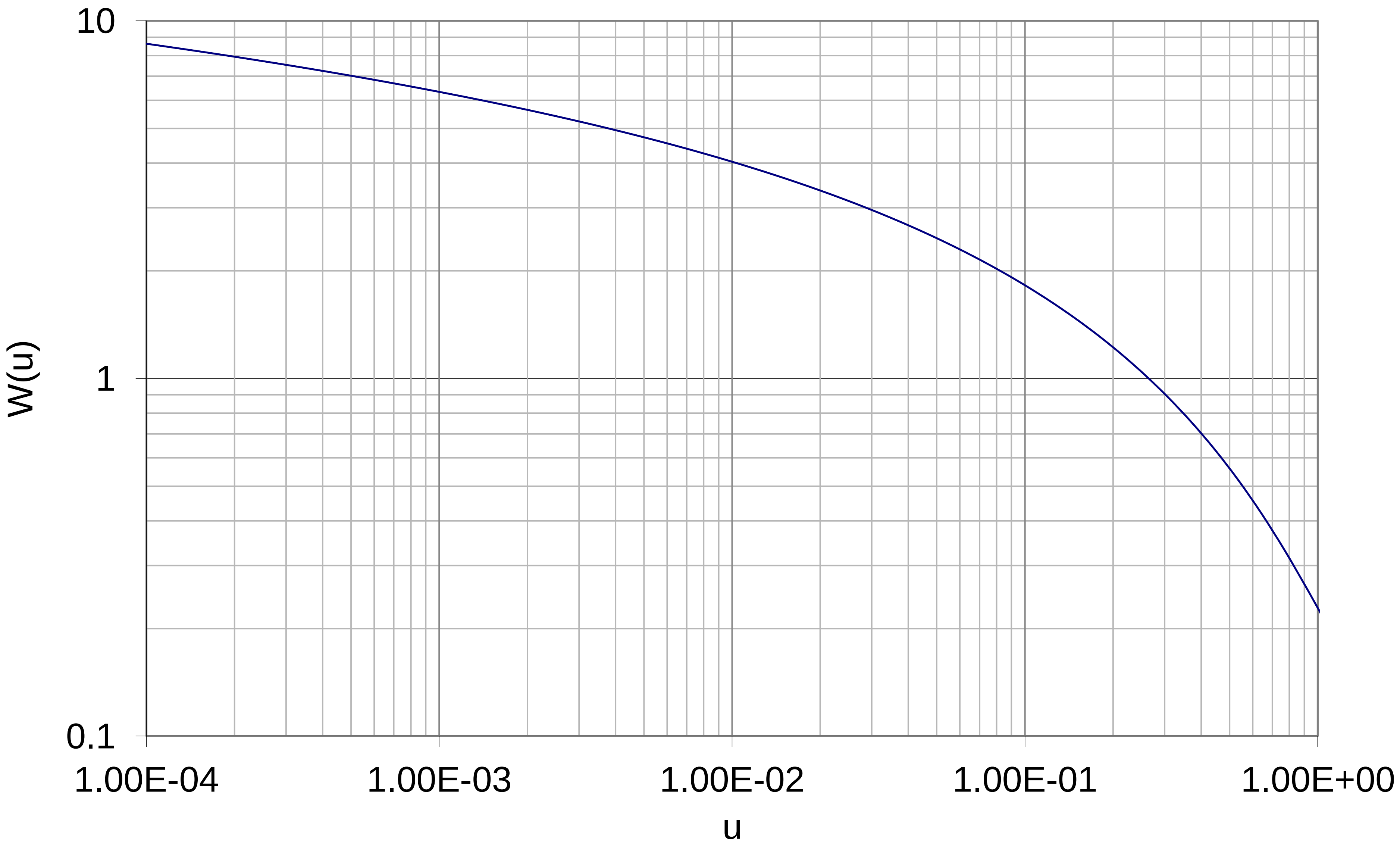
$$u = \frac{r^2 S}{4Tt}$$

$S =$ Storage coefficient of the aquifer (dimensionless)

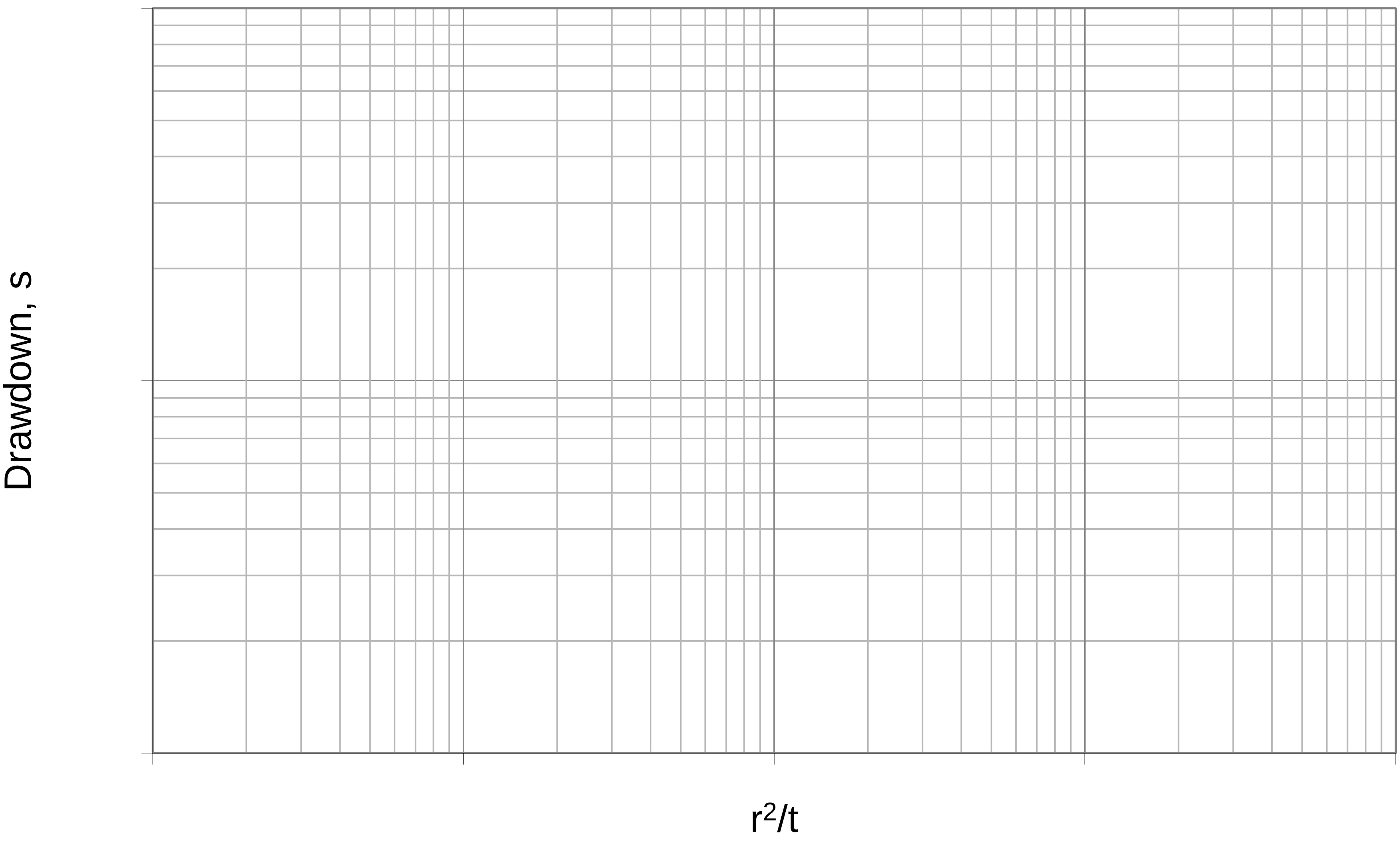
Table 4.4.1 Values of $W(u)$ for Values of u

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

Type Curve



Pumping test data



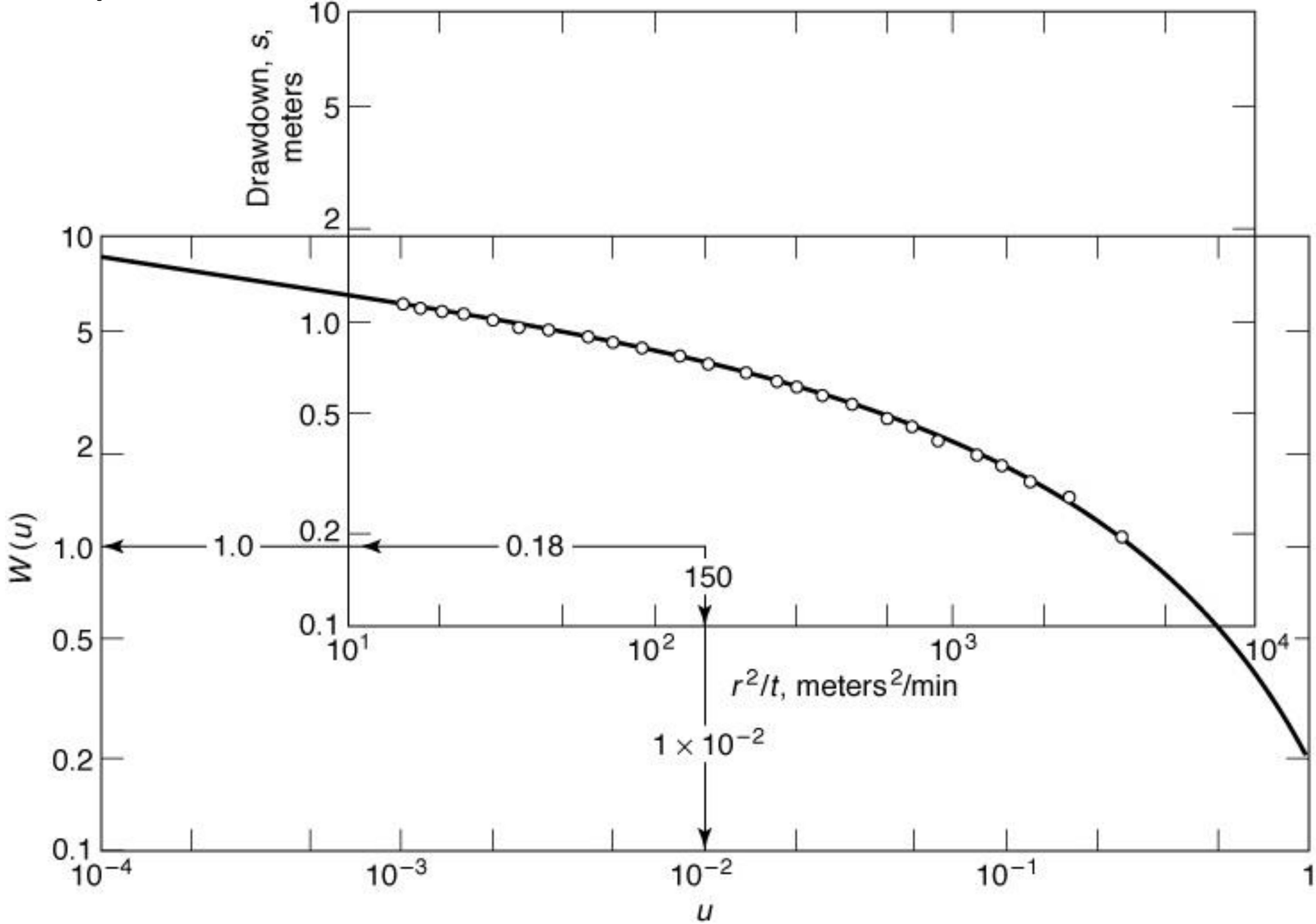
Example 4.4.2

Table 4.4.2 Pumping Test Data

(r = 60 m)

t, min	s, m	r^2/t, m^2/min	t, min	s, m	r^2/t, m^2/min
0	0	∞	18	0.67	200
1	0.20	3,600	24	0.72	150
1.5	0.27	2,400	30	0.76	120
2	0.30	1,800	40	0.81	90
2.5	0.34	1,440	50	0.85	72
3	0.37	1,200	60	0.90	60
4	0.41	900	80	0.93	45
5	0.45	720	100	0.96	36
6	0.48	600	120	1.00	30
8	0.53	450	150	1.04	24
10	0.57	360	180	1.07	20
12	0.60	300	210	1.10	17
14	0.63	257	240	1.12	15

Example 4.4.2



Drawdown, s, meters
10
5
2
1.0
0.5
0.2
0.1
10^1
10^2
10^3
10^4
r^2/t, meters^2/min
W(u)
10
5
2
1.0
0.5
0.2
0.1
10^-4
10^-3
10^-2
10^-1
1
u
1.0
0.18
150
1 × 10^-2

Values of r^2/t in m^2/min are computed and appear in the right column of Table 4.4.2. Values of s and r^2/t are plotted on logarithmic paper. Values of $W(u)$ and u from Table 4.4.1 are plotted on another sheet of logarithmic paper of the same size and scale, and a curve is drawn through the points. The two sheets are superposed and shifted with coordinate axes parallel until the observational points coincide with the curve, as shown in Figure 4.4.1. A convenient match point is selected with $W(u) = 1.00$ and $u = 1 \times 10^{-2}$, so that $s = 0.18$ m and $r^2/t = 150$ m^2/min = 216,000 m^2/day. Thus, from Equation 4.4.5,

$$T = \frac{Q}{4\pi s} W(u) = \frac{2500(1.00)}{4\pi(0.18)} = 1110 \text{ m}^2 / \text{day}$$

and from Equation 4.4.6,

$$S = \frac{4Tu}{r^2/t} = \frac{4(1110)\left(1\times10^{-2}\right)}{216{,}000} = 0.000206$$

■

Cooper-Jacob Method of solution

$$s = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left(\frac{r^2 S}{4Tt}\right)\right]$$

$$= \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2 S}\right)$$

$$T = \frac{2.3Q}{4\pi\Delta s}\log\left(\frac{t_2}{t_1}\right)$$

$$0 = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt_0}{r^2 S}\right) \Rightarrow \frac{2.25Tt_0}{r^2 S} = 1 \Rightarrow S = \frac{2.25Tt_0}{r^2}$$

Example 4.4.3

Rework Example 4.4.2 using the Cooper–Jacob method.

From the pumping test data in Table 4.4.2, s and t are plotted on semilogarithmic paper, as shown in Figure 4.4.2. A straight line is fitted through the points, and $\Delta s = 0.40$ m and $t_0 = 0.39 \text{ min} = 2.70 \times 10^{-4}$ day are read. Thus,

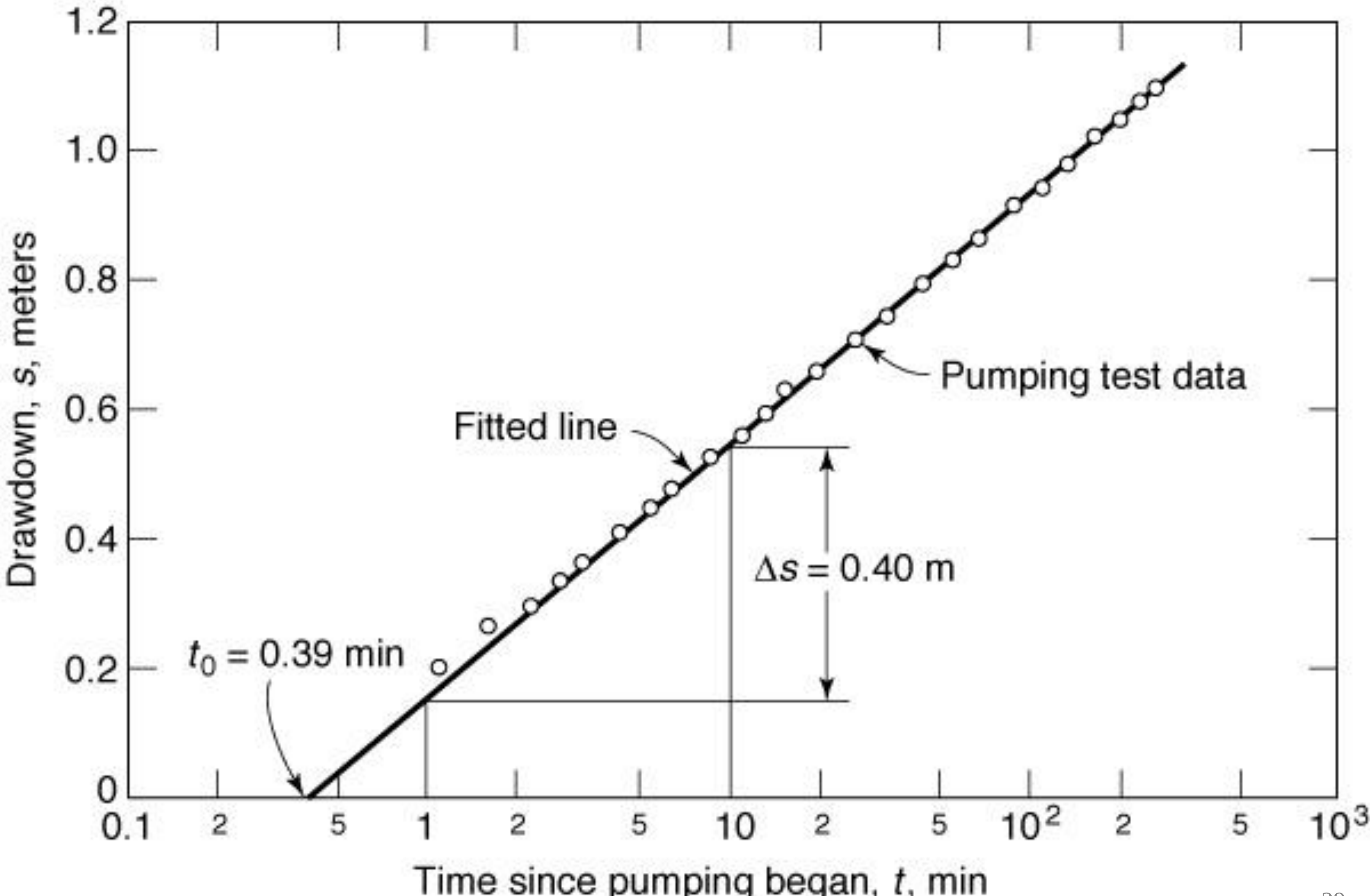
$$T = \frac{2.30(2500)}{4\pi(0.40)} = 1144 \text{ m}^2/\text{day}$$

and

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25(1144)\left(2.70\times10^{-4}\right)}{(60)^2} = 0.000193$$

■

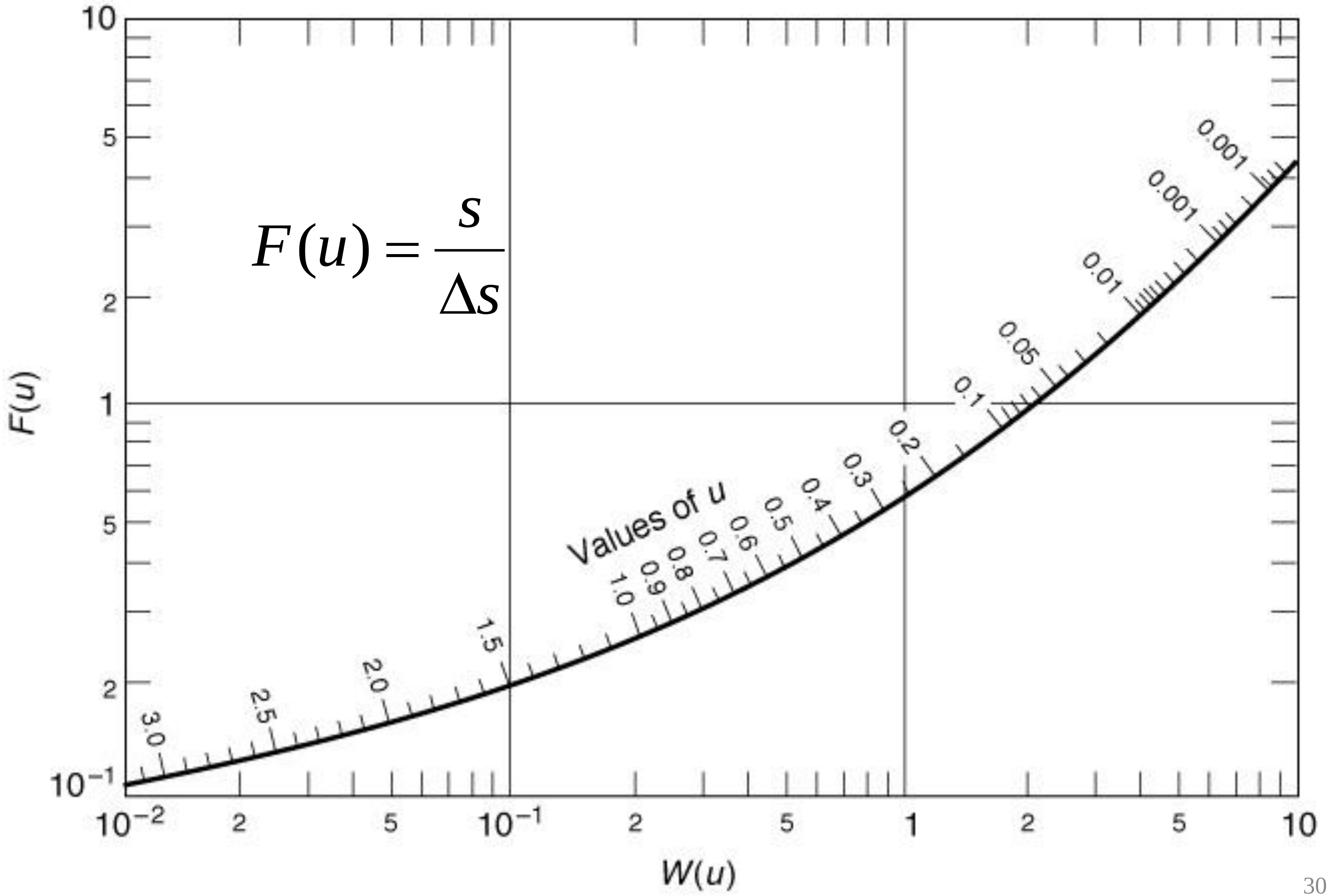
Example 4.4.3



1.2
1.0
0.8
0.6
0.4
0.2
0
Drawdown, s, meters
0.1
2
5
1
2
5
10
2
5
10^2
2
5
10^3
Time since pumping began, t, min
Pumping test data
Fitted line
Δs = 0.40 m
t0 = 0.39 min

Chow Method of solution

$$F(u) = \frac{s}{\Delta s}$$



Example 4.4.5

Repeat Example 4.4.2 using the Chow method.

In Figure 4.4.4 data are plotted from Table 4.4.2 and point A is selected on the curve where $t = 6$ min $= 4.2 \times 10^{-3}$ day and $s = 0.47$ m. A tangent is constructed as shown; the drawdown difference per log cycle of time is $\Delta s = 0.38$ m. Then $F(u) = 0.47/0.38 = 1.24$, and from Figure 4.4.3, $W(u) = 2.75$ and $u = 0.038$. Hence,

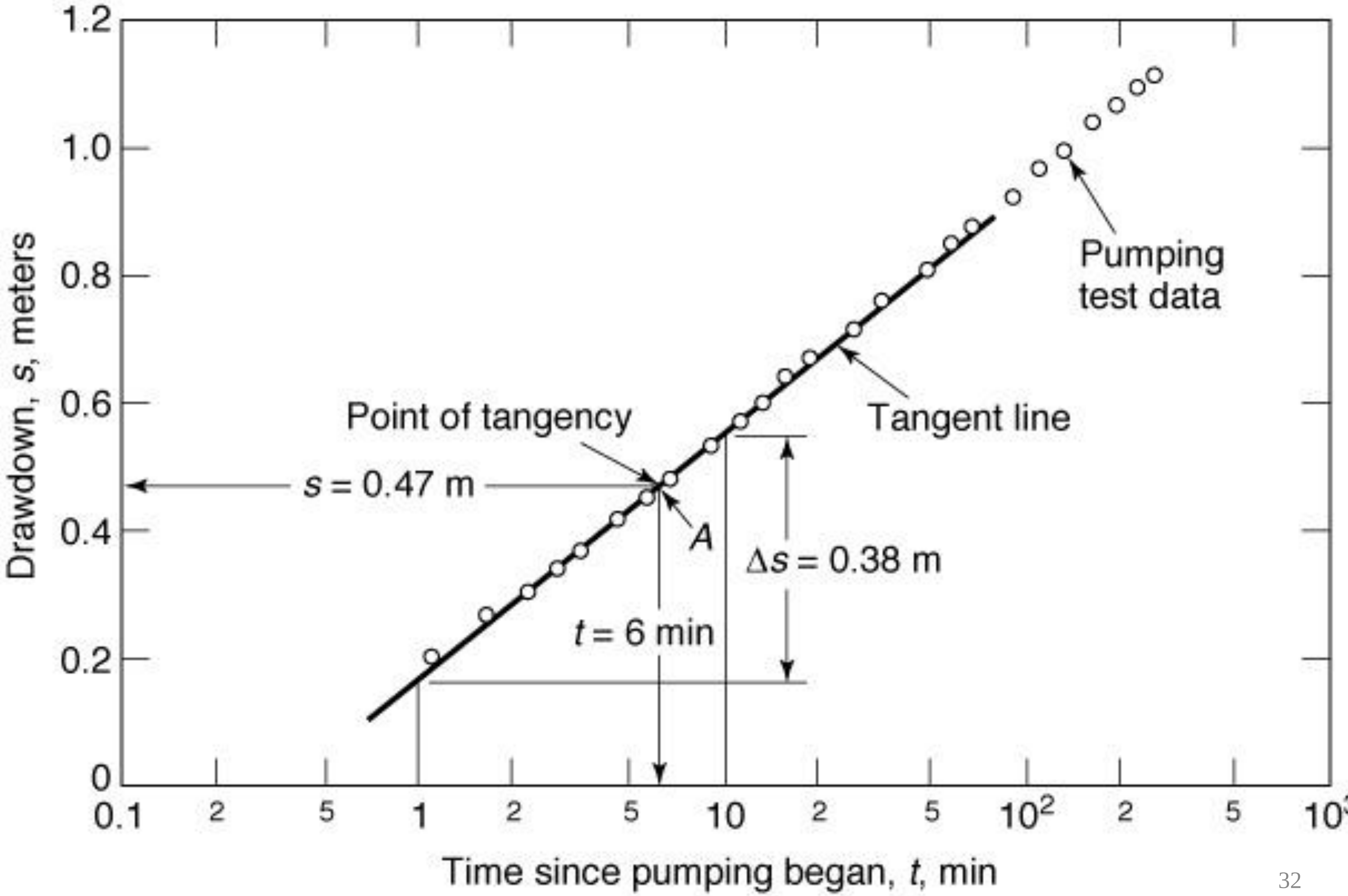
$$T = \frac{Q}{4\pi s} W(u) = \frac{2500}{4\pi(0.47)} 2.75 = 1160 \text{ m}^2/\text{day}$$

and

$$S = \frac{4Ttu}{r^2} = \frac{4(1160)\left(4.2\times10^{-3}\right)(0.038)}{(60)^2} = 0.000206$$

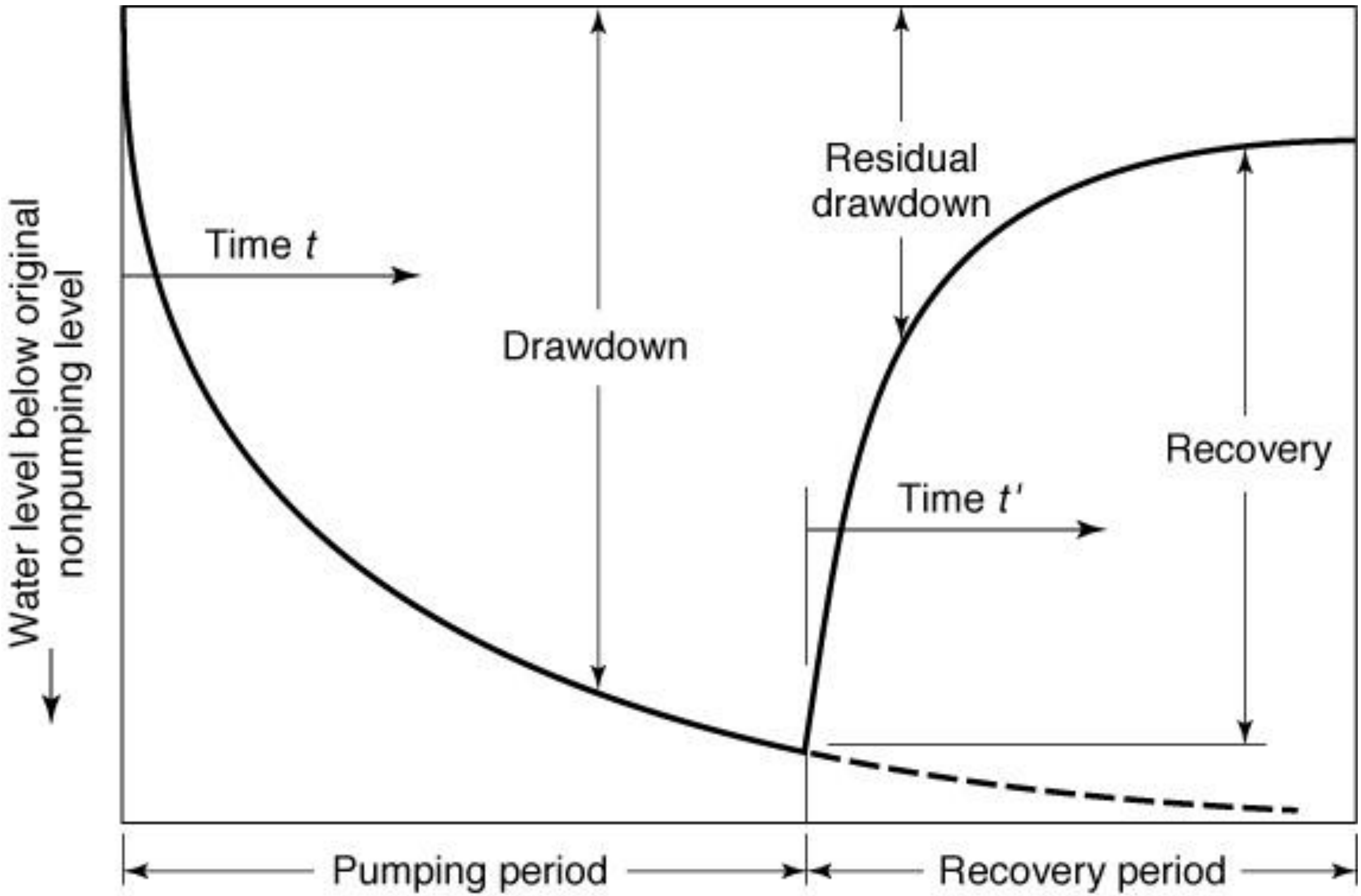
■

Example 4.4.5



Drawdown, s, meters
1.2
1.0
0.8
0.6
0.4
0.2
0
Pumping test data
Tangent line
Point of tangency
$s = 0.47$ m
A
$\Delta s = 0.38$ m
$t = 6$ min
0.1
2
5
1
2
5
10
2
5
10^2
2
5
10^3
Time since pumping began, t, min

Recovery test



Recovery test

$$s = \frac{Q}{4\pi T}\left[W(u) - W(u')\right]$$

$$u = \frac{r^2 S}{4Tt} \qquad u' = \frac{r^2 S}{4Tt'}$$

$$s' = \frac{2.3Q}{4\pi T}\log\left(\frac{t_2}{t_1}\right)$$

$$T = \frac{2.3Q}{4\pi\Delta s'}$$

Example 4.4.6

A well pumping at a uniform rate of 2,500 m^3/day was shut down after 240 min; thereafter, measurements of s' and t' tabulated in Table 4.4.3 were made in an observation well. Determine the transmissivity.

Values of t/t' are computed, as shown in Table 4.4.3, and then plotted versus s' on semilogarithmic paper (see Figure 4.4.6). A straight line is fitted through the points and $\Delta s' = 0.40$ m is determined; then,

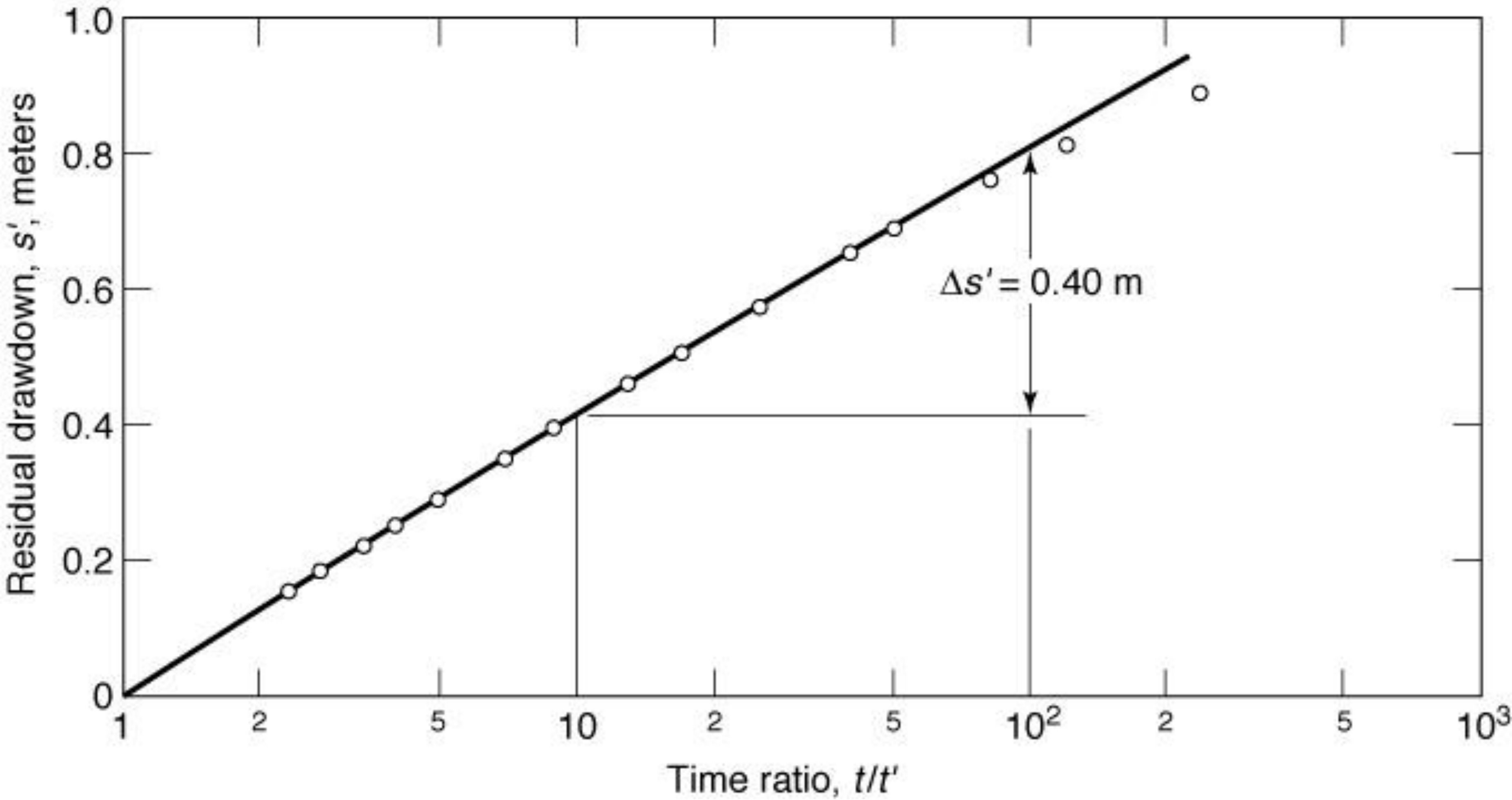
$$T = \frac{2.30Q}{4\pi\Delta s'} = \frac{2.30(2500)}{4\pi(0.40)} = 1140 \text{ m}^2/\text{day}$$

Example 4.4.6

Table 4.4.3 Recovery Test Data (pump shut down at $t = 240$ min)

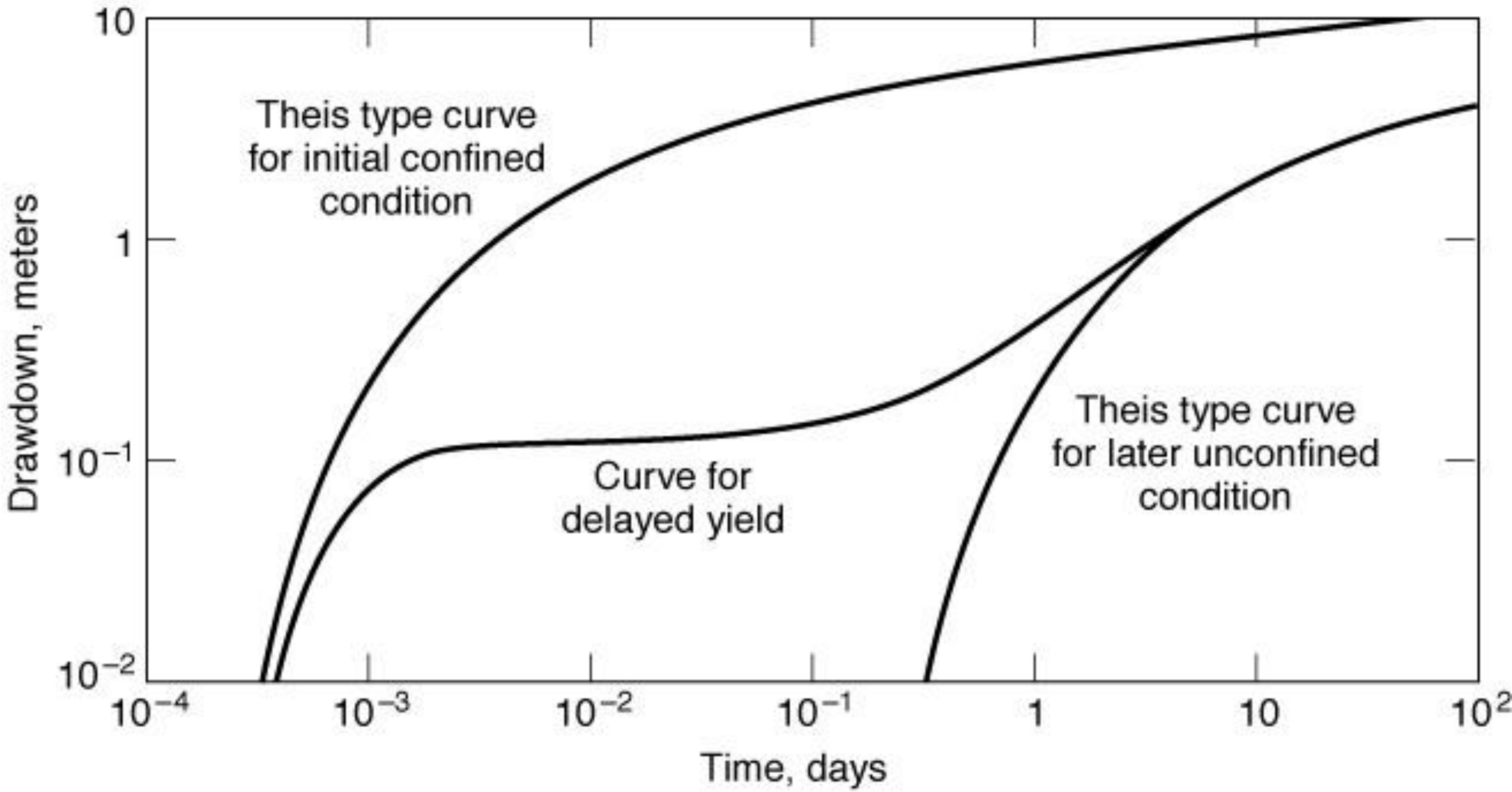
t', min	t, min	t/t'	s', m
1	241	241	0.89
2	242	121	0.81
3	243	81	0.76
5	245	49	0.68
7	247	35	0.64
10	250	25	0.56
15	255	17	0.49
20	260	13	0.55
30	270	9	0.38
40	280	7	0.34
60	300	5	0.28
80	320	4	0.24
100	340	3.4	0.21
140	380	2.7	0.17
180	420	2.3	0.14

Example 4.4.6



1.0
0.8
0.6
0.4
0.2
0
Residual drawdown, s′, meters
Δs′ = 0.40 m
1
2
5
10
2
5
10²
2
5
10³
Time ratio, t/t′

Unsteady radial flow in an unconfined aquifer



$$s = \frac{Q}{4\pi T}\left[W(u_a, u_y, \eta)\right]$$

$W(u_a, u_y, \eta) =$ Unconfined aquifer well function

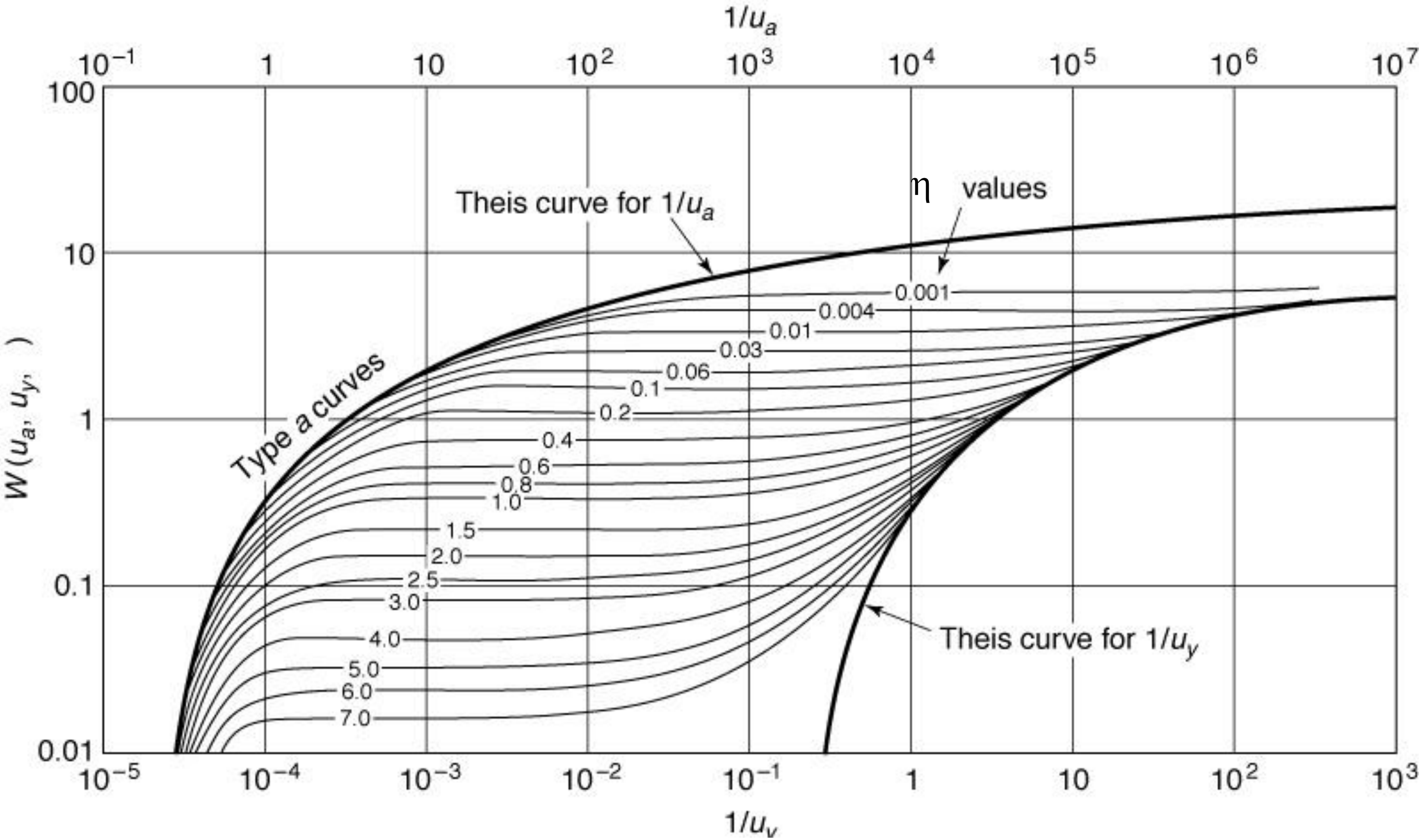
$$u_a = \frac{r^2 S}{4Tt}$$ (For early drawdown)

$$u_y = \frac{r^2 S_y}{4Tt}$$ (For later drawdown)

$$\eta = \frac{r^2 K_v}{b^2 K_h}$$

b= initial saturated thickness of the unconfined aquifer
K_h, K_v = Horizontal and vertical hydraulic conductivity, respectively

Type curve for unconfined aquifer



Example 4.5.1

A well pumping at 144.4 ft^3/min fully penetrates an unconfined aquifer with a saturated thickness of 25 ft. Determine the transmissivity, storativity, specific yield, and horizontal and vertical hydraulic conductivities using the tabulated time–drawdown data in Table 4.5.1 for an observation well located 73 ft away.

Time–drawdown data (Table 4.5.1) are plotted in Figure 4.5.4, which shows the typical three phases of drawdown for unconfined aquifers. The early drawdown versus time data fit best on the type-a curves for $\eta = 0.06$. The selected match point in Figure 4.5.4 has the following coordinates: ($t = 0.17$ min, $s = 0.57$ ft) and ($1/u_a = 1.0$, $W(u_a, u_y, \eta) = 1.0$). Using Equation 4.5.1 with a discharge of $Q = 144.4$ ft^3/min, we find the transmissivity to be

$$T = \frac{Q}{4\pi s} W(u_a, u_y, \eta) = \frac{(144.4 \text{ ft}^3/\text{min})}{4\pi(0.57 \text{ ft})}(1.0) = 20.16 \text{ ft}^2/\text{min} \cong 29{,}900 \text{ ft}^2/\text{day}$$

Next, the storativity value is computed using Equation 4.5.2:

$$S = \frac{4Tu_a t}{r^2} = \frac{4(20.16 \text{ ft}^2/\text{min})(1.0)(0.17 \text{ min})}{(73 \text{ ft})^2} = 0.00257$$

Moving the data curve to the right on the type curve to the best late-time match (for $\eta = 0.06$) where $s =$ 0.57 ft (see the match point on Figure 4.5.5) yields ($t = 13$ min, $s = 0.57$ ft) and ($1/u_y = 0.1$, $W(u_y, \eta) =$ 1). Inserting the appropriate values in Equation 4.5.1 does not change the transmissivity estimate, but using Equation 4.5.3 yields

$$S_y = \frac{4Tu_y t}{r^2} = \frac{4(20.16 \text{ ft}^2/\text{min})(0.1)(13 \text{ min})}{(73 \text{ ft})^2} = 0.02$$

Example 4.5.1

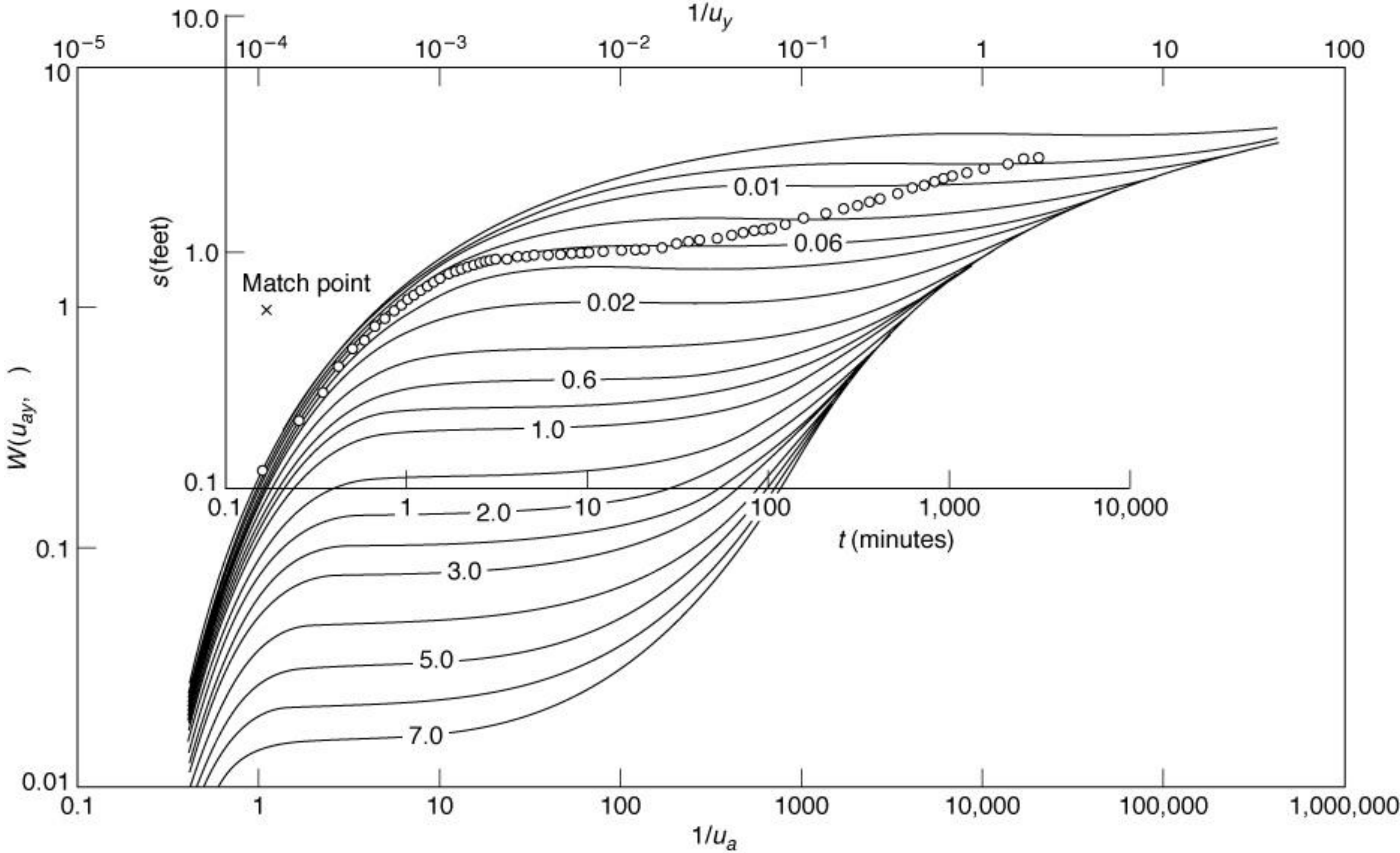
The horizontal hydraulic conductivity, K_r or K_h, is computed using

$$K_h = \frac{T}{b} = \frac{20.16\ \text{ft}^2/\text{min}}{25\ \text{ft}^2} = 0.806\ \text{ft/min or } 1160\ \text{ft/day}$$

and the vertical hydraulic conductivity, K_z or K_v, is computed using Equation 4.5.4:

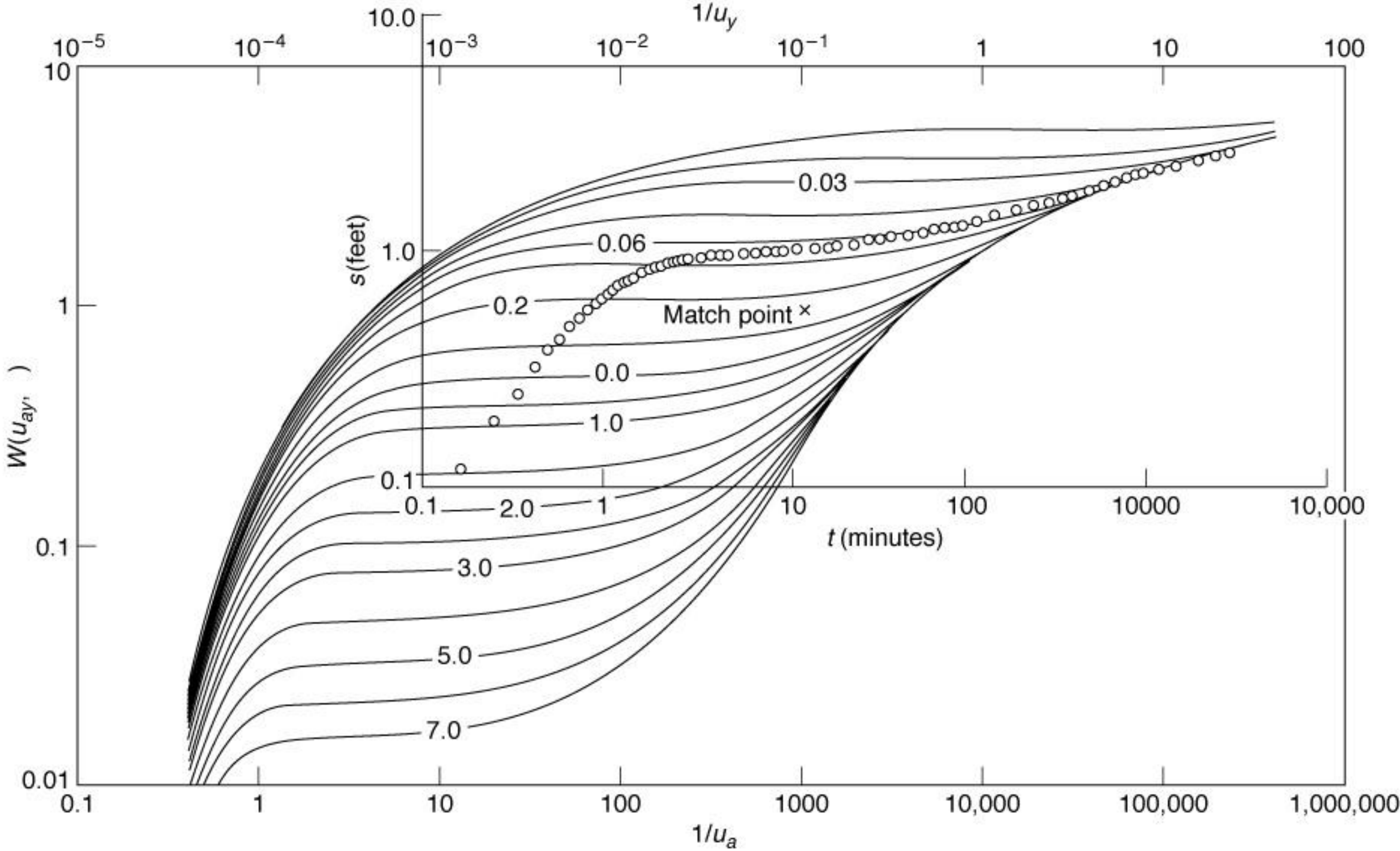
$$K_v = \frac{\eta b^2 K_h}{r^2} = \frac{(0.06)(25\ \text{ft})^2(1160\ \text{ft/day})}{(73\ \text{ft})^2} = 8.2\ \text{ft/day}$$

Example 4.5.1



1/u_y
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
100
10.0
1.0
0.1
s(feet)
Match point
0.01
0.06
0.02
0.6
1.0
2.0
3.0
5.0
7.0
0.1
1
10
100
1,000
10,000
t (minutes)
W(u_{ay},)
10
1
0.1
0.01
0.1
1
10
100
1000
10,000
100,000
1,000,000
1/u_a

Example 4.5.1



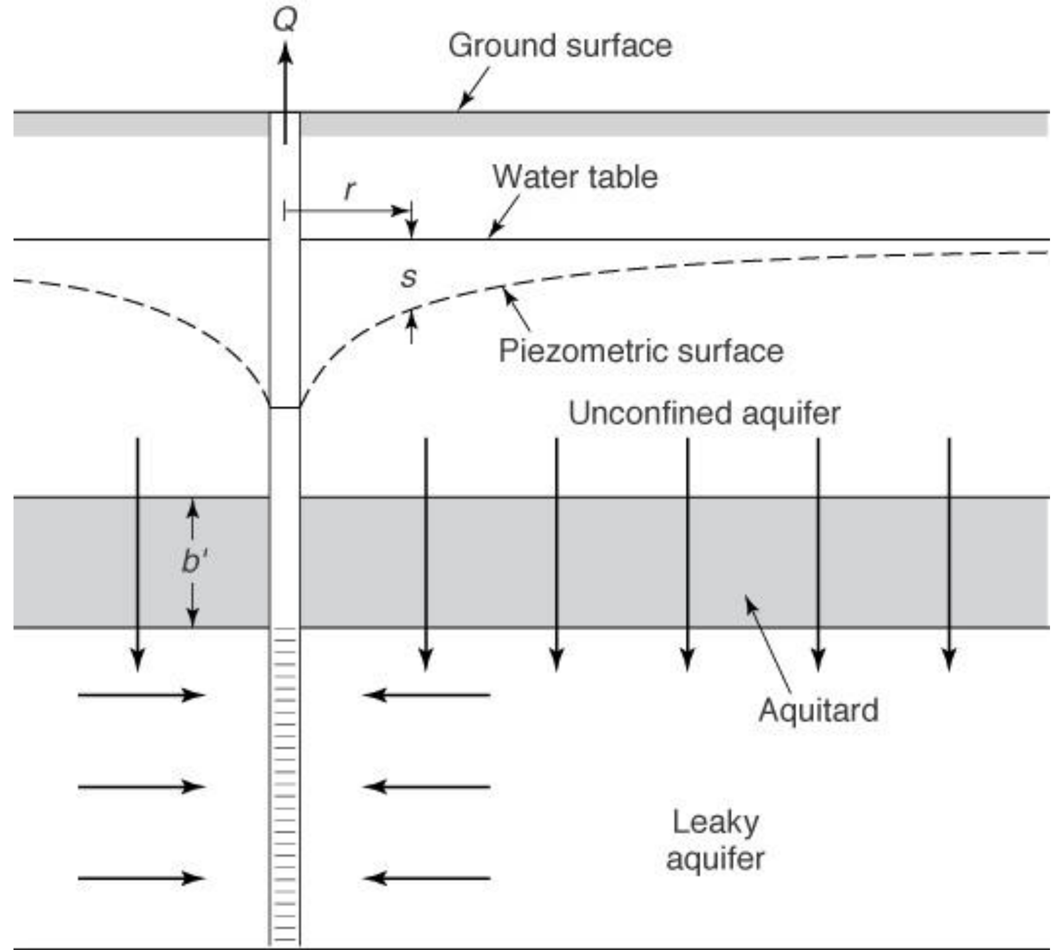
1/u_y
10.0
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
100
10
0.03
s(feet)
1.0
0.06
1
0.2
Match point ×
0.0
1.0
0.1
0.1
2.0
1
10
100
10000
10,000
t (minutes)
0.1
3.0
5.0
7.0
W($u_{ay'}$)
0.01
0.1
1
10
100
1000
10,000
100,000
1,000,000
1/u_a

Unsteady radial flow in a leaky aquifer

$$s = \frac{Q}{4\pi T}\left[W(u, r/B)\right]$$

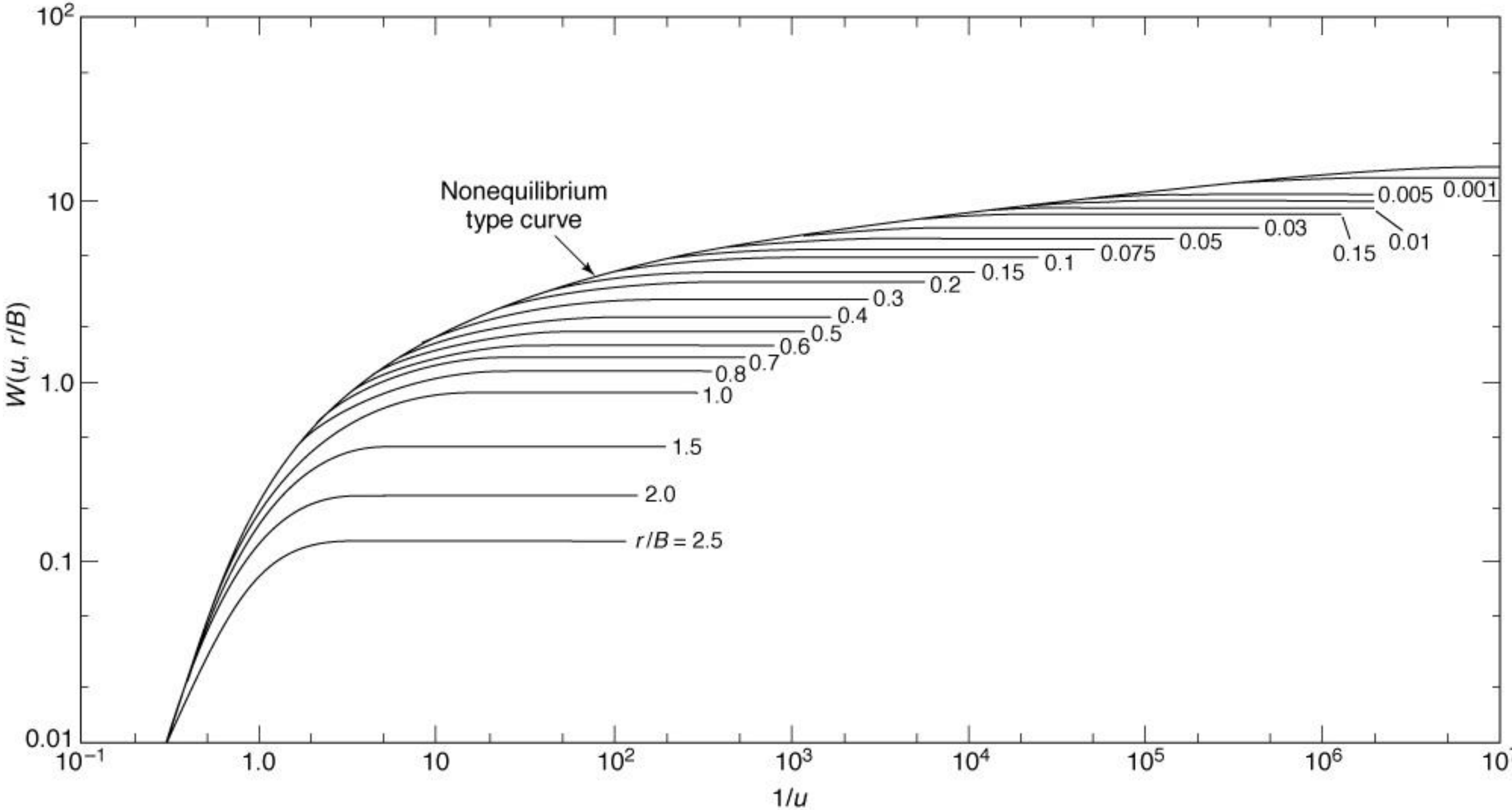
$$u = \frac{r^2 S}{4Tt}$$

$$\frac{r}{B} = \frac{r}{\sqrt{T/\left(K'/b'\right)}}$$



b' = thickness of the saturated semi-pervious layer
K' = vertical hydraulic conductivity of the aquitard

Type curve for leaky aquifer



Example 4.6.1

A well pumping at 600 ft^3/min fully penetrates a confined aquifer overlain by a leaky confining layer of 14-ft thickness. Using the tabulated time–drawdown data for an observation well 40 ft away from the pumping well, estimate the transmissivity and storage coefficient of the confined aquifer, and the permeability of the aquitard. Assume that the confining layer does not release water from storage.

Time (min)	Drawdown (ft)	Time (min)	Drawdown (ft)
0	0.00	80	12.02
2	5.65	90	12.26
4	6.96	100	12.33
6	7.72	110	12.37
8	8.00	120	12.41
10	8.71	150	12.69
15	9.47	180	12.85
20	9.99	210	13.09
25	10.35	240	13.13
30	10.70	270	13.25
40	11.14	300	13.33
50	11.46	360	13.37
60	11.62	420	13.41
70	11.86		

Example 4.6.1

The time–drawdown field data were superimposed on the family type curves for leaky aquifers (Figure 4.6.3). Comparison shows that the best fit occurs for $r/B = 0.03$. The coordinates of the match point selected are

$$\frac{1}{u} = 1000, \qquad W\left(u, \frac{r}{B}\right) = 1.0$$

$$t = 59 \text{ min}, \qquad s = 1.93 \text{ ft}$$

Next we must perform the following unit conversions in order to obtain the transmissivity in units of ft^2/day and hydraulic conductivity of the aquitard in units of ft/day for $Q = 600\ \text{ft}^3/\text{min} = 864{,}000\ \text{ft}^3/\text{day}$ and $t = 59$ min $= 0.041$ days. The transmissivity and storage coefficient of the confined aquifer are computed using Equations 4.6.1 and 4.6.2 rearranged respectively as

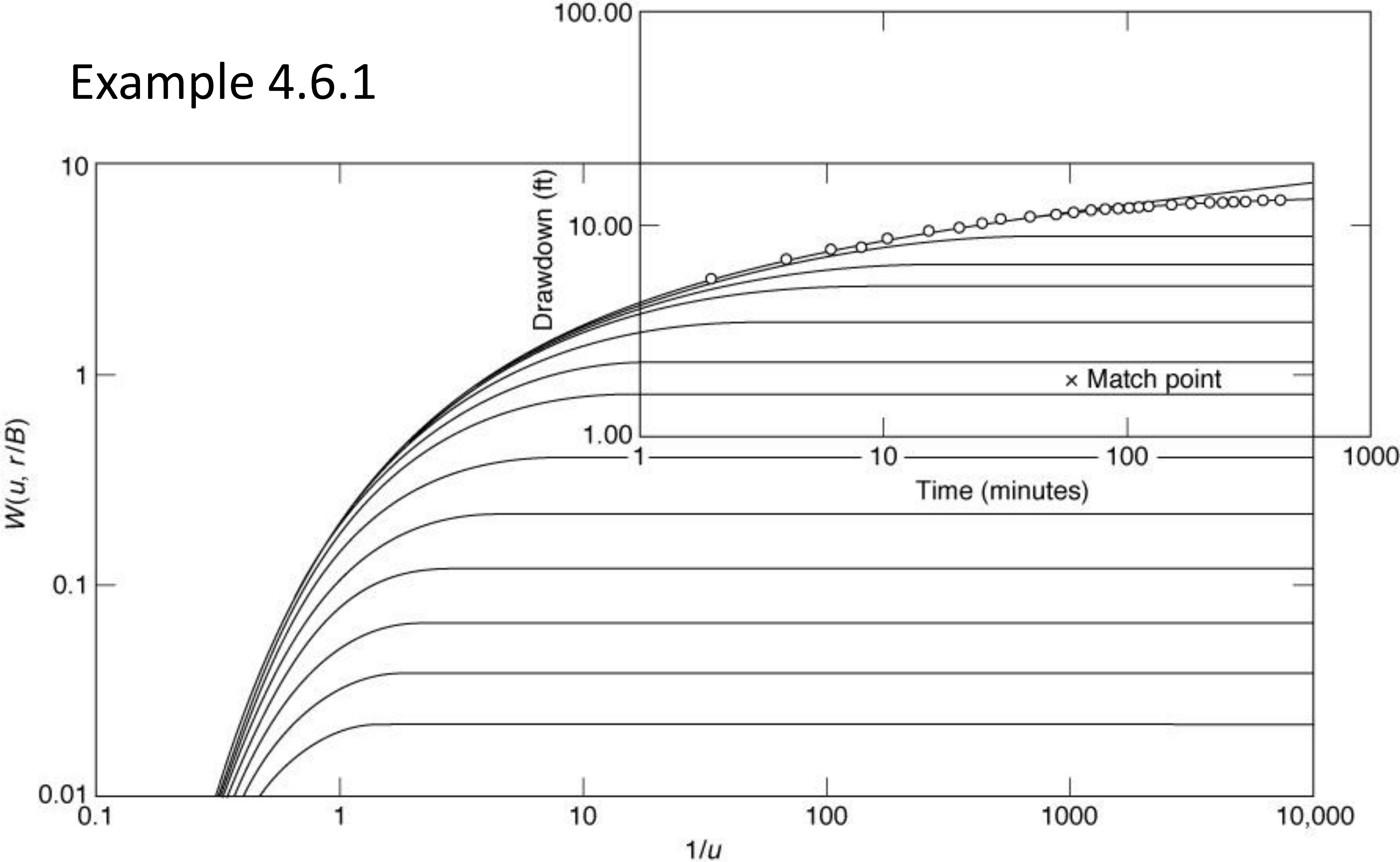
$$T = \frac{Q}{4\pi s} W(u, r/B) = \frac{864{,}000\ \text{ft}^3/\text{day}}{4\pi(1.93\ \text{ft})}(1.0) = 35{,}624\ \text{ft}^2/\text{day}$$

$$S = \frac{4Tu}{r^2} = \frac{4(35{,}624\ \text{ft}^2/\text{day})(0.041\ \text{days})(0.001)}{(40\ \text{ft})^2} = 0.00365$$

The hydraulic conductivity of the aquitard is computed by rearranging Equation 4.6.3

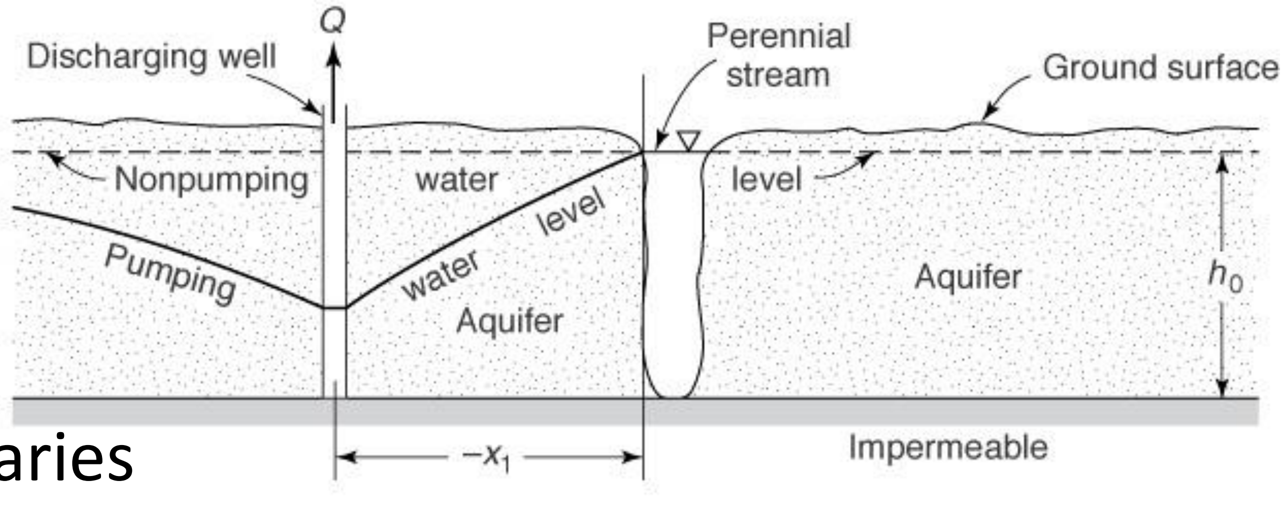
$$K' = \frac{Tb'(r/B)^2}{r^2} = \frac{(35{,}624\ \text{ft}^2/\text{day})(14\ \text{ft})(0.03)^2}{(40\ \text{ft})^2} = 0.28\ \text{ft/day}$$

Example 4.6.1



100.00
10.00
1.00
Drawdown (ft)
1
10
100
1000
Time (minutes)
× Match point
10
1
0.1
0.01
W(u, r/B)
0.1
1
10
100
1000
10,000
1/u

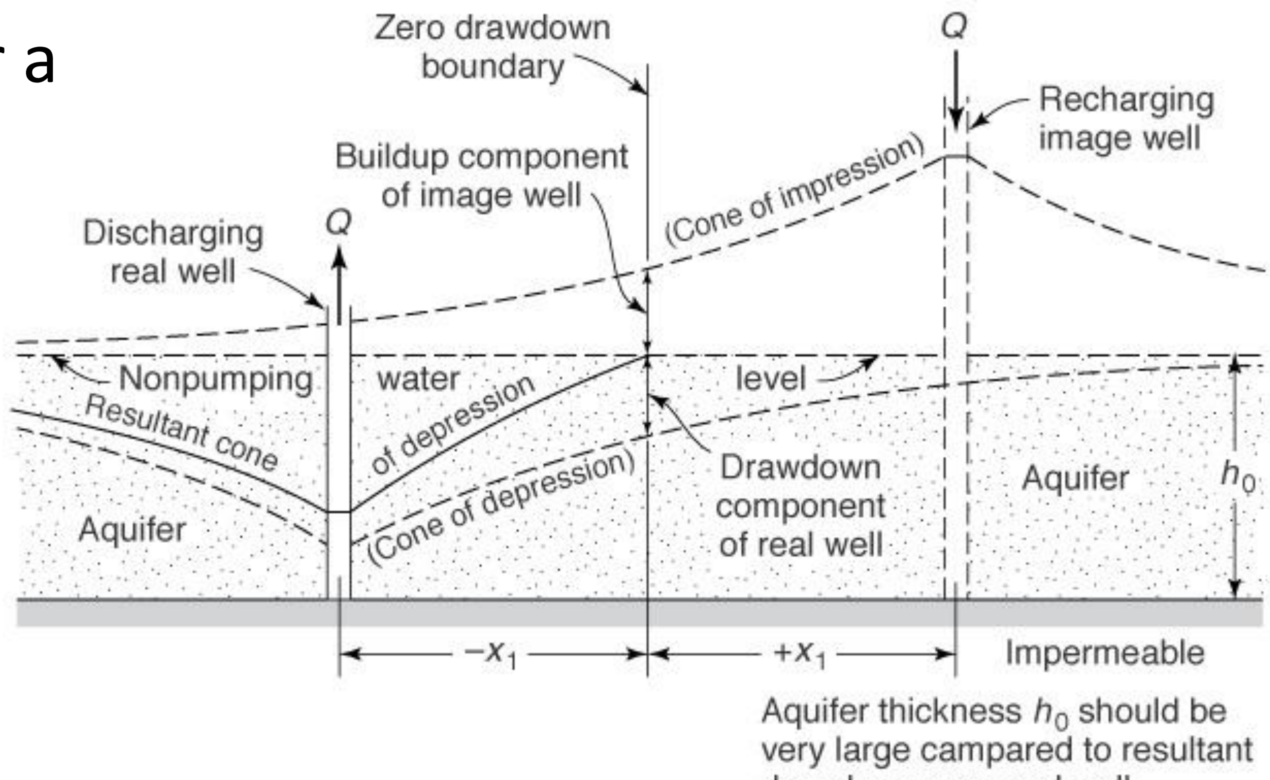
Well flow near aquifer boundaries



(a)

Well flow near a stream

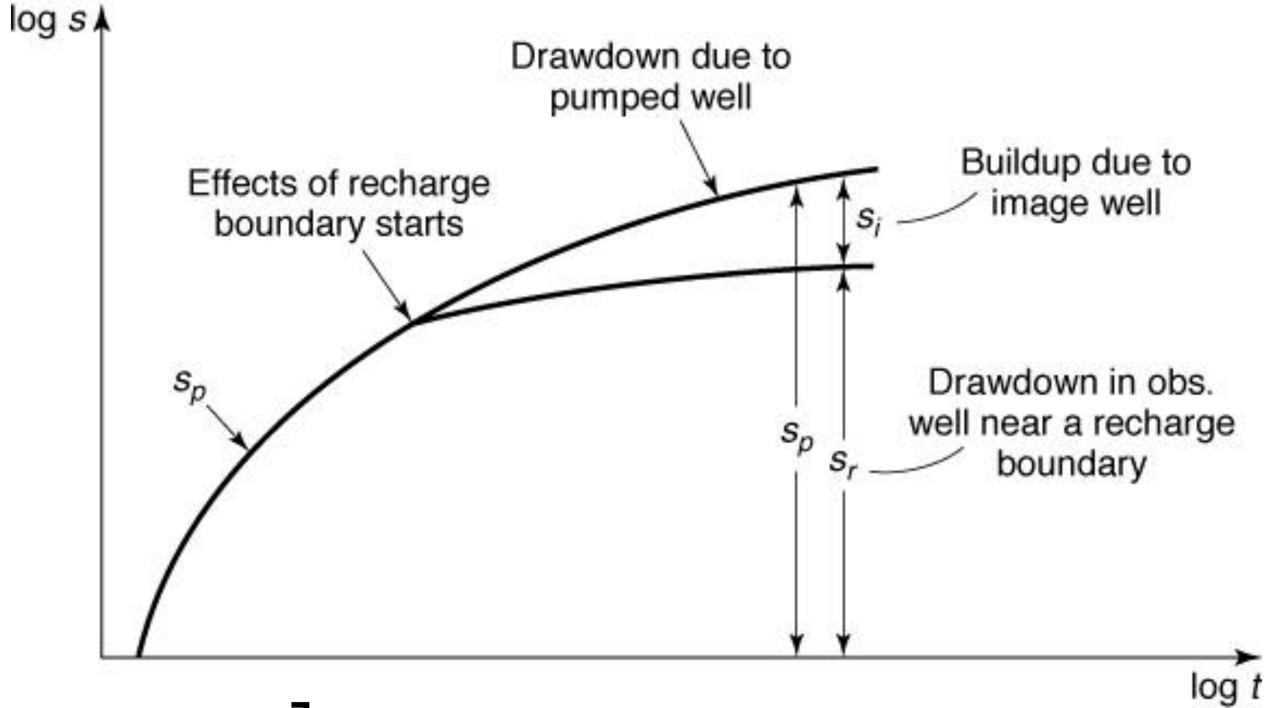
$$s_r = s_p - s_i$$



(b)

$$u_p = \frac{r_p^2 S}{4Tt_p}$$

$$u_i = \frac{r_i^2 S}{4Tt_i}$$



$$s_r = \frac{Q}{4\pi T}\left[W(u_p) - W(u_i)\right]$$

For large values of time t

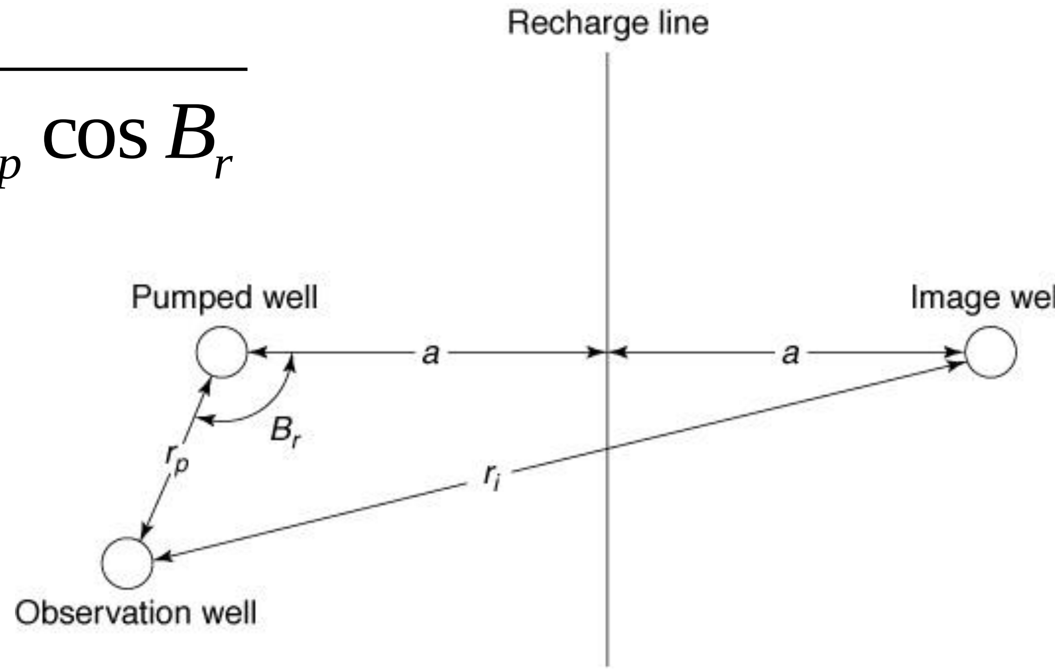
$$W(u_p) = \left[-0.5772 - \ln u_p\right]$$

$$W(u_i) = \left[-0.5772 - \ln u_i\right]$$

$$s_r = \frac{Q}{4\pi T}\left[-\ln(u_p) + \ln(u_i)\right]$$

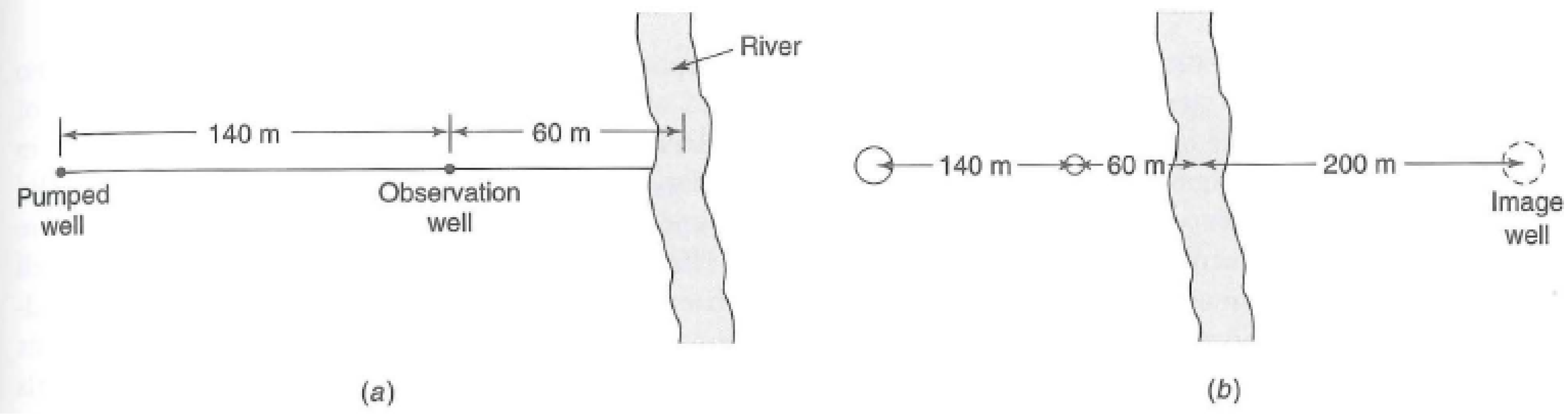
$$s_r = \frac{Q}{4\pi T}\ln\left(\frac{u_i}{u_p}\right) = \frac{Q}{4\pi T}\ln\left(\frac{r_i^2}{r_p^2}\right) = \frac{Q}{2\pi T}\ln\left(\frac{r_i}{r_p}\right)$$

$$r_i = \sqrt{4a^2 + r_p^2 - 4ar_p\cos B_r}$$



Example 4.7.1

A 0.5-m diameter well (200 m from a river) is pumping at an unknown rate from a confined aquifer (see Figure 4.7.6). The aquifer properties are $T = 432\ \mathrm{m^2/day}$ and $S = 4.0 \times 10^{-4}$. After eight hours of pumping, the drawdown in the observation well (60 m from the river) is 0.8 m. Compute the rate of pumping and the drawdown in the pumped well. What is the effect of the river on drawdown in the observation well and in the pumped well?



Example 4.7.1

The following information is given in the above statement: $r_w = 0.25$ m, $T = 432$ m^2/day $= 5.0 \times 10^{-3}$ m^2/s, $S = 4 \times 10^{-5}$, $t = 8$ hr $= 28{,}800$ s, and $s = 0.8$ m. A recharging image well is placed at the same distance from the river as the pumped well as shown in Figure 4.7.6b.

Equation 4.7.5 is used to compute the discharge from the pumped well knowing the above information:

$$s = \frac{Q}{4\pi T} W(u_p) - \frac{Q}{4\pi T} W(u_i)$$

$$u_p = \frac{r_p^2 S}{4Tt} = \frac{(140)^2 (4 \times 10^{-4})}{4(5 \times 10^{-3})(28800)} = 1.36 \times 10^{-2}$$

$$u_i = \frac{r_i^2 S}{4Tt} = \frac{(260)^2 (4 \times 10^{-4})}{4(5 \times 10^{-3})(28800)} = 4.69 \times 10^{-2}$$

$W(u_p) = 3.79$ for $u_p = 1.36 \times 10^{-2}$ and $W(u_i) = 2.54$ for $u_i = 4.69 \times 10^{-2}$

Thus the discharge is computed using

$$0.8 = \frac{Q}{4\pi(5 \times 10^{-3})}(3.79) - \frac{Q}{4\pi(5 \times 10^{-3})}(2.54)$$

so that $Q = 0.04$ m^3/s.

Example 4.7.1

The drawdown in the pumped well is computed using equation 4.7.5:

$$u_w = \frac{r_w^2 S}{4Tt} = \frac{(0.25)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 4.34\times10^{-8}$$

$$u_i = \frac{(400)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 0.111$$

$$W\left(u_w\right) = 16.38 \text{ for } u_w = 4.39\times10^{-8} \text{ and } W\left(u_i\right) = 1.75 \text{ for } u_i = 0.111$$

Thus the drawdown is

$$s_w = \frac{0.04}{4\pi\left(5\times10^{-3}\right)}(16.38) - \frac{0.04}{4\pi\left(5\times10^{-3}\right)}(1.75) = 9.31\text{ m}$$

The effect of the river on the wells is to decrease the drawdown, so the reduced drawdown in the observation well is

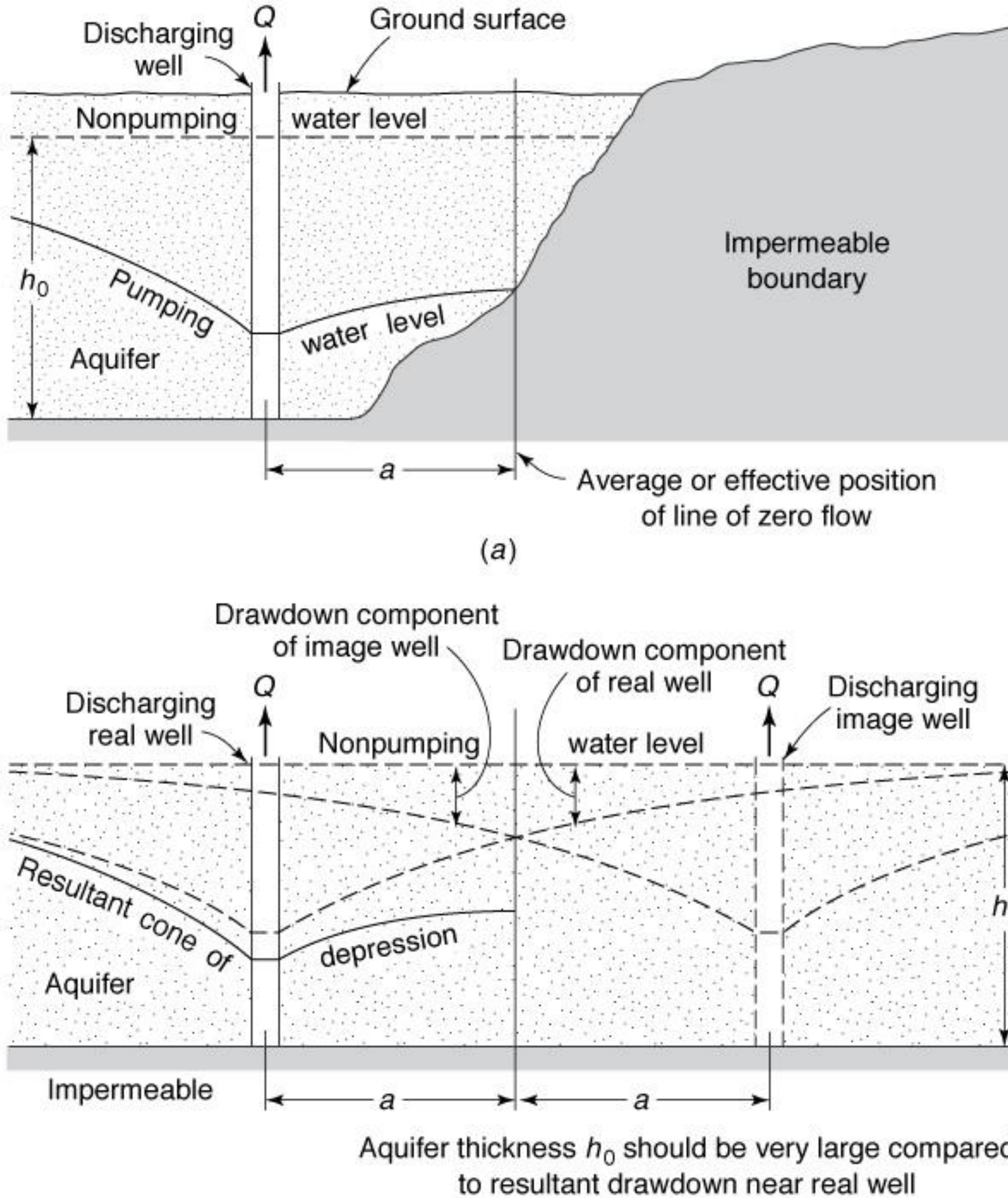
$$s_{\text{river}} = -\frac{Q}{4\pi T}W\left(u_i\right) = -\frac{0.04}{4\pi\left(5\times10^{-3}\right)}(2.54) = -1$$

Similarly, in the pumped well, the reduced drawdown is

$$s_{\text{river}} = -\frac{0.04}{4\pi\left(5\times10^{-3}\right)}(1.75) = -1.11\text{ m}$$

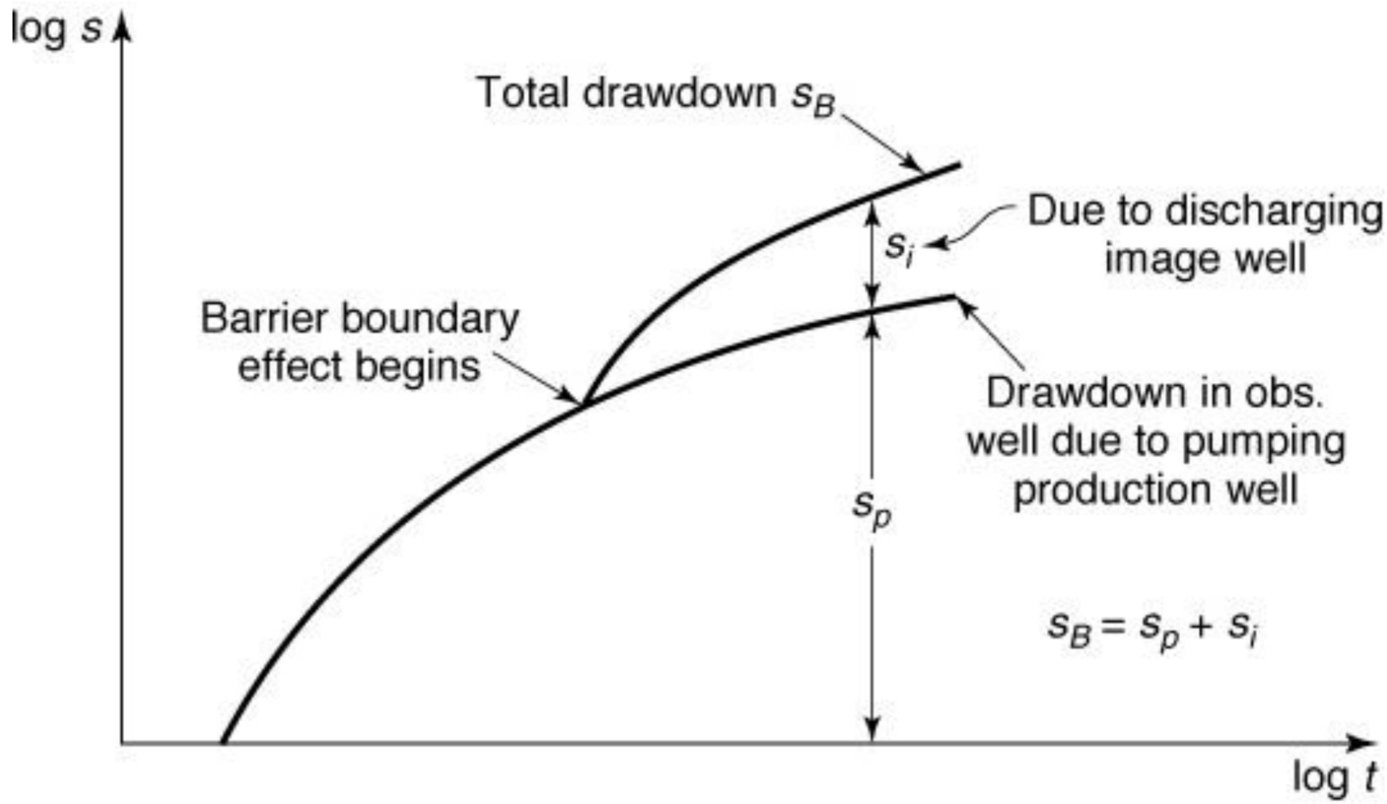
Well flow near an impermeable boundary

$$s_b = s_p + s_i$$



(a)

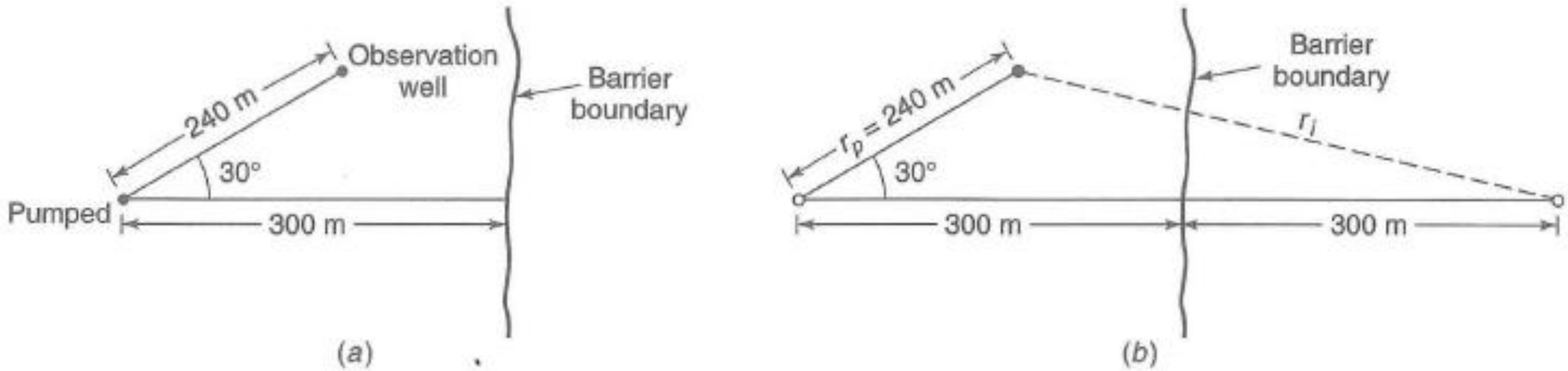
(b)



$$s_b = \frac{Q}{4\pi T}\left[W(u_p) + W(u_i)\right]$$

Example 4.7.2

A well is pumping near a barrier boundary (see Figure 4.7.9) at a rate of 0.03 m^3/s from a confined aquifer 20 m thick. The hydraulic conductivity of the aquifer is 27.65 m/day and its storativity is 3×10^{-5}. Determine the drawdown in the observation well after 10 hours of continuous pumping. What is the fraction of the drawdown attributable to the barrier boundary?



The following information is given in the above problem statement: $Q = 0.03$ m^3/s, $b = 20$ m, $K = 27.65$ m/day $= 3.2 \times 10^{-4}$ m/s, $S = 3 \times 10^{-5}$, $t = 10$ hrs $= 36{,}000$ s. An image well is placed across the boundary at the same distance from the boundary as the pumped well (as shown in Figure 4.7.9b). The drawdown in the observation well is due to the real well and the imaginary well (which accounts for the barrier boundary). Hence, using Equation 4.7.17

$$s = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i)$$

$$u_p = \frac{r_p^2 S}{4Tt} = \frac{(240)^2 (3 \times 10^{-5})}{4(20)(3.2 \times 10^{-4})(36{,}000)} = 1.88 \times 10^{-3}$$

Example 4.7.2

Next compute the distance from the observation well to the image well: $r_i^2 = 600^2 + 240^2 - 2(600)(300)\cos 30° = 16{,}8185\ \text{m}^2$ so $r_i = 410$ m. Using r_i, compute

$$u_i = \frac{168185\left(3\times10^{-5}\right)}{4(20)\left(3.2\times10^{-4}\right)(36{,}000)} = 5.47\times10^{-3}$$

The well functions are now computed or obtained from Table 4.4.1 as $W(u_p) = 5.72$ for $u_p = 1.88 \times 10^{-3}$ and $W(u_i) = 4.64$ for $u_i = 5.47 \times 10^{-3}$.

The drawdown at the observation well is computed as

$$s = \frac{0.03}{4\pi(20)\left(3.2\times10^{-4}\right)} = (5.72 + 4.64) = 3.86\ \text{m.}$$

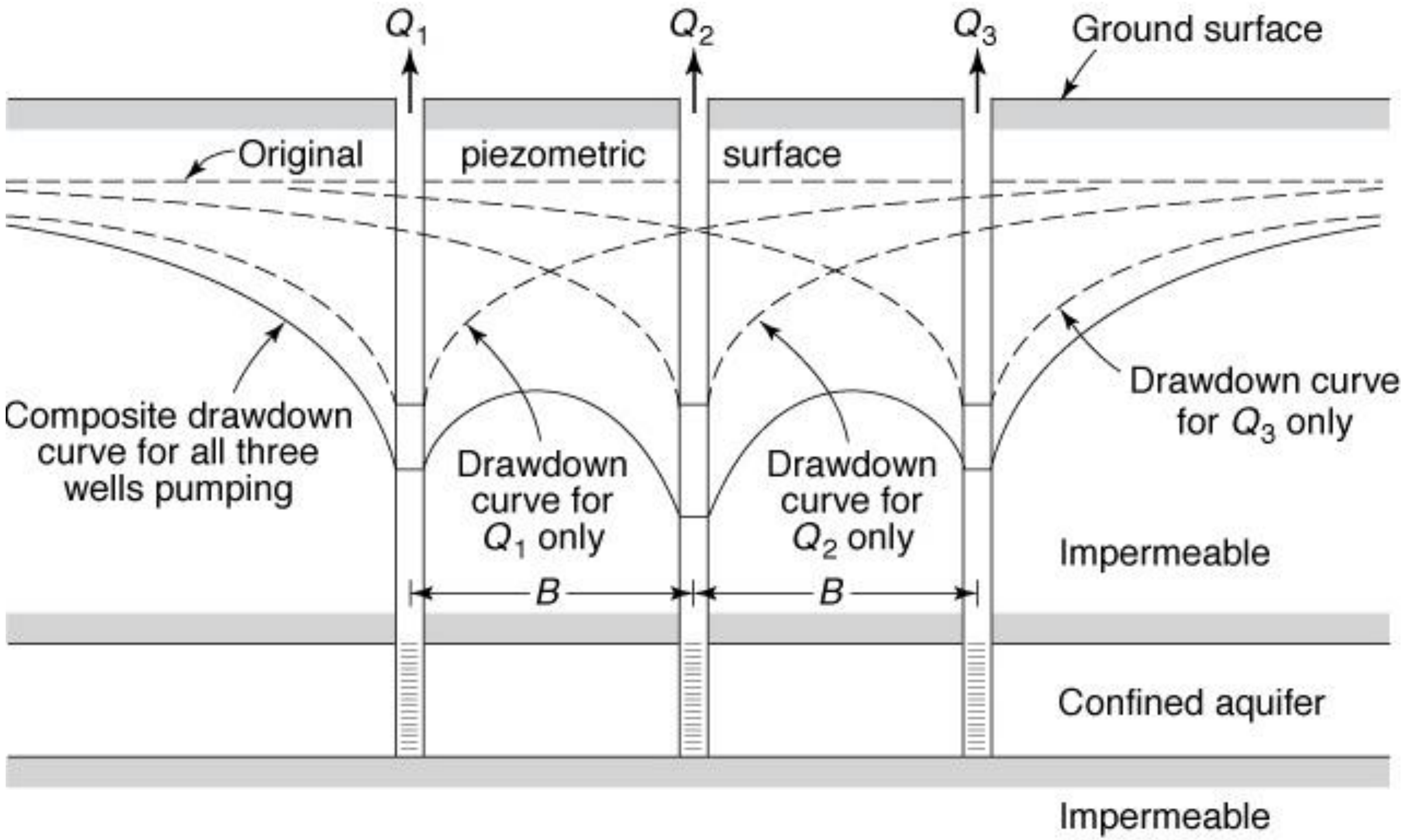
The drawdown attributable to the barrier boundary is computed as

$$s_i = \frac{Q}{4\pi T}W(u_i) = \frac{0.03}{4\pi(20)\left(3.2\times10^{-4}\right)}(4.64) = 1.73\ \text{m}$$

and the fraction of drawdown attributable to the impermeable boundary is

$$\frac{s_i}{s} = \frac{1.73}{3.86} = 0.45\ (45\%).$$

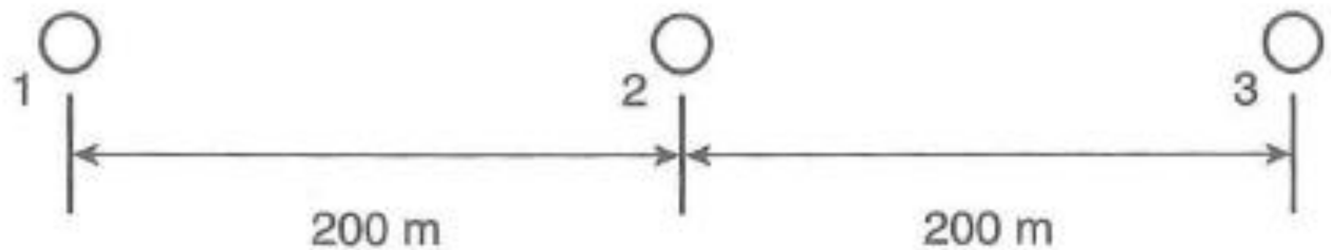
Multiple well systems



$$S_T = S_1 + S_2 + S_3 \cdots + S_n$$

Example 4.8.1

Three pumping wells located along a straight line are spaced at 200 m apart. What should be the steady-state pumping rate from each well so that the near steady-state drawdown in each well will not exceed 2 m? The transmissivity of the confined aquifer, which all the wells fully penetrate, is 2400 m^2/day and all the wells are 40 cm in diameter. The thickness of the aquifer is 40 m and the radius of influence of each well is 800 m.



Example 4.8.1

The following information is given in the above problem statement: $s_1 \le 2$ m, $s_2 \le 2$ m, and $s_3 \le 2$ m, $T =$ 2,400 m^2/day = 27.8×10^{-3} m^2/s, $r_w = 0.2$ m, $b = 40$ m, $r_0 = 800$ m, and $r = 200$ m. Let Q be the pumping rate from each well and h_0 be the head before pumping started. For well 1, $s_1 = s_{11} + s_{12} + s_{13}$ where s_{ij} is the drawdown in well i due to pumping in well j. Thus, for the other wells, $s_2 = s_{21} + s_{22} + s_{23}$, and $s_3 = s_{31} + s_{32} + s_{33}$. By symmetry, $s_1 = s_3$. The drawdowns in well 1 due to pumping in wells 1, 2, and 3 are respectively

$$s_{11} = \frac{Q \ln\left(\frac{r_0}{r_w}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{0.2}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 47.48Q$$

$$s_{12} = \frac{Q \ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{200}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 7.94Q$$

$$s_{13} = \frac{Q \ln\left(\frac{r_0}{r_{13}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{400}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 3.97Q$$

Example 4.8.1

The drawdowns in wells 1 and 3 are identical so total drawdown in the wells is $s_1 = s_3 = 47.48Q + 7.94Q + 3.97Q = 59.39Q$. The drawdowns in well 2 due to pumping in wells 1, 2, and 3 are respectively

$$s_{21} = \frac{Q\ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{200}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 7.94Q$$

$$s_{22} = s_{11} = 47.48Q$$

$$s_{23} = s_{21} = 7.94Q$$

The total drawdown in well 2 is $s_2 = 7.94Q + 47.48Q + 7.94Q = 63.36Q$. The relationships for $s_1 = 59.39Q$ and $s_2 = 63.36Q$ show that for the same discharge from all the wells, more drawdown results at the middle well; therefore, the drawdown in this well governs. So using $s_2 \le 2$ or $63.36Q \le 2$, then the steady-state pumping rate from each well should be $Q \le 3.16 \times 10^{-2}\ \mathrm{m^3/s} = 113\ \mathrm{m^3/hr}$. ■