Chapter 3: Groundwater Movement

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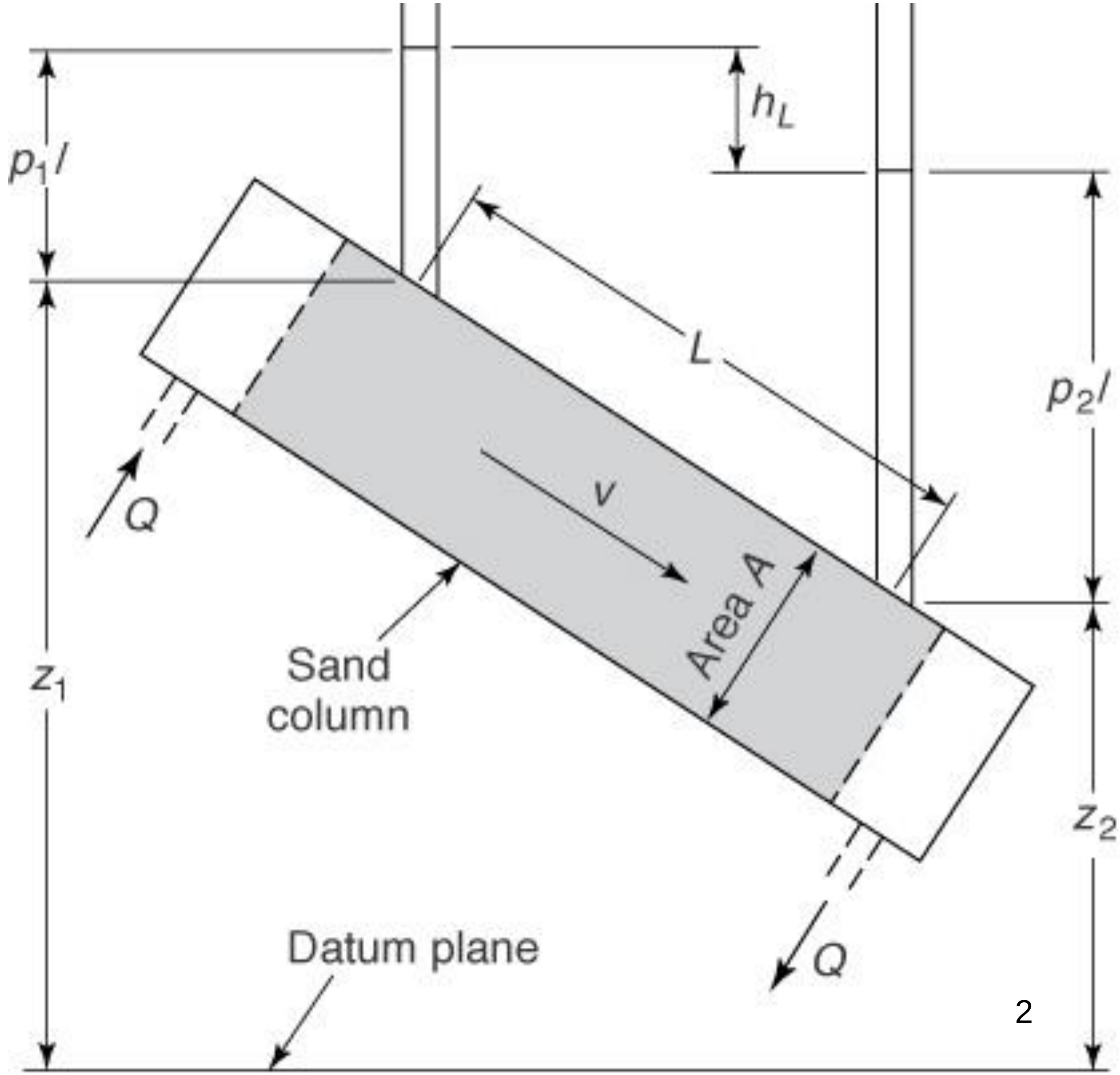
Darcy's law

$$v = \frac{Q}{A} = -K\frac{dh}{dl}$$

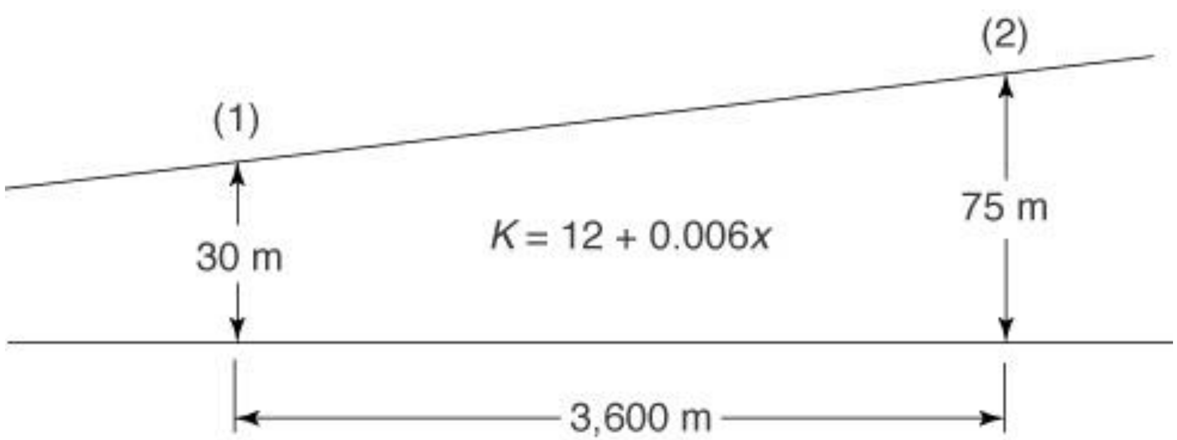


Example 3.1.2

A confined aquifer with a horizontal bed has a varying thickness as shown in Figure 3.1.2. The aquifer is inhomogeneous with $K = 12 + 0.006x$, where $x = 0$ at section (1), and the piezometric heads at sections (1) and (2) are 14.2 m and 18.8 m, respectively measured above the upper confining layer. Assuming the flow in the aquifer is essentially horizontal, determine the flow rate per unit width.

Darcy's law for a constant thickness aquifer is given by Equation 3.1.4,

$$Q = -KA\frac{dh}{dl}$$



Since the aquifer thickness is variable in this problem, we must also write the cross-sectional area and the hydraulic gradient as a function of the distance x. Assuming a unit width, $A = b_1 + \frac{(b_2 - b_1)x}{L}$, where $b_1 = 30$ m, $b_2 = 75$ m, and $L = 3{,}600$ m, then we have

$$A = 30 + \frac{(75-30)x}{3{,}600} = 30 + 0.0125x$$

Substituting the expressions for A and K into Darcy's equation yields the expression for Q in following form:

$$Q = -(12 + 0.006x)(30 + 0.0125x)\frac{dh}{dx}$$

Rearranging this equation and integrating from section (1) to section (2) yields

$$\int_0^{3600} \frac{1}{(12+0.006x)(30+0.0125x)}\,dx = \int_{14.2}^{18.8} -\frac{1}{Q}\,dh$$

This equation is integrated using partial fraction decomposition to obtain

$$\int_0^{3600}\left[\frac{0.2}{(12+0.006x)}-\frac{0.416}{(30+0.0125x)}\right]dx=\int_{14.2}^{18.8}-\frac{1}{Q}\,dh$$

$$\left[33.333\ln(12+0.006x)-33.28\ln(30+0.0125x)\right]_{x=0}^{x=3,600}=-\frac{1}{Q}h\Big]_{h_1=14.2}^{h_2=18.8}$$

$$-26.54-(-30.36)=-\frac{1}{Q}(18.8-14.2)$$

$$Q=-1.20\ (\text{m}^3/\text{day/m})$$

The minus sign implies that the flow is from section (2) to (1). ■

Darcy Velocity

$$v_a = \frac{Q}{\alpha A}$$

Validity of Darcy's law

$$N_R = \frac{\rho v D}{\mu}$$

$$\text{Darcy's law is valid for} \quad N_R < 1$$

Permeability

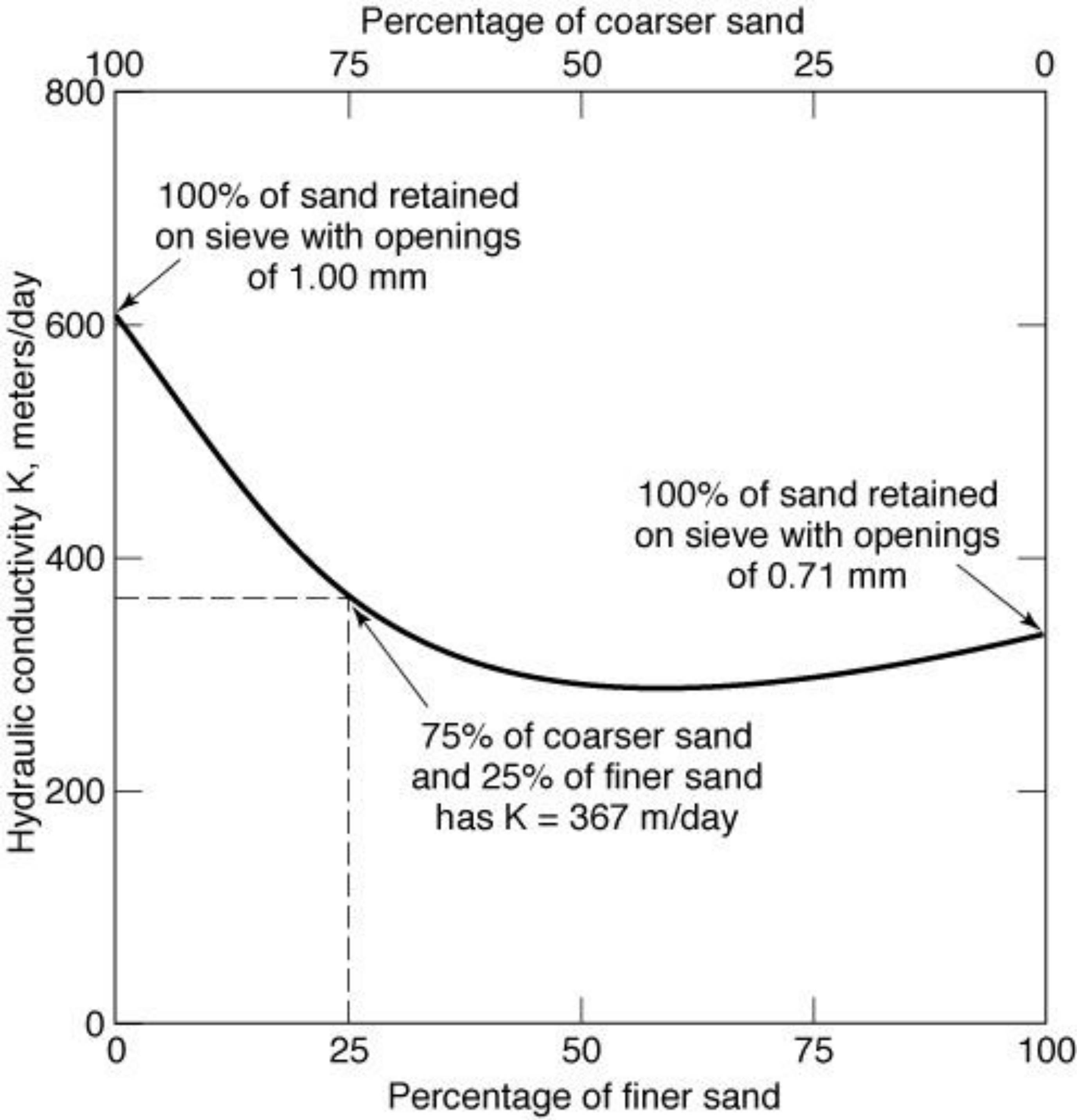
Intrinsic Permeability $$k = \frac{K\mu}{\rho g}: \quad \mathrm{m}^2 \; or \; (\mu\mathrm{m})^2$$

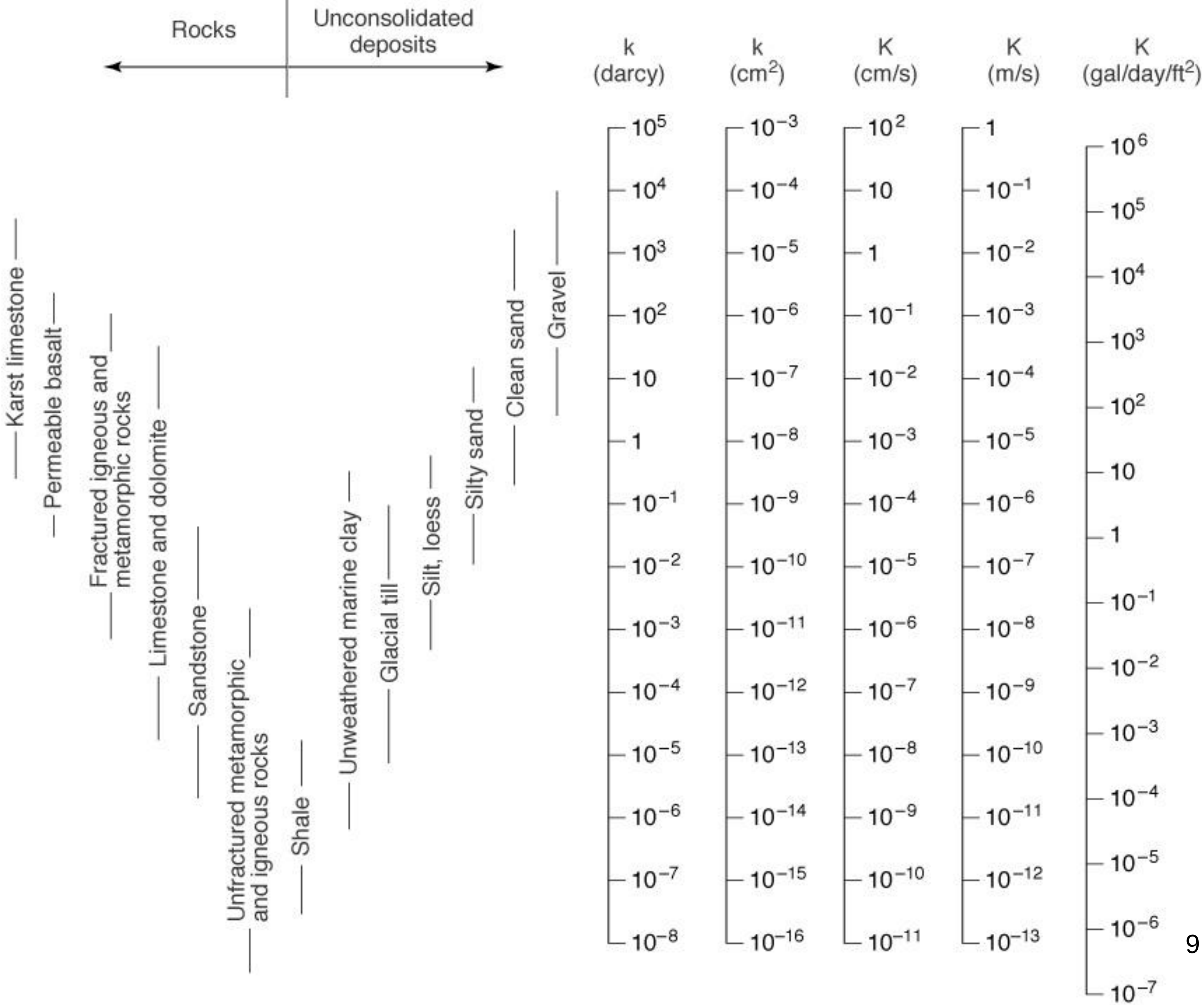
$$1\,\mathrm{darcy} = \frac{(1\,\mathrm{centipoise})(1\,\mathrm{cm}^3/\mathrm{s})/(1\,\mathrm{cm}^2)}{1\,\mathrm{atm./cm}} 0.987(\mu\mathrm{m})^2$$

Hydraulic Conductivity $$K = -\frac{v}{dh/dl}: \quad \mathrm{m/day}$$

Transmissivity $$T = Kb: \quad \mathrm{m}^2/\mathrm{day}$$

$$b = \text{saturated thickness of aquifer}$$


Percentage of coarser sand
100
75
50
25
0
800
600
400
200
0
Hydraulic conductivity K, meters/day
100% of sand retained on sieve with openings of 1.00 mm
100% of sand retained on sieve with openings of 0.71 mm
75% of coarser sand and 25% of finer sand has K = 367 m/day
0
25
50
75
100
Percentage of finer sand


Rocks
Unconsolidated deposits
k (darcy)
k (cm2)
K (cm/s)
K (m/s)
K (gal/day/ft2)
Karst limestone
Permeable basalt
Fractured igneous and metamorphic rocks
Limestone and dolomite
Sandstone
Unfractured metamorphic and igneous rocks
Shale
Unweathered marine clay
Glacial till
Silt, loess
Silty sand
Clean sand
Gravel
10^5 10^4 10^3 10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8
10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12 10^-13 10^-14 10^-15 10^-16
10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11
1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12 10^-13
10^6 10^5 10^4 10^3 10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7

Determination of permeability

Empirical relationships

$$k = cd^2 \quad or \quad k = f_s f_\alpha d^2$$

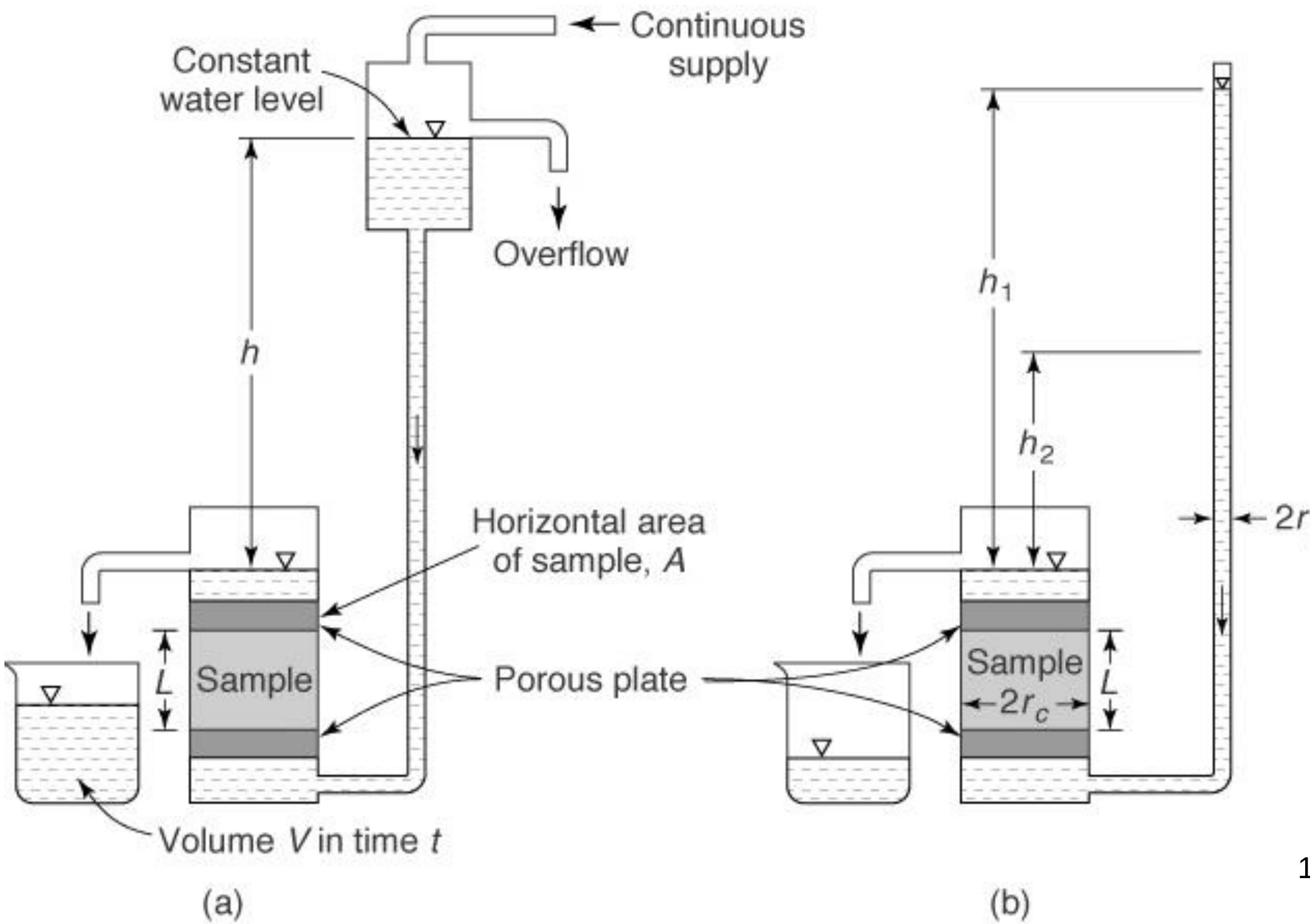
$$c = \text{dimensionless coefficient}$$

$$d = \text{characteristic grain diameter}$$

$$f_s = \text{grain or pore shape factor}$$

$$f_\alpha = \text{porosity factor}$$

Laboratory methods:
Constant head and Falling head permeameters



Constant head permeameter: $$K = \frac{(\text{Volume/time}) \times L}{Ah}$$

Falling head permeameter: $$K = \left(\frac{r_t}{r_c}\right)^2 \left(\frac{L}{\text{time}}\right) \ln\left(\frac{h_1}{h_2}\right)$$

Example 3.3.3

A 20-cm long field sample of silty, fine sand with a diameter of 10 cm is tested using a falling-head permeameter. The falling-head tube has a diameter of 3.0 cm and the initial head is 8.0 cm. Over a period of 8 hr, the head in the tube falls to 1.0 cm. Estimate the hydraulic conductivity of the sample.

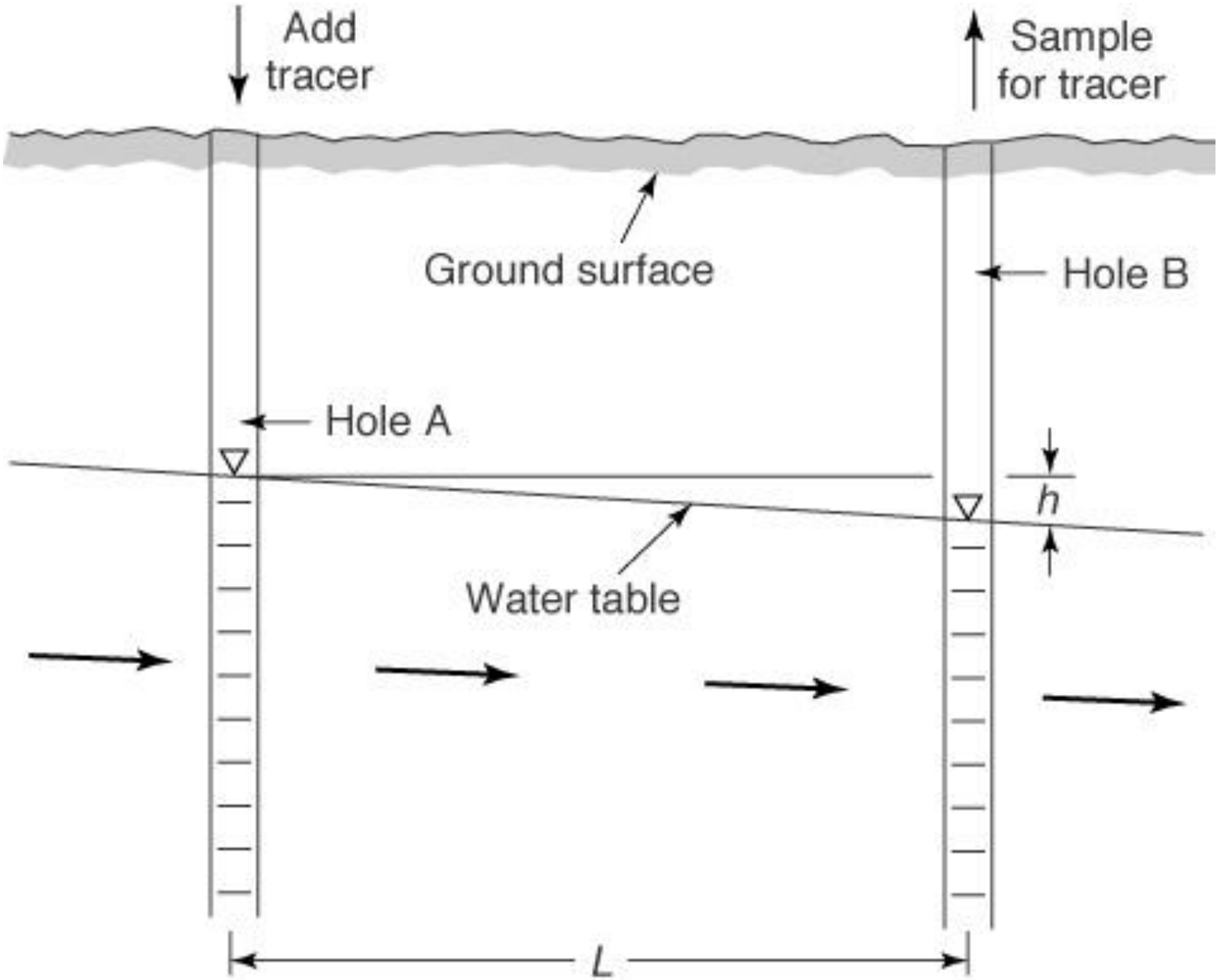
Equation 3.3.6 is used to compute the hydraulic conductivity in a falling-head permeameter test:

$$K = \frac{r_t^2 L}{r_c^2 t} \ln \frac{h_1}{h_2} = \frac{(1.5 \text{ cm})^2 (20 \text{ cm})}{(5.0 \text{ cm})^2 (8 \times 3600 \text{ sec})} \ln \frac{8.0 \text{ cm}}{1.0 \text{ cm}} = 1.3 \times 10^{-4} \text{ cm/s} = 0.112 \text{ m/day}$$

■

Tracer tests

$$K = \frac{\alpha L^2}{h \times \text{time}}$$



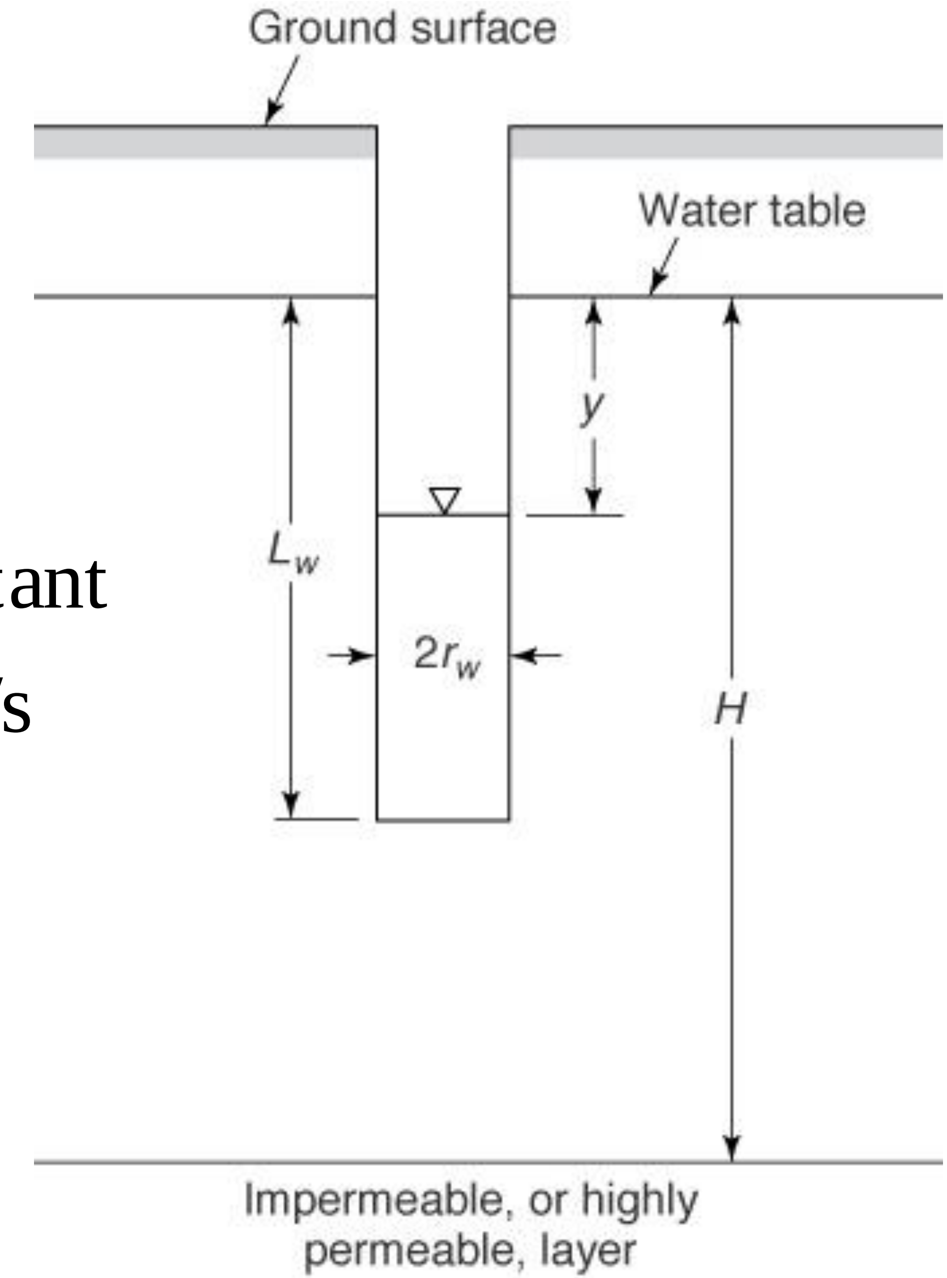
Auger hole tests

$$K = \frac{C}{864}\frac{dy}{dt}$$

K in m/day

C is a dimensionless constant

$dy/dt =$ rate of rise in cm/s



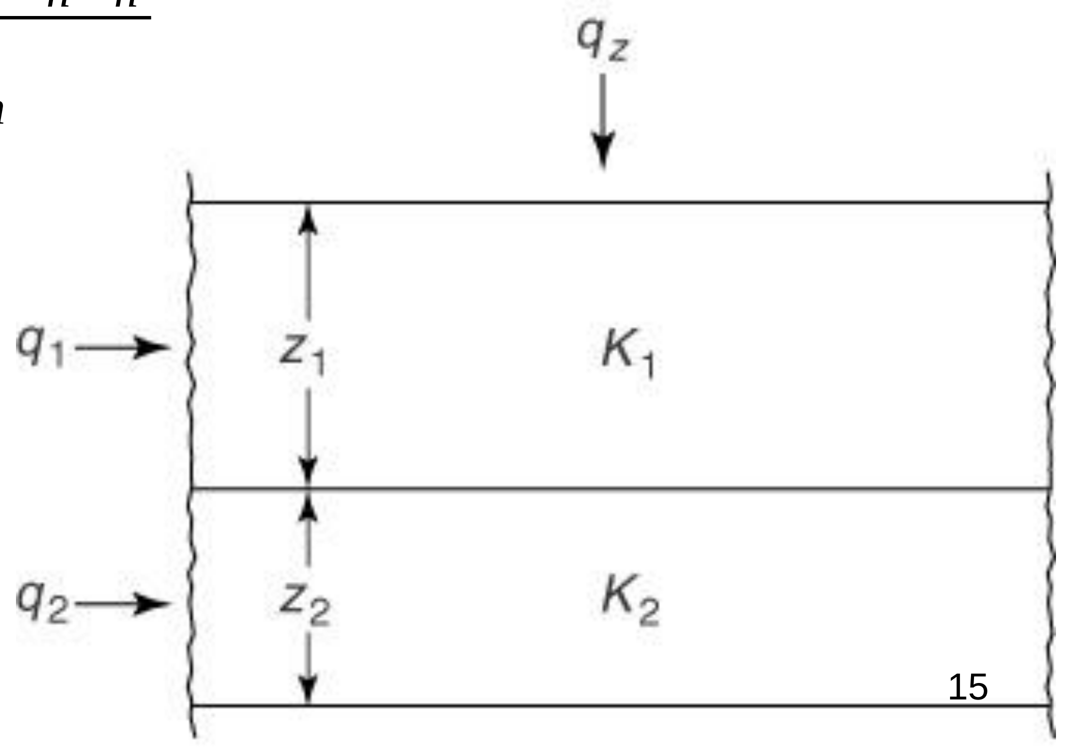
Pumping tests (Chapter 4)

Anisotropic aquifers

$$q_1 = K_1 i z_1 \quad q_2 = K_2 i z_2 \quad q_x = K_x i (z_1 + z_2) = q_1 + q_2$$

$$K_x = \frac{K_1 z_1 + K_2 z_2}{z_1 + z_2} \qquad \left(i = dh / dl \right)$$

$$K_x = \frac{K_1 z_1 + K_2 z_2 \cdots + K_n z_n}{z_1 + z_2 \cdots + z_n}$$



$$q_z = K_1 (dh_1 / z_1) \quad dh_1 = \frac{z_1}{K_1} q_z$$

$$dh_1 + dh_2 = \left(\frac{z_1}{K_1} + \frac{z_2}{K_2} \right) q_z$$

for homogeneous system

$$q_z = K_z \left(\frac{dh_1 + dh_2}{z_1 + z_2} \right) \quad or \quad dh_1 + dh_2 = \left(\frac{z_1 + z_2}{K_z} \right) q_z$$

$$K_z = \frac{z_1 + z_2}{z_1 / K_1 + z_2 / K_2}$$

$$K_z = \frac{z_1 + z_2 \cdots + z_n}{z_1 / K_1 + z_2 / K_2 \cdots + z_n / K_n}$$

Example 3.4.1

An unconfined aquifer consists of three horizontal layers, each individually isotropic. The top layer has a thickness of 10 m and a hydraulic conductivity of 11.6 m/day. The middle layer has a thickness of 4.4 m and a hydraulic conductivity of 4.5 m/day. The bottom layer has a thickness of 6.2 m and a hydraulic conductivity of 2.2 m/day. Compute the equivalent horizontal and vertical hydraulic conductivities.

Equation 3.4.5 is used to compute the equivalent horizontal hydraulic conductivity:

$$K_x = \frac{K_1 z_1 + K_2 z_2 + K_3 z_3}{z_1 + z_2 + z_3}$$

$$= \frac{(11.6 \text{ m/day})(10 \text{ m}) + (4.5 \text{ m/day})(4.4 \text{ m}) + (2.2 \text{ m/day})(6.2 \text{ m})}{(10 \text{ m} + 4.4 \text{ m} + 6.2 \text{ m})} = 7.25 \text{ m/day}$$

The equivalent vertical hydraulic conductivity is computed using Equation 3.4.12:

$$K_z = \frac{z_1 + z_2 + z_3}{\dfrac{z_1}{K_1} + \dfrac{z_2}{K_2} + \dfrac{z_3}{K_3}}$$

$$= \frac{10 \text{ m} + 4.4 \text{ m} + 6.2 \text{ m}}{\dfrac{10 \text{ m}}{11.6 \text{ m/day}} + \dfrac{4.4 \text{ m}}{4.5 \text{ m/day}} + \dfrac{6.2 \text{ m}}{2.2 \text{ m/day}}} = 4.42 \text{ m/day}$$

Note that the equivalent hydraulic conductivities above are computed based on the assumption that each layer is individually isotropic, that is, $K_x = K_z$ in each layer. ■

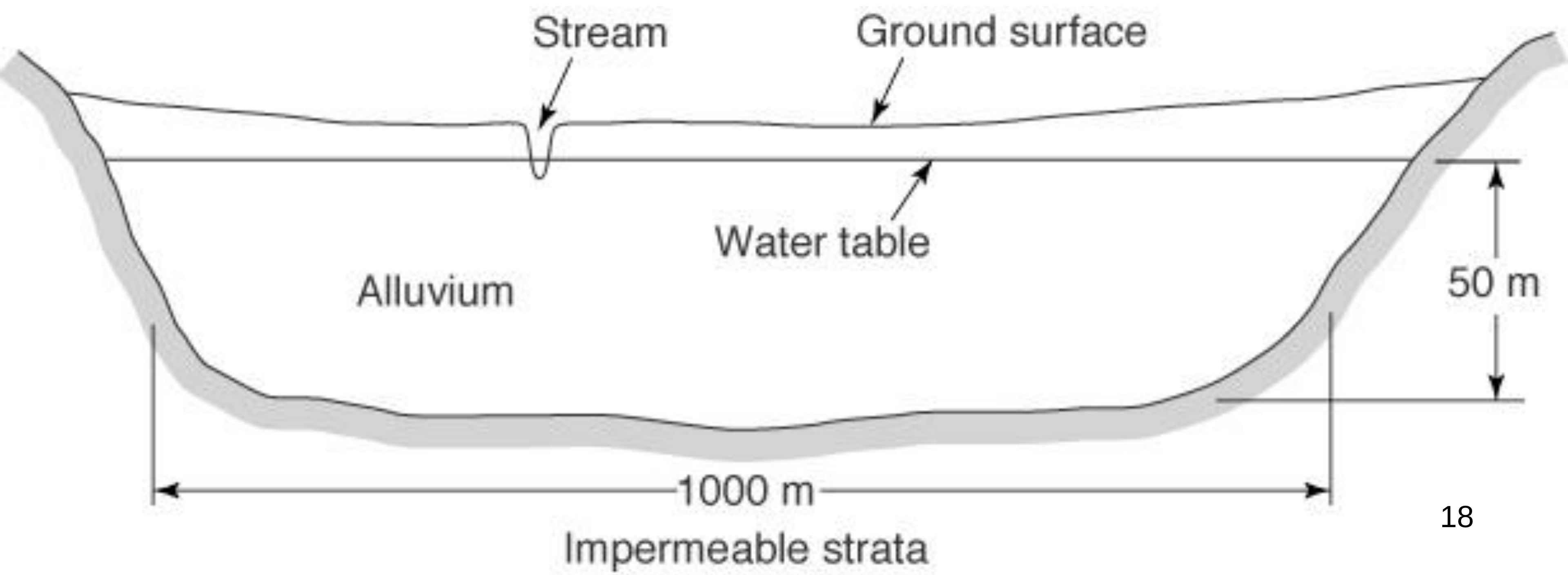
Groundwater flow rates

Assume a productive alluvial aquifer with K = 75 m/day and a hydraulic gradient = -10 m/1000 m = -0.01. From Darcy's law,

$$v = -K\frac{dh}{dl} = 75(0.01) = 0.75\text{m/day}$$

$$Q = Av = (50)(1000)(0.75) = 37500\text{m}^3/\text{day}$$

$$= 0.43\text{m}^3/\text{s}$$



Groundwater flow directions: Flow nets

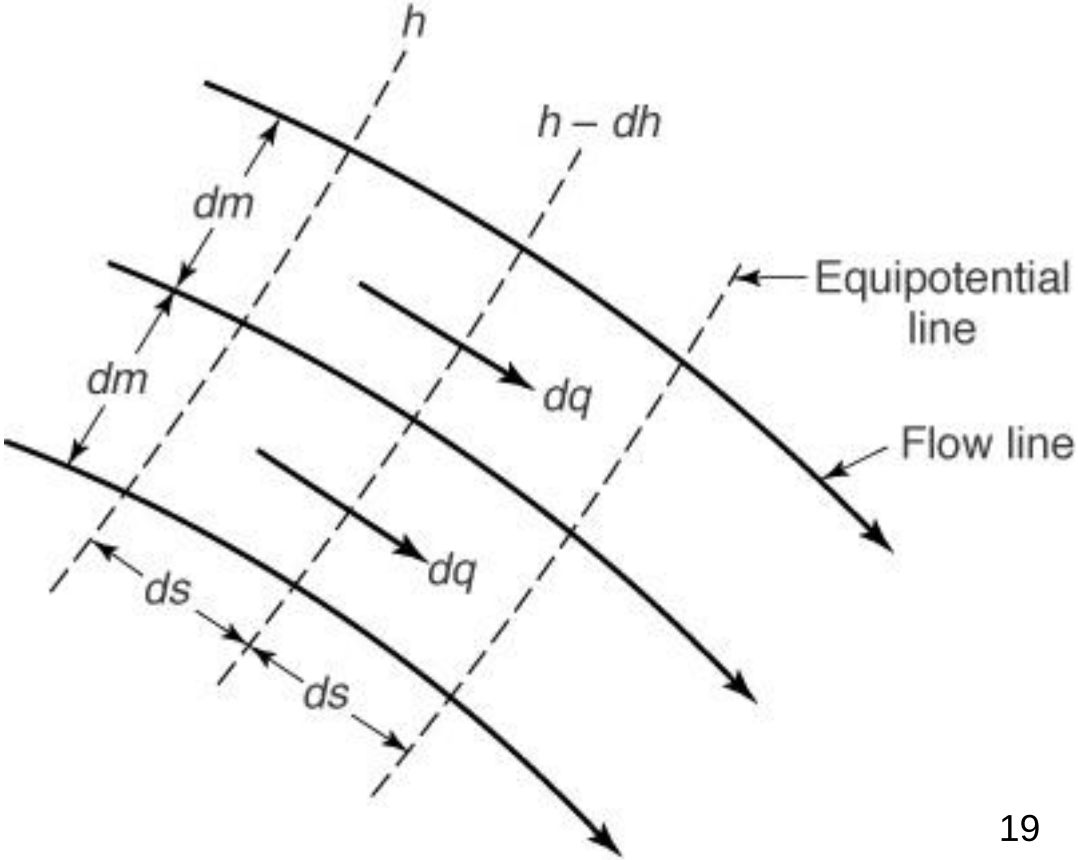
$$i = \frac{dh}{ds}$$

$$q = K\frac{dh}{ds}dm$$

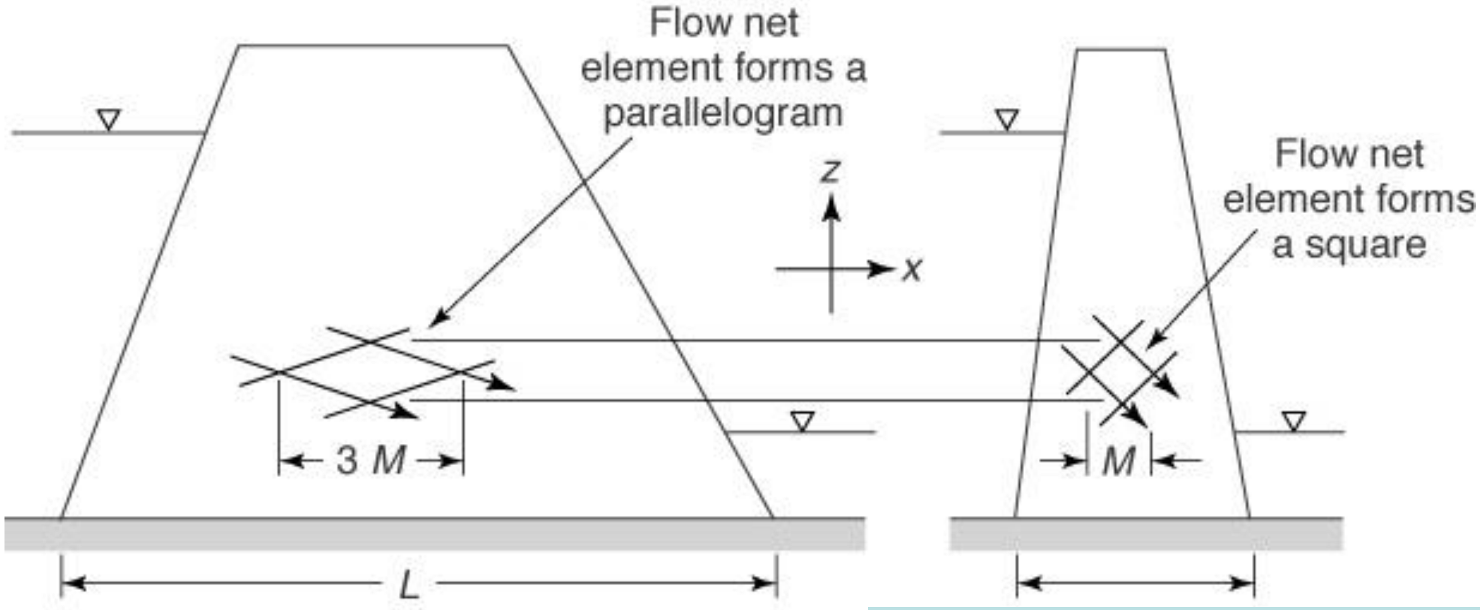
$$q = Kdh \quad (ds = dm)$$

$$dh = \frac{h}{n}$$

$$Q = mq = Km\frac{h}{n}$$



Flow nets for anisotropic hydraulic conductivity



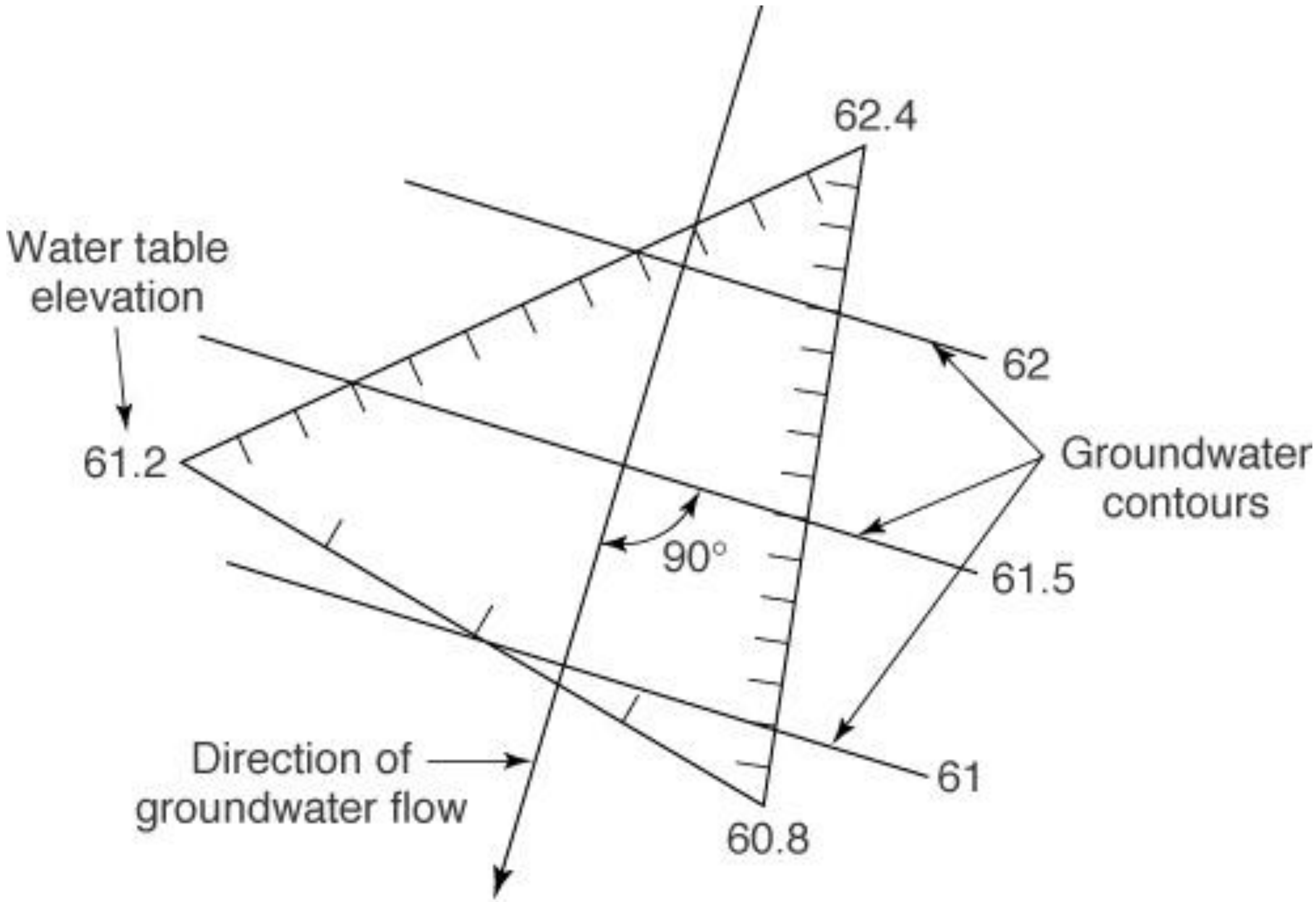
(a)

$$L\sqrt{\frac{K_z}{K_x}} = L\sqrt{\frac{1}{9}} = \frac{L}{3} \qquad (K_x = 9K_z)$$

$$for\ K_x > K_z$$

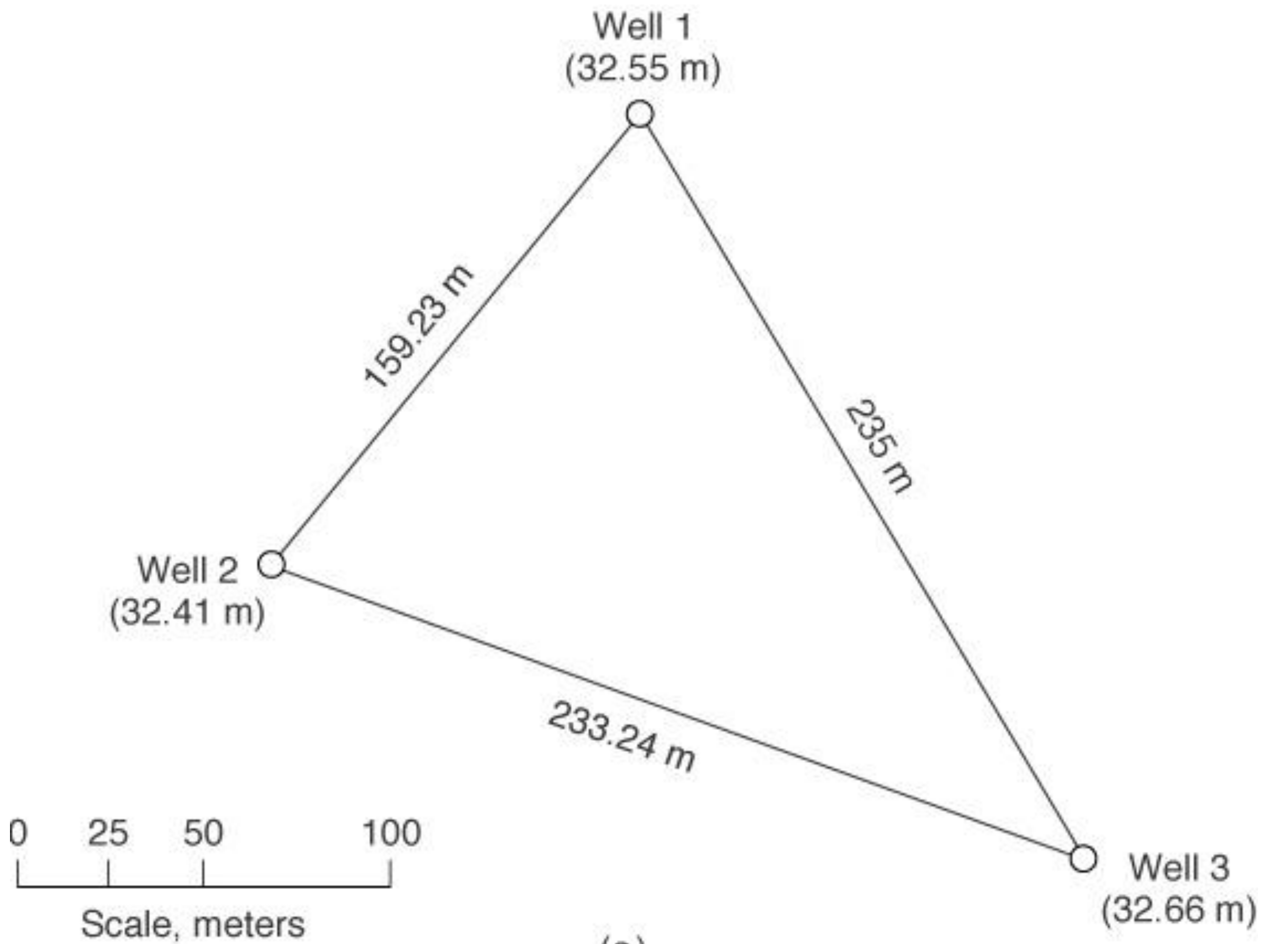
$$K' = \sqrt{K_x K_z}$$

Estimating groundwater flow direction from three well observations



Example 3.6.1

Three observation wells are installed to determine the direction of groundwater movement and the hydraulic gradient in a regional aquifer. The distance between the wells and the total head at each well are shown in Figure.

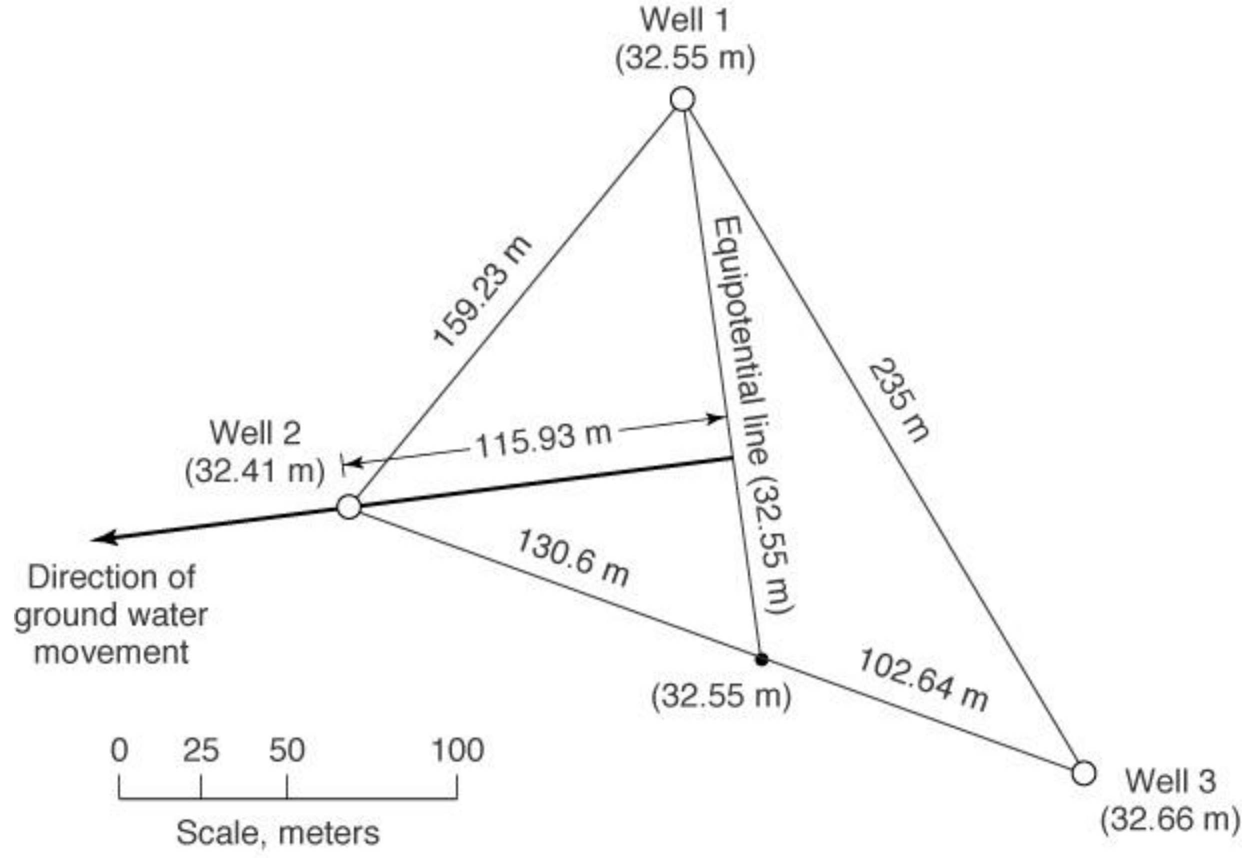


(a)

Step 1: Identify the well with the intermediate water level- Well in this case.

Step 2: Along the straight line between the wells with the highest head and the lowest head, identify the location of the same head of the well from Step I. Note that this is accomplished by locating the elevation of 32.55 m between Well 2 and Well 3 in the graphical solution.

Step 3: Draw a straight line between the intermediate well from Step 1 and the point identified in Step 2.



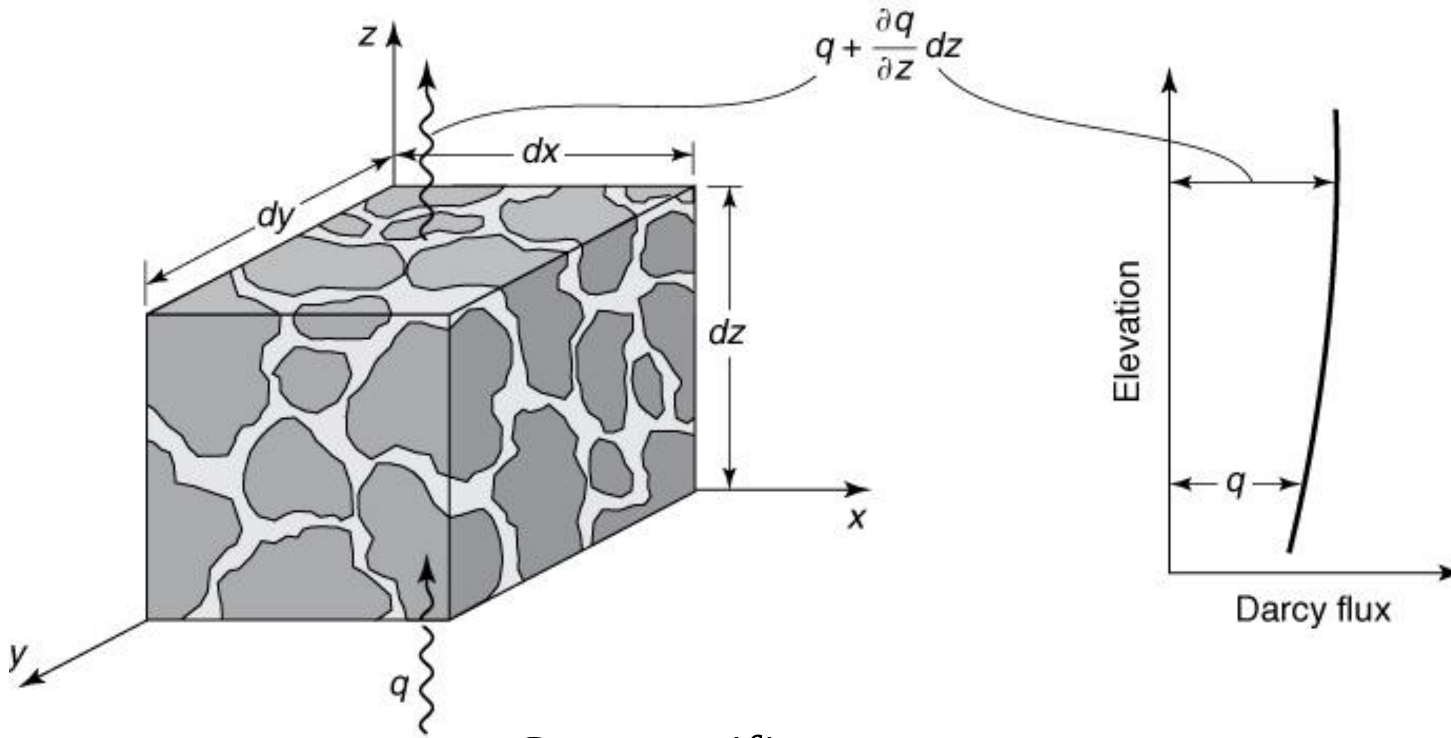
(b)

This is a segment of the equipotential line along which the total head is the same as that in the intermediate well (i.e., equipotential line of 32.55 m head in this case).

Step 4: Draw a line perpendicular to the equipotential line passing through the well with the lowest head. The hydraulic gradient is the slope of that perpendicular line. Also, the direction of the line indicates the direction of ground water movement. The graphical procedure above is illustrated in the Figure. The hydraulic gradient is then computed as

$$i = \frac{32.55\ \text{m} - 32.41\ \text{m}}{115.93\ \text{m}} = 0.0012$$

General Flow Equations



S_S = specific storage

$W = Q/(dxdydz)$

= flow into or out of control volume

$$\int_{CS} \rho V.dA = -\frac{d}{dt}\int_{CV} \rho.d\forall$$

$$\frac{d}{dt}\int_{CV} \rho.d\forall = \rho S_S \frac{\partial h}{\partial t}\left(dxdydz\right) + \rho W\left(dxdydz\right)$$

$$\int_{CS} \rho V.dA = \rho\left(q + \frac{\partial q}{\partial z} dz \right) dxdy - qdxdy = \rho dxdydz \frac{\partial q}{\partial z}$$

$$\int_{CS} \rho V.dA = \rho dxdydz \frac{\partial q}{\partial x} + \rho dxdydz \frac{\partial q}{\partial y} + \rho dxdydz \frac{\partial q}{\partial z}$$

$$\rho dxdydz \frac{\partial q}{\partial x} + \rho dxdydz \frac{\partial q}{\partial y} + \rho dxdydz \frac{\partial q}{\partial z}$$

$$= -\left(\rho S_S \frac{\partial h}{\partial t} (dxdydz) + \rho W (dxdydz) \right)$$

$$S_S \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} + \frac{\partial q}{\partial z} + W = 0$$

$$q_x = -K_x \frac{\partial h}{\partial x} \quad q_y = -K_y \frac{\partial h}{\partial y} \quad q_z = -K_z \frac{\partial h}{\partial z}$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z} \right) = S_S \frac{\partial h}{\partial t} + W$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_S}{K} \frac{\partial h}{\partial t} + \frac{W}{K} \quad for \quad K = K_x = K_y = K_z$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{W}{K} \quad \text{for steady state}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \quad (S = S_S b, T = Kb)$$

For radial flow

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)=\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}$$

For steady state

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)=0$$