Chapter 4

Groundwater and Well Hydraulics

Darcy's law and the fundamental equations governing groundwater movement can now be applied to particular situations. Solutions of groundwater flow to wells rank highest in importance. From pumping tests of wells, storage coefficients and transmissivities of aquifers can be determined; furthermore, with these aquifer characteristics known, future declines in groundwater levels associated with pumpage can be calculated. Well flow equations have been developed for steady and unsteady flows, for various types of aquifers, and for several special boundary conditions. For practical application, most solutions have been reduced to convenient graphic or mathematical form, or computer programs.

4.1 STEADY UNIDIRECTIONAL FLOW

Steady flow implies that no change occurs with time. Flow conditions differ for confined and unconfined aquifers and hence need to be considered separately, beginning with flow in one direction.

4.1.1 Confined Aquifer

Let groundwater flow with a velocity v in the x-direction of a confined aquifer of uniform thickness. Then for one-dimensional, steady flow, Equation 3.9.10 reduces to

$$\frac{\partial^2 h}{\partial x^2} = 0 \tag{4.1.1}$$

which has for its solution

$$h = C_1 x + C_2 \tag{4.1.2}$$

where h is the head above a given datum and C_1 and C_2 are constants of integration. Assuming $h = 0$ when $x = 0$ and $\partial h/\partial x = -(v/K)$ from Darcy's law, then we have

$$h = -\frac{vx}{K} \tag{4.1.3}$$

This states that the head decreases linearly, as sketched in Figure 4.1.1, with flow in the x-direction.

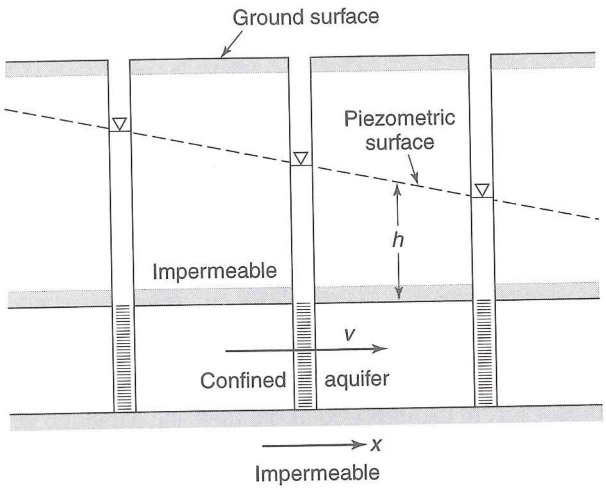


Figure 4.1.1. Steady unidirectional flow in a confined aquifer of uniform thickness.

EXAMPLE 4.1.1

Referring to Figure 4.1.1, if the distance and the observed piezometric surface drop between two adjacent wells are 1,000 m and 3 m, respectively, find an estimate of the time it takes for a molecule of water to move from one well to the other. Assume steady unidirectional flow in a homogeneous silty sand confined aquifer with a hydraulic conductivity $K = 3.5$ m/day and an effective porosity of 0.35.

SOLUTION

First compute the discharge velocity:

$$v = -\frac{hK}{x} = -\frac{(-3\text{m})(3.5\text{ m/d})}{(1000\text{m})} = 0.0105\text{ m/d}$$

The pore (seepage) velocity is computed using the velocity:

$$v_p = v/n_e = (0.0105\text{ m/d})/(0.35) = 0.03\text{ m/d}$$

It would take 1000 m/(0.03 m/d × 365) ≈ 91.3 years. ■

4.1.2 Unconfined Aquifer

For the similar flow situation in an unconfined aquifer, direct analytic solution of the Laplace equation is not possible. The difficulty arises from the fact that the water table in the two-dimensional case represents a flow line. The shape of the water table determines the flow distribution, but at the same time the flow distribution governs the water table shape. To obtain a solution, Dupuit[13] assumed (1) the velocity of the flow to be proportional to the tangent of the hydraulic gradient instead of the sine as defined in Darcy's law and (2) the flow to be horizontal and uniform everywhere in a vertical section. These assumptions, although permitting a solution to be obtained, limit the application of the results. For unidirectional flow, as sketched in Figure 4.1.2, the discharge per unit width q at any vertical section can be given as

$$q = -Kh\frac{dh}{dx} \tag{4.1.4}$$

where K is hydraulic conductivity, h is the height of the water table above an impervious base, and x is the direction of flow. Integrating yields

$$qx = -\frac{K}{2}h^2 + C \tag{4.1.5}$$

and, if $h = h_0$ where $x = 0$, then the *Dupuit equation*

$$q = \frac{K}{2x}\left(h_0^2 - h^2\right) \tag{4.1.6}$$

results, which indicates that the water table is parabolic in form.

For flow between two fixed bodies of water of constant heads h_0 and h_1 as in Figure 4.1.2, the water table slope at the upstream boundary of the aquifer (neglecting the capillary zone) is

$$\frac{dh}{dx} = -\frac{q}{Kh_0} \tag{4.1.7}$$

But the boundary $h = h_0$ is an equipotential line because the fluid potential in a water body is constant; consequently, the water table must be horizontal at this section, which is inconsistent with Equation 4.1.7. In the direction of the flow, the parabolic water table described by Equation 4.1.6 increases in slope. By so doing, the two Dupuit assumptions, previously stated, become increasingly poor approximations to the actual flow; therefore, the actual water table deviates more and more from the computed position in the direction of flow, as indicated in Figure 4.1.2. The fact that the actual water table lies above the computed one can be explained by the fact that the Dupuit flows are all assumed horizontal, whereas the actual velocities of the same magnitude have a downward vertical component so that a greater saturated thickness is required for the same discharge. At the downstream boundary a discontinuity in flow forms because no consistent flow pattern can connect a water table directly to a downstream freewater surface. The water table actually approaches the boundary tangentially above the water body surface and forms a *seepage face.*

The above discrepancies indicate that the water table does not follow the parabolic form of Equation 4.1.6; nevertheless, for flat slopes, where the sine and tangent are nearly equal, it closely predicts the water table position except near the outflow. The equation, however, accurately determines q or K for given boundary heads.[41]

EXAMPLE 4.1.2

A stratum of clean sand and gravel between two channels (see Figure 4.1.2) has a hydraulic conductivity $K = 10^{-1}$ cm/sec, and is supplied with water from a ditch ($h_0 = 6.5$ m deep) that penetrates to the bottom of the stratum. If the water surface in the second channel is 4 m above the bottom of the stratum and its

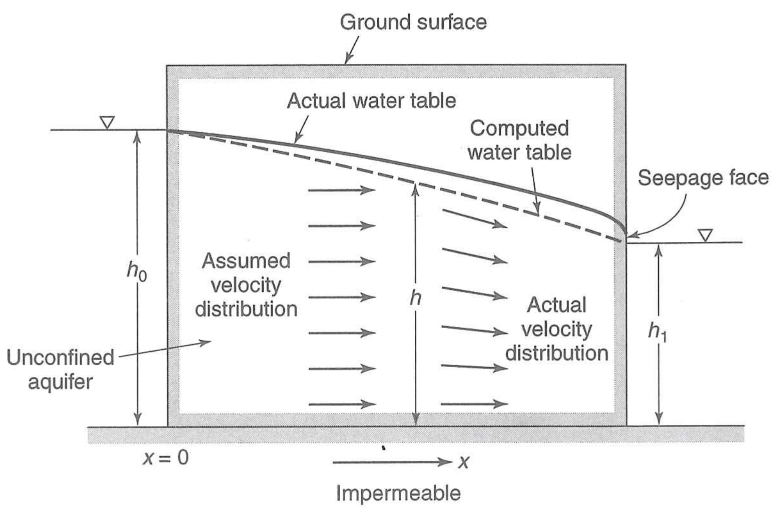


Figure 4.1.2. Steady flow in an unconfined aquifer between two water bodies with vertical boundaries.

distance to the ditch is $x = 150$ m (which is also the thickness of the stratum), estimate the unit flow rate into the gallery.

SOLUTION

The flow is computed using the Dupuit equation (4.1.6) for unit flow, where

$$K = 10^{-1} \text{ cm/sec} = 86.4 \text{ m/day}$$

$$q = \frac{K}{2x}\left(h_0^{\,2} - h^2\right) = \frac{86.4 \text{ m/day}}{2(150 \text{ m})}\left(6.5^2 - 4^2\right)\text{m}^2 = 7.56 \text{ m}^2\text{/day}$$

■

4.1.3 Base Flow to a Stream

Estimates of the base flow to streams (see Chapter 6) or average groundwater recharge can be computed by applying the above analysis of one-directional flow in an unconfined aquifer. For example, picture the idealized boundaries shown in Figure 4.1.3 of two long parallel streams completely penetrating an unconfined aquifer with a continuous recharge rate W occurring uniformly over the aquifer. With the Dupuit assumptions, the flow per unit thickness is

$$q = -Kh\frac{dh}{dx} \qquad (4.1.8)$$

and by continuity

$$q = Wx \qquad (4.1.9)$$

Combining these equations and integrating leads to the result

$$h^2 = h_a^{\,2} + \frac{W}{K}\left(a^2 - x^2\right) \qquad (4.1.10)$$

where h, h_a, a, and x are as defined in Figure 4.1.3, and K is the hydraulic conductivity. From symmetry and continuity

$$Q_b = 2aW \qquad (4.1.11)$$

where Q_b is the base flow entering each stream per unit length of stream channel. If h is known at any point, Q_b or W can be computed provided K is known.

Extensions of this analysis have been applied to design the spacing of parallel drains on agricultural land for specified soil, crop, and irrigation conditions.

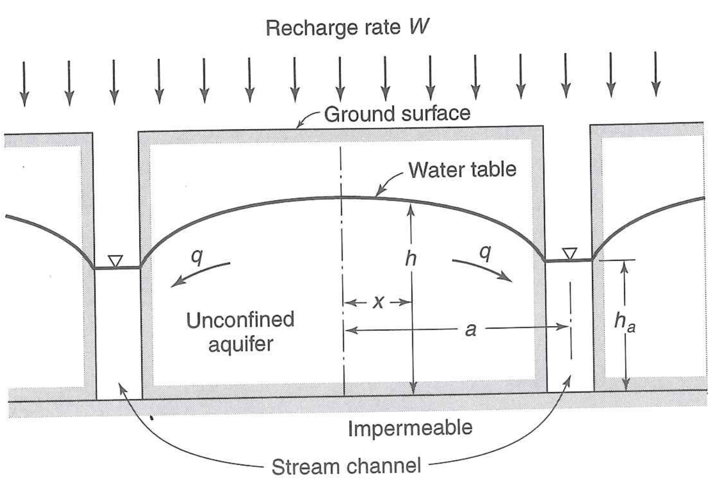


Figure 4.1.3. Steady flow to two parallel streams from a uniformly recharged unconfined aquifer.

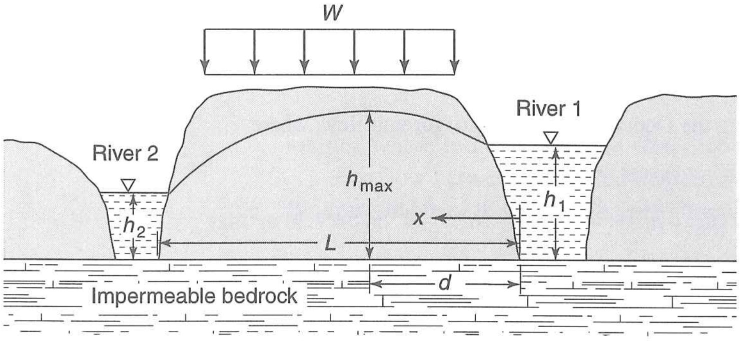


Figure 4.1.4. Unconfined aquifer between two rivers ($x = 0$, $h = h_1$ and $x = L$, $h = h_2$).

Consider the unconfined aquifer between two rivers as shown in Figure 4.1.4 with recharge rate of W. The flow is only in one direction so that the x-axis is aligned parallel to the flow. The flow is then determined by

$$\frac{d}{dx}\left(Kh\frac{dh}{dx}\right) = -W \tag{4.1.12}$$

or

$$\frac{d^2(h^2)}{dx^2} = -\frac{2W}{K} \tag{4.1.13}$$

Integration of Equation 4.1.13 yields

$$h^2 = -\frac{Wx^2}{K} + c_1 x + c_2 \tag{4.1.14}$$

where c_1 and c_2 are constants of integration. Boundary conditions ($h = h_1$ at $x = 0$ and $h = h_2$ at $x = L$) are applied to obtain

$$h^2 = h_1^2 - \frac{(h_1^2 - h_2^2)x}{L} + \frac{W}{K}(L - x)x \tag{4.1.15}$$

The discharge q_x per unit width at any distance from the origin (see Figure 4.1.4) can be computed using Equation 4.1.8 with *dh/dx* computed by differentiating Equation 4.1.15:

$$q_x = \frac{K(h_1^2 - h_2^2)}{2L} - W\left(\frac{L}{2} - x\right) \tag{4.1.16}$$

where the units are ft^2/day or m^2/day for q_x, ft/day or m/day for K and W, and ft or m for x, h_1, h_2, and L.

Figure 4.1.4 shows the location where $h = h_{max}$ (a crest in the water table), for the case of infiltration, which is essentially a water divide where $q_x = 0$. The distance d from the origin to the water divide is computed using Equation 4.1.16 with $q_x = 0$ and $x = d$ to obtain

$$d = \frac{L}{2} - \frac{K}{W}\frac{(h_1^2 - h_2^2)}{2L} \tag{4.1.17}$$

At $x = d$, $h = h_{max}$ which can then be substituted into Equation 4.1.15 to obtain the following expression for h_{max}:

$$h_{max}^2 = h_1^2 - \frac{(h_1^2 - h_2^2)d}{L} + \frac{W}{K}(L - d)d \tag{4.1.18}$$

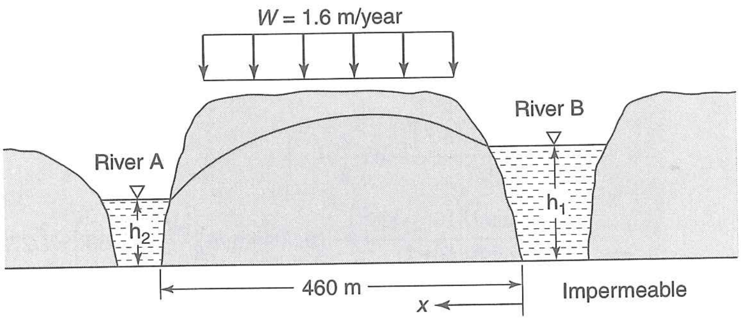


Figure 4.1.5. Example 4.1.3

EXAMPLE 4.1.3

An unconfined aquifer of clean sand and gravel is located between two fully penetrating rivers (see Figure 4.1.5) and has a hydraulic conductivity of $K = 10^{-2}$ cm/sec. The aquifer is subject to a uniform recharge of 1.6 m/year. The water surface elevations in rivers A and B are 8.5 m and 10 m, respectively, above the bottom. Estimate (a) the maximum elevation of the water table and the location of groundwater divide, (b) the travel times from groundwater divide to both rivers ($n_e = 0.35$), and (c) the daily discharge per kilometer from the aquifer into both rivers.

SOLUTION

(a) The maximum elevation of the water table occurs at the location of the groundwater divide computed using Equation 4.1.17 with $W = 1.6$ m/year = 0.0044 m/day and $K = 10^{-2}$ cm/s = 8.64 m/day:

$$d = \frac{L}{2} - \frac{K}{W}\frac{\left(h_1^2 - h_2^2\right)}{2L} = \frac{460 \text{ m}}{2} - \frac{8.64 \text{ m/d}}{0.0044 \text{ m/d}}\frac{\left(10^2 - 8.5^2\right)\text{m}^2}{2(460 \text{ m})} = 171 \text{ m from river B}$$

The maximum head at the divide is computed using Equation 4.1.18:

$$h_{\max} = \sqrt{h_1^2 - \frac{\left(h_1^2 - h_2^2\right)d}{L} + \frac{W}{K}(L-d)d}$$

$$= \sqrt{10^2 - \frac{\left(10^2 - 8.5^2\right)(171)}{460} + \frac{0.0044}{8.64}(460-171)(171)} = 10.7\text{m}$$

(b) The average pore velocity is computed using Darcy's law with the Dupuit assumptions:

$$v_A = \left(\frac{K}{n_e}\right)\left(\frac{\Delta h}{\Delta x}\right) = \left(\frac{8.64 \text{ m/d}}{0.35}\right)\left(\frac{10.7-8.5}{460-171}\right)\frac{\text{m}}{\text{m}} = 0.190 \text{ m/day}$$

So the travel time from the groundwater divide to river A is

$$t = \frac{L_A}{v_A} = \frac{460 \text{ m} - 171 \text{ m}}{0.190 \text{ m/day}} = 1524 \text{ days} = 4.18 \text{ years}$$

Similarly, the travel time from the groundwater divide to river B is computed as

$$v_B = \left(\frac{K}{n}\right)\left(\frac{\Delta h}{\Delta x}\right) = \left(\frac{8.64 \text{ m/d}}{0.35}\right)\left(\frac{10.7-10}{171}\right) = 0.101 \text{ m/day}$$

$$t = \frac{L_A}{v_A} = \frac{171\text{m}}{0.104 \text{ m/day}} = 1692 \text{ days} = 4.64 \text{ years}$$

(c) From Equation 4.1.16, for $x = 0$:

$$q_x = \frac{K\left(h_1^2 - h_2^2\right)}{2L} - W\left(\frac{L}{2} - x\right)$$

$$= \frac{(8.64 \text{ m/d})\left(10^2 - 8.5^2\right)\text{m}^2}{2(460)\text{m}} - (0.0044 \text{ m/d})\left(\frac{460}{2} - 0\right)\text{m} = -0.751\left(\text{m}^3/\text{day}\right)/\text{m}$$

The minus sign occurs due to opposite flow direction to the x-axis (see Figure 4.1.5). So, (0.751 × 1000 m) = 751 m^3/day is the daily discharge from the aquifer per kilometer into river B.

Similarly, for $x = 460$ m:

$$q_x = \frac{K\left(h_1{}^2 - h_2{}^2\right)}{2L} - W\left(\frac{L}{2} - x\right)$$
$$= \frac{(8.64 \text{ m/d})\left(10^2 - 8.5^2\right)}{2(460)} - (0.0044 \text{ m/d})\left(\frac{460}{2} - 460\right) = 1.27\left(\text{m}^3/\text{day}\right)/\text{m}$$

The daily discharge from the aquifer per kilometer into river A is (1.27 × 1000 m) = 1270 m^3/day. ■

4.2 STEADY RADIAL FLOW TO A WELL

When a well is pumped, water is removed from the aquifer surrounding the well, and the water table or piezometric surface, depending on the type of aquifer, is lowered. The *drawdown* at a given point is the distance the water level is lowered. A *drawdown curve* (or *cone*) shows the variation of drawdown with distance from the well (see Figure 4.2.1). In three dimensions, the drawdown curve describes a conic shape known as the *cone of depression*, as shown in Figure 4.2.2. Also, the outer limit of the cone of depression (zero drawdown) defines the *area of influence* of the well.

4.2.1 Confined Aquifer

To derive the radial flow equation (which relates the well discharge to drawdown) for a well completely penetrating a confined aquifer, referring to Figure 4.2.1 will prove helpful. The flow is assumed two-dimensional to a well centered on a circular island and penetrating a homogeneous and isotropic aquifer. Because the flow is everywhere horizontal, the Dupuit assumptions apply without error. Using plane polar coordinates with the well as the origin, we obtain the well discharge Q at any distance r as

$$Q = Av = -2\pi rbK\frac{dh}{dr} \tag{4.2.1}$$

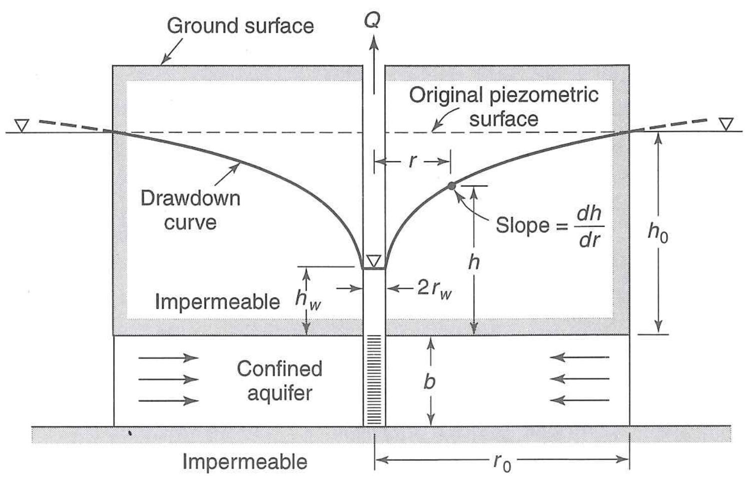


Figure 4.2.1. Steady radial flow to a well penetrating a confined aquifer on an island.

for steady radial flow to the well. Rearranging and integrating 4.2.1 for the boundary conditions at the well, $h = h_w$ and $r = r_w$, and at the edge of the island, $h = h_0$ and $r = r_0$, yield

$$h_0 - h_w = \frac{Q}{2\pi Kb} \ln \frac{r_0}{r_w} \tag{4.2.2}$$

or

$$Q = 2\pi Kb \frac{h_0 - h_w}{\ln(r_0/r_w)} \tag{4.2.3}$$

with the negative sign neglected.

In the more general case of a well penetrating an extensive confined aquifer, as in Figure 4.2.2, there is no external limit for r. From the above derivation at any given value of r,

$$Q = 2\pi Kb \frac{h - h_w}{\ln(r/r_w)} \tag{4.2.4}$$

which shows that h increases indefinitely with increasing r. Yet, the maximum h is the initial uniform head h_0. Thus, from a theoretical aspect, steady radial flow in an extensive aquifer does not exist because the cone of depression must expand indefinitely with time. However, from a practical standpoint, h approaches h_0 with distance from the well, and the drawdown varies with the logarithm of the distance from the well.

The flow net in Figure 4.2.3 illustrates the distribution of flow in a confined aquifer for a fully penetrating well and a 100 percent open hole. Figure 4.2.4 illustrates the flow distribution to a discharging well in a confined aquifer. The well is a fully penetrating, 100 percent open hole. Figure 4.2.5 illustrates the flow net for a well that penetrates 50 percent of the confined aquifer with an open hole. The flow net in Figure 4.2.6 illustrates the distribution of flow in a confined aquifer for a well that penetrates through the upper confining bed but not into the artesian aquifer.

Equation 4.2.4, known as the *equilibrium,* or *Thiem,*[60] *equation,* enables the hydraulic conductivity or the transmissivity of a confined aquifer to be determined from a pumped well

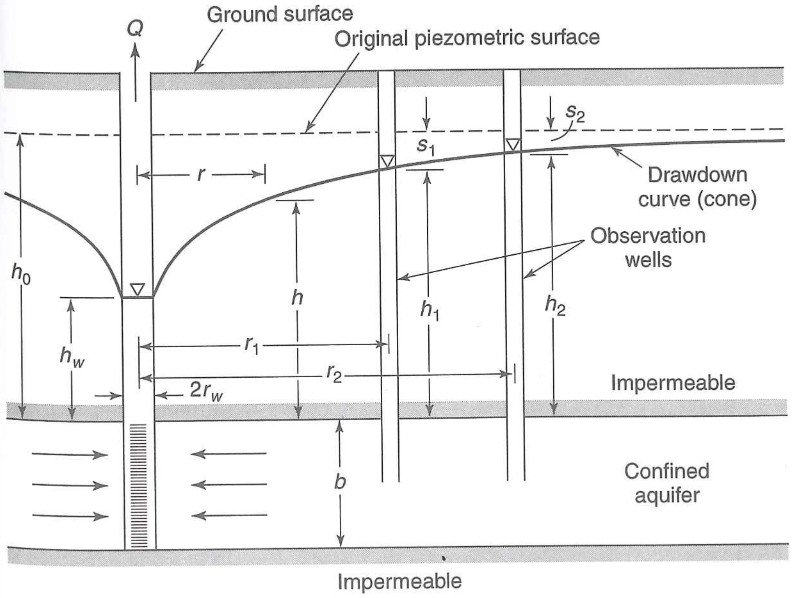


Figure 4.2.2. Radial flow to a well penetrating an extensive confined aquifer.

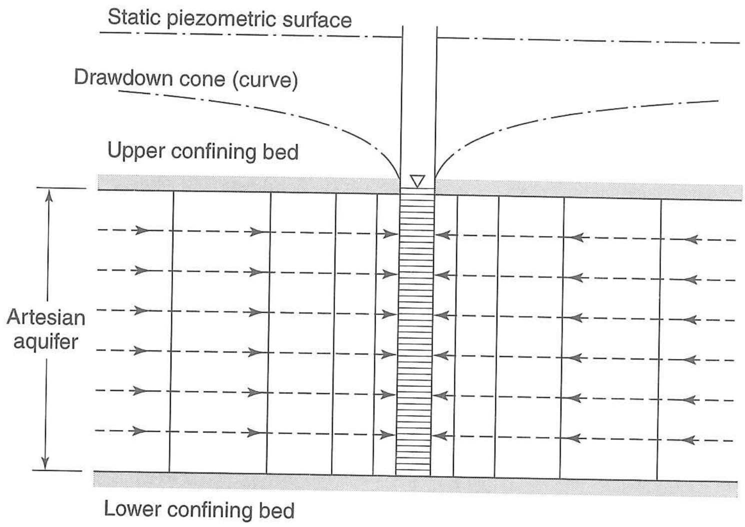


Figure 4.2.3. Distribution of flow to a discharging well in an artesian aquifer—a fully penetrating and 100 percent open hole (from the U.S. Bureau of Reclamation[62]).

that fully penetrates the aquifer. Because any two points define the logarithmic drawdown curve, the method consists of measuring drawdowns in two observation wells at different distances from a well pumped at a constant rate. Theoretically, h_w at the pumped well can serve as one measurement point; however, well losses caused by flow through the well screen and inside the well introduce errors so that h_w should be avoided. The transmissivity is given by

$$T = Kb = \frac{Q}{2\pi(h_2 - h_1)} \ln \frac{r_2}{r_1} \tag{4.2.5}$$

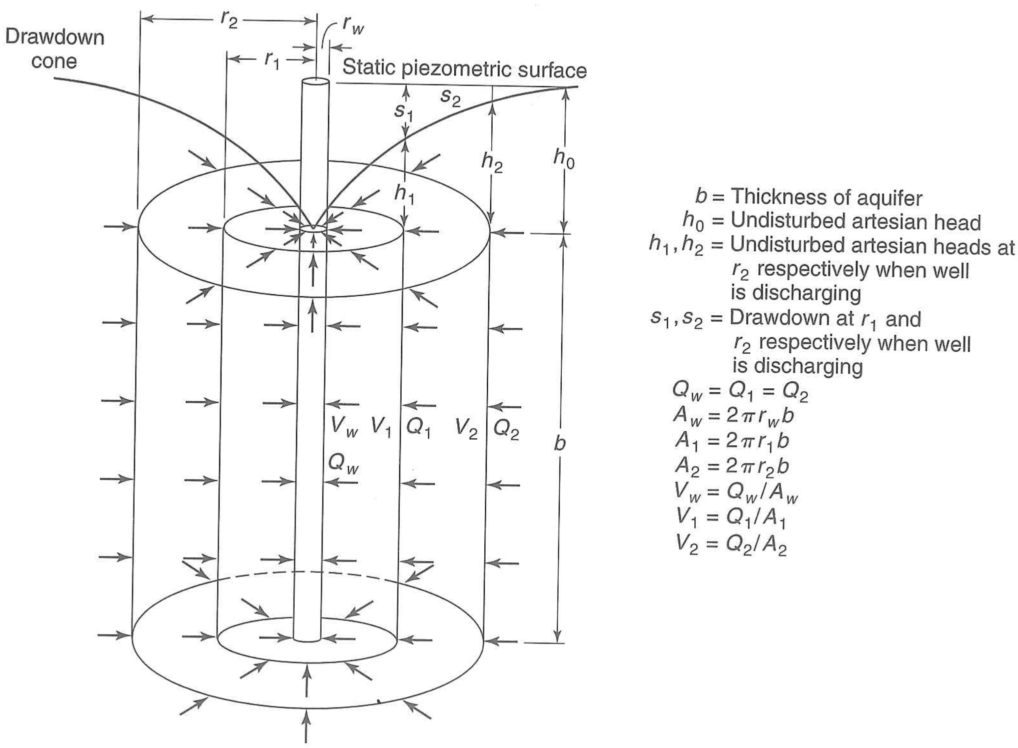


Figure 4.2.4. Flow distribution to a discharging well in an artesian aquifer—a fully penetrating and 100 percent open hole (from the U.S. Bureau of Reclamation[62]).

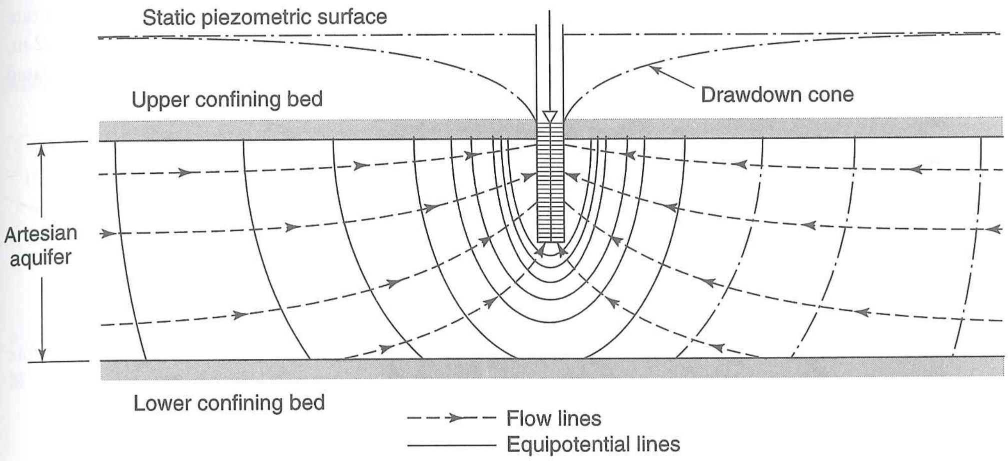


Figure 4.2.5. Distribution of flow to a well in an artesian aquifer—a 50-percent penetrating and open hole (from U.S. Bureau of Reclamation[62]).

where r_1 and r_2 are the distances and h_1 and h_2 are the heads of the respective observation wells.

From a practical standpoint, the drawdown s rather than the head h is measured so that Equation 4.2.5 can be rewritten

$$T = \frac{Q}{2\pi(s_1 - s_2)} \ln \frac{r_2}{r_1} \tag{4.2.6}$$

where s_1 and s_2 are shown in Figure 4.2.2. In order to apply Equation 4.2.6, pumping must continue at a uniform rate for a sufficient time to approach a steady-state condition—that is, one in which the drawdown changes negligibly with time.* The observation wells should be located close enough to the pumping well so that their drawdowns are appreciable and can be readily measured. The derivation assumes that the aquifer is homogeneous and isotropic, is of uniform thickness, and is of infinite areal extent; that the well penetrates the entire aquifer; and that initially the piezometric surface is nearly horizontal.

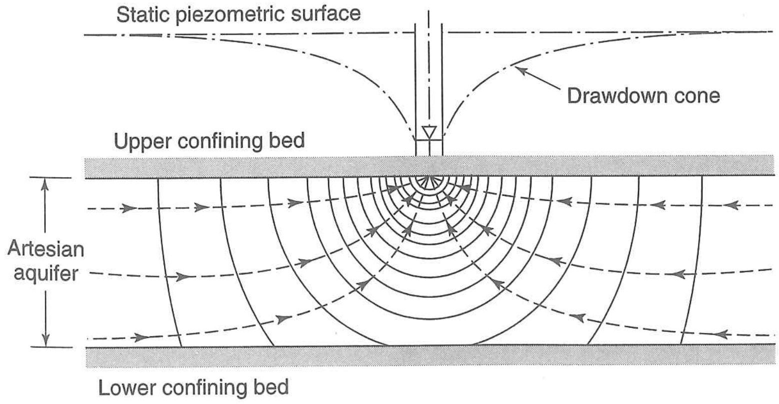


Figure 4.2.6. Distribution of flow to a discharging well—just penetrating to the top of an artesian aquifer. A strong vertical component of flow is established out to a distance approximately equal to the thickness of the aquifer (from U.S. Bureau of Reclamation[62]).

*In fact, the difference in drawdowns ($s_1 - s_2$) becomes essentially constant while both values are still increasing so that Equation 4.2.6 generally gives good results after only a few days of pumping.

EXAMPLE 4.2.1

A well fully penetrates a 25-m thick confined aquifer. After a long period of pumping at a constant rate of 0.05 m^3/s, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Determine the hydraulic conductivity and the transmissivity. What type of unconsolidated deposit would you expect this to be?

SOLUTION

Use Equation 4.2.5 to compute the hydraulic conductivity with $Q = 0.05\ m^3/s$, $r_1 = 50$ m, $r_2 = 150$ m, $s_1 = h_0 - h_1$, and $s_2 = h_0 - h_2$, so $s_1 - s_2 = h_2 - h_1 = 3 - 1.2 = 1.8$ m. $Q = 0.05\ m^3/s = 4320\ m^3/day$, and

$$K = \frac{Q}{2\pi b(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ \text{m}^3/\text{day}}{2\pi(25\ \text{m})(1.8\ \text{m})} \ln\left(\frac{150}{50}\right) = 16.8\ \text{m/day}$$

The transimissivity is $T = Kb = (16.8\ \text{m/day})(25\ \text{m}) = 420\ \text{m}^2/\text{day}$. Referring to Figure 3.2.2 and Table 3.2.1 with $K = 1.94 \times 10^{-4}$ m/s shows that this aquifer is probably a medium clean sand. ■

EXAMPLE 4.2.2

A 1-m diameter well penetrates vertically through a confined aquifer 30 m thick. When the well is pumped at 113 m^3/hr, the drawdown in a well 15 m away is 1.8 m; in another well 50 m away, it is 0.5 m. What is the approximate head in the pumped well for steady-state conditions and what is the approximate drawdown in the well? Also compute the transmissivity of the aquifer and the radius of influence of the pumping well. Take the initial piezometric level as 40 m above the datum.

SOLUTION

First determine the hydraulic conductivity using Equation 4.2.5: $Q = 113\ m^3/hr = 2712\ m^3/day$. Then

$$K = \frac{Q}{2\pi b(s_1 - s_2)} \ln\left(\frac{r_2}{r_1}\right) = \frac{2712\ \text{m}^3/\text{day}}{2\pi(30\ \text{m})(1.8\ \text{m} - 0.5\ \text{m})} \ln\left(\frac{50}{15}\right) = 13.3\ \text{m/day}$$

The transmissivity is $T = Kb = 13.3\ \text{m/day} \times 30\ \text{m} = 400\ \text{m}^2/\text{day}$.

To compute the approximate head, h_w, in the pumped well, rearrange Equation 4.2.5 and use $h_2 = h_0 - s_2 = 40 - 0.5 = 39.5$ m

$$h_w = h_2 - \frac{Q}{2\pi Kb} \ln\left(\frac{r_2}{r_w}\right) = 39.5\ \text{m} - \frac{2712\ \text{m}^3/\text{day}}{2\pi(13.3\ \text{m/day})(30\ \text{m})} \ln\left(\frac{50\ \text{m}}{0.5\ \text{m}}\right) = 34.5\ \text{m}$$

Drawdown is then

$$s_w = h_0 - h_w = 40\ \text{m} - 34.5\ \text{m} = 5.5\ \text{m}$$

The radius of influence (R) of pumping well can be found by rearranging Equation 4.2.5 and solving for r_0 which is R:

$$R = (r_1)\exp\left[\frac{2\pi Kb(h_0 - h_1)}{Q}\right] = (15\ \text{m})\exp\left[\frac{2\pi(13.3\ \text{m/day})(30\ \text{m})(40\ \text{m} - 38.2\ \text{m})}{2712\ \text{m}^3/\text{day}}\right] = 79\ \text{m}$$

■

4.2.2 Unconfined Aquifer

An equation for steady radial flow to a well in an unconfined aquifer also can be derived with the help of the Dupuit assumptions. As shown in Figure 4.2.7, the well completely penetrates the aquifer to the horizontal base and a concentric boundary of constant head surrounds the well. The well discharge is

$$Q = -2\pi r K h \frac{dh}{dr} \tag{4.2.7}$$

which, when integrated between the limits $h = h_w$ at $r = r_w$ and $h = h_0$ at $r = r_0$, yields

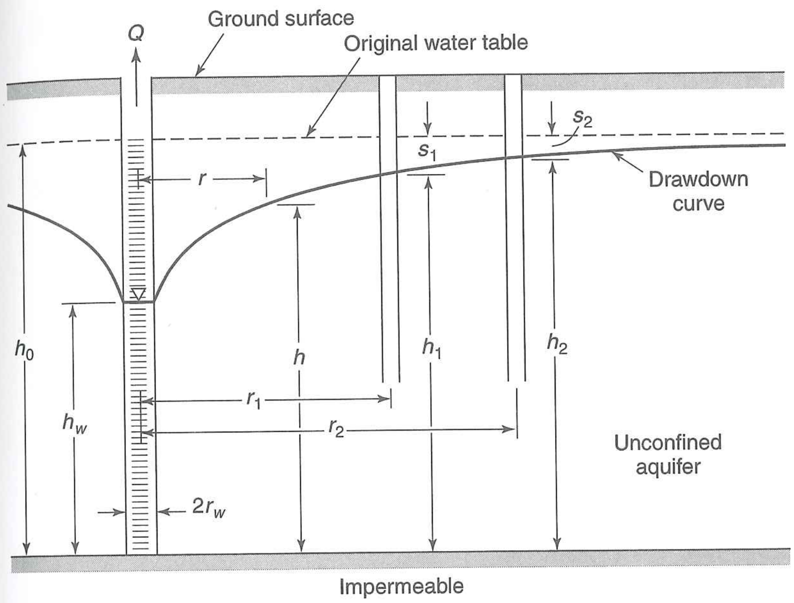


Figure 4.2.7. Radial flow to a well penetrating an unconfined aquifer.

$$Q = \pi K \frac{h_0^2 - h_w^2}{\ln(r_0 / r_w)} \tag{4.2.8}$$

Converting to heads and radii at two observation wells (see Figure 4.2.7),

$$Q = \pi K \frac{h_2^2 - h_1^2}{\ln(r_2/r_1)} \tag{4.2.9}$$

and rearranging to solve for the hydraulic conductivity

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln \frac{r_2}{r_1} \tag{4.2.10}$$

This equation fails to accurately describe the drawdown curve near the well because the large vertical flow components contradict the Dupuit assumptions; however, estimates of K for given heads are good. In practice, drawdowns should be small in relation to the saturated thickness of the unconfined aquifer.

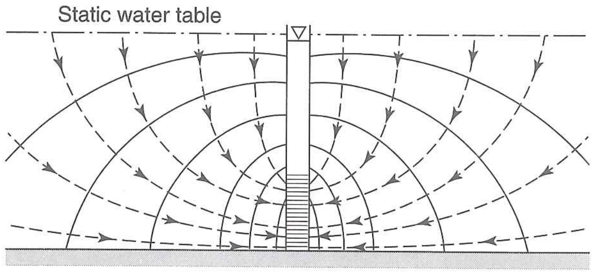
The transmissivity can be approximated from Equation 4.2.10 by

$$T \cong K \frac{h_1 + h_2}{2} \tag{4.2.11}$$

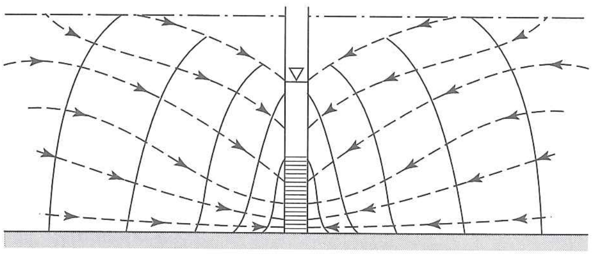
Where drawdowns are appreciable, the heads h_1 and h_2 in Equation 4.2.10 can be replaced by $(h_0 - s_1)$ and $(h_0 - s_2)$, respectively, as shown in Figure 4.2.7. Then the transmissivity for the full thickness becomes[3, 37]

$$T = Kh_0 = \frac{Q}{2\pi\left[\left(s_1 - \frac{s_1^2}{2h_0}\right) - \left(s_2 - \frac{s_2^2}{2h_0}\right)\right]} \ln \frac{r_2}{r_1} \tag{4.2.12}$$

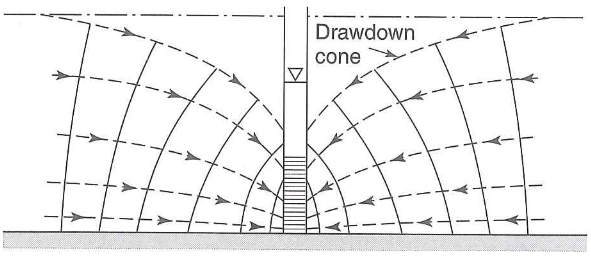
Figure 4.2.8 illustrates the flow distribution in an unconfined (free) aquifer, for a fully penetrating well that has the openings in the lower one-third of the aquifer.



(*a*) Initial stage in pumping a free aquifer. Most water follows a path with a high vertical component from the water table to the screen.



(*b*) Intermediate stage in pumping a free aquifer. Radial component of flow becomes more pronounced but contribution from drawdown cone in immediate vicinity of well is still important.



(*c*) Approximate steady-state stage in pumping a free aquifer. Profile of cone of depression is established. Nearly all water originating near outer edge of area of influence and stable primarily radial flow pattern established.

Figure 4.2.8. Development of flow distribution about a discharging well in a free aquifer—a fully penetrating and 33 percent open hole (from U.S. Bureau of Reclamation[62]).

EXAMPLE 4.2.3

A well penetrates an unconfined aquifer. Prior to pumping the water level (head) is $h_0 = 25$ m. After a long period of pumping at a constant rate of 0.05 m^3/s, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Compute the hydraulic conductivity of the aquifer and the radius of influence of pumping well. What type of deposit is the aquifer material?

SOLUTION

Use Equation 4.2.10 to compute K with $Q = 0.05\ m^3/s = 4320\ m^3/day$, $r_1 = 50$ m, $r_2 = 150$ m, $h_1 = 25 - 3 = 22$ m, and $h_2 = 25 - 1.2 = 23.8$ m.

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ \text{m}^3/\text{day}}{\pi\left(23.8^2 - 22^2\right)} \ln\left(\frac{150\ \text{m}}{50\ \text{m}}\right) = 18.3\ \text{m/day}$$

The deposit is probably a medium clean sand. Equation 4.2.10 is used to compute the radius of influence:

$$R = (r_1)\exp\left[\frac{K\pi\left(h_0^2 - h_1^2\right)}{Q}\right] = (50\ \text{m})\exp\left[\frac{(18.3\ \text{m/day})\pi\left(25^2 - 22^2\right)}{4320\ \text{m}^3/\text{day}}\right] = 327\ \text{m}$$

■

EXAMPLE 4.2.4

A well 0.5 m in diameter penetrates 33 m below the static water table. After a long period of pumping at a rate of 80 m^3/hr, the drawdowns in wells 18 and 45 m from the pumped well were found to be 1.8 and 1.1 m respectively. (a) What is the transmissivity of the aquifer? (b) What is the approximate drawdown in the pumped well? (c) Determine the radius of influence of the pumping well.

SOLUTION

(a) Use Equation 4.2.10 for steady-state radial flow to a well in an unconfined aquifer to compute the hydraulic conductivity, where $Q = 80\ m^3/hr = 1920\ m^3/day$; $h_1 = 33 - 1.8 = 31.2$ m.; $h_2 = 33 - 1.1 = 31.9$ m; $r_2 = 45$ m and $r_1 = 18$ m:

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{1920\ \text{m}^3/\text{day}}{\pi\left(31.9^2 - 31.2^2\right)} \ln\left(\frac{45}{18}\right) = 12.7\ \text{m/day}$$

The transmissivity is computed as $T = Kb = 12.7$ m/day $\times$ 33 m = 418 m^2/day.

(b) Next compute the head and drawdown at the well. First rearrange Equation 4.2.10 to solve for the head at the well:

$$h_w = \sqrt{h_2^2 - \frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right)} = \sqrt{31.2^2 - \frac{1920\ \text{m}^3/\text{day}}{\pi(12.68\ \text{m/day})} \ln\left(\frac{18\ \text{m}}{0.25\ \text{m}}\right)} = 27.7\ \text{m}$$

The drawdown is computed as $s_w = 33$ m $- 27.7$ m $= 5.3$ m.

(c) The radius of influence of the pumping well is computed by rearranging Equation 4.2.5:

$$R = (r_1)\exp\left(\frac{\pi K\left(h_0^2 - h_1^2\right)}{Q}\right) = (45\ \text{m})\exp\left(\frac{\pi(12.68\ \text{m/day})\left(33^2 - 31.9^2\right)}{1920\ \text{m}^3/\text{day}}\right) = 198\ \text{m}$$

■

4.2.3 Unconfined Aquifer with Uniform Recharge

Figure 4.2.9 shows a well penetrating an unconfined aquifer that is recharged uniformly at rate W from rainfall, excess irrigation water, or other surface-water sources. The flow Q toward the well increases as the well is approached, reaching a maximum of Q_w at the well. The increase

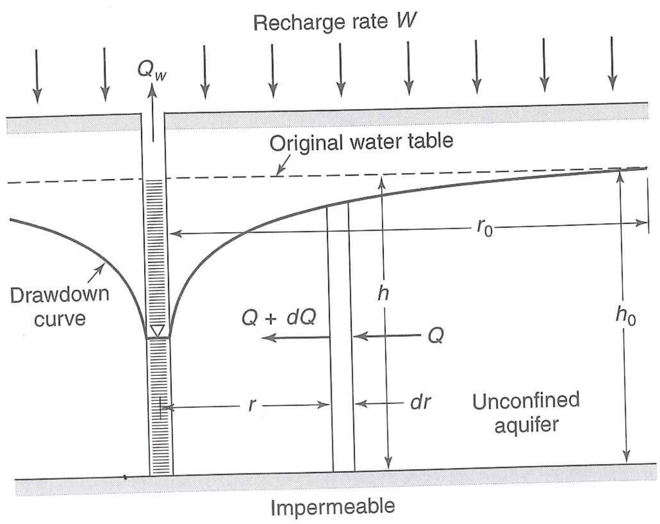


Figure 4.2.9. Steady flow to a well penetrating a uniformly recharged unconfined aquifer.

in flow dQ through a cylinder of thickness dr and radius r comes from the recharged water entering the cylinder from above; hence,

$$dQ = -2\pi r\, dr\, W \tag{4.2.13}$$

Integrating, we obtain

$$Q = -\pi r^2 W + C \tag{4.2.14}$$

but at the well $r \to 0$ and $Q = Q_w$, so that

$$Q = -\pi r^2 W + Q_w \tag{4.2.15}$$

Substituting this flow in the equation for flow to the well (Equation 4.2.7) gives

$$-2\pi r K h \frac{dh}{dr} = -\pi r^2 W + Q_w \tag{4.2.16}$$

Integrating, and noting that $h = h_0$ at $r = r_0$, yield the equation for the drawdown curve:

$$h_0^2 - h^2 = \frac{W}{2K}\left(r^2 - r_0^2\right) + \frac{Q_w}{\pi K}\ln\frac{r_0}{r} \tag{4.2.17}$$

By comparing Equation 4.2.17 with Equation 4.2.8, the effect of the vertical recharge becomes apparent.

It follows that when $r = r_0$, $Q = 0$, so that from Equation 4.2.15

$$Q_w = \pi r_0^2 W \tag{4.2.18}$$

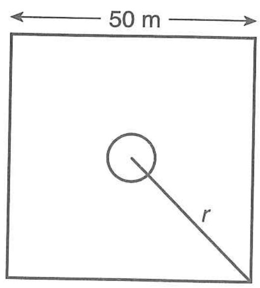
Thus, the total flow of the well equals the recharge within the circle defined by the radius of influence; conversely, the radius of influence is a function of the well pumpage and the recharge rate only. This results in a steady-state drawdown; however, the analysis assumes an idealized circular outer boundary with a constant head and no flow–conditions that rarely occur in the field.

EXAMPLE 4.2.5

A pumping well is to be used to maintain a lowered water table at a construction site. The site is square, 50 m on a side, and the 25-cm diameter well is located at the center of the square, as shown in the figure. The hydraulic conductivity of the unconfined aquifer is estimated to be about 1×10^{-5} m/s or 0.864 m/day. The bottom of the aquifer is approximately horizontal at a depth of 20 m below the ground surface. Under natural conditions, the water table is nearly horizontal at a depth of 1 m below the ground surface and the unconfined aquifer is uniformly recharged at a rate $W = 0.06$ m/day. During the construction period, the water table must be lowered a minimum of 3 m over the site. Assuming steady-state conditions, compute the minimum pumping rate required.

SOLUTION

The given condition is satisfied if the drawdown at any of the corner points is 3 m.



The well discharge is expressed in terms of the radius of influence using Equation 4.2.17 as $Q_w = \pi r_0^2 W = (0.06)\,\pi r_0^2$. Substitute this relationship along with $h_0 = 19$ m, $h = 16$ m, $W = 0.06$ m/day, $K = 0.864$ m/day, and $r = \sqrt{25^2 + 25^2} = 35.35$ m into Equation 4.2.17 to obtain

$$19^2 - 16^2 = \frac{0.06}{2 \times 0.864}\left(35.35^2 - r_0^2\right) + \frac{\left(0.06\pi r_0^2\right)}{\pi \times 0.864}\ln\left(\frac{r_0}{35.35}\right)$$

Solving the above equation using an iterative procedure yields $r_0 \approx 70$ m. The minimum pumping rate is $Q_w = \pi(70^2)\,(0.06) \approx 924\ \text{m}^3\text{/day}$ or $0.01069\ \text{m}^3\text{/s}$. ■

4.3 WELL IN A UNIFORM FLOW

Drawdown curves for well flow presented heretofore have assumed an initially horizontal groundwater surface. A practical situation is that of a well pumping from an aquifer having a uniform flow field, as indicated by a uniformly sloping piezometric surface or water table. Figure 4.3.1 shows sectional and plan views of a well penetrating a confined aquifer with a sloping piezometric surface. It is apparent that the circular area of influence associated with a radial flow pattern becomes distorted; however, for most relatively flat natural slopes the Dupuit radial flow equation can be applied without appreciable error.

For wells pumping from an area with a sloping hydraulic gradient, the hydraulic conductivity can be determined from Equation 4.2.7 by inserting average heads and hydraulic gradients. The resulting expression has the form

$$K = \frac{2Q}{\pi r\left(h_u + h_d\right)\left(i_u + i_d\right)} \tag{4.3.1}$$

for an unconfined aquifer where Q is the pumping rate, h_u and h_d are the saturated thicknesses, and i_u and i_d are the water table slopes at distance r upstream and downstream, respectively, from the well. For a confined aquifer, piezometric slopes replace water table slopes, and $(h_u + h_d)$ is replaced by $2b$ where b is the aquifer thickness.

In Figure 4.3.1, the groundwater divide marking the boundary of the region producing inflow to the well is shown. For a well pumping for an infinite time, the boundary would extend up to the limit of the aquifer. The expression for the boundary of the region producing inflow can be derived by superposition of radial and one-dimensional flow fields to yield

$$-\frac{y}{x} = \tan\left(\frac{2\pi Kbi}{Q}y\right) \tag{4.3.2}$$

where the rectangular coordinates are as shown in Figure 4.3.1 with the origin at the well, b is the aquifer thickness, Q is the discharge rate, i is the natural piezometric slope, and K is hydraulic conductivity. From Equation 4.3.2, the boundary asymptotically approaches the finite limits

$$y_L = \pm\frac{Q}{2Kbi} \tag{4.3.3}$$

as $x \to \infty$. The boundary of the contributing area extends downstream to a stagnation point where

$$x_L = -\frac{Q}{2\pi Kbi} \tag{4.3.4}$$

It follows that the upstream inflow zone equals $2\pi x_L$.

Equations 4.3.1 to 4.3.3 also apply to unconfined aquifers by replacing b by the uniform saturated aquifer thickness h_0, providing the drawdown is small in relation to the aquifer thickness. An important practical application of these equations concerns determining whether an upstream pollution source will affect a nearby pumping well (see Chapter 8).

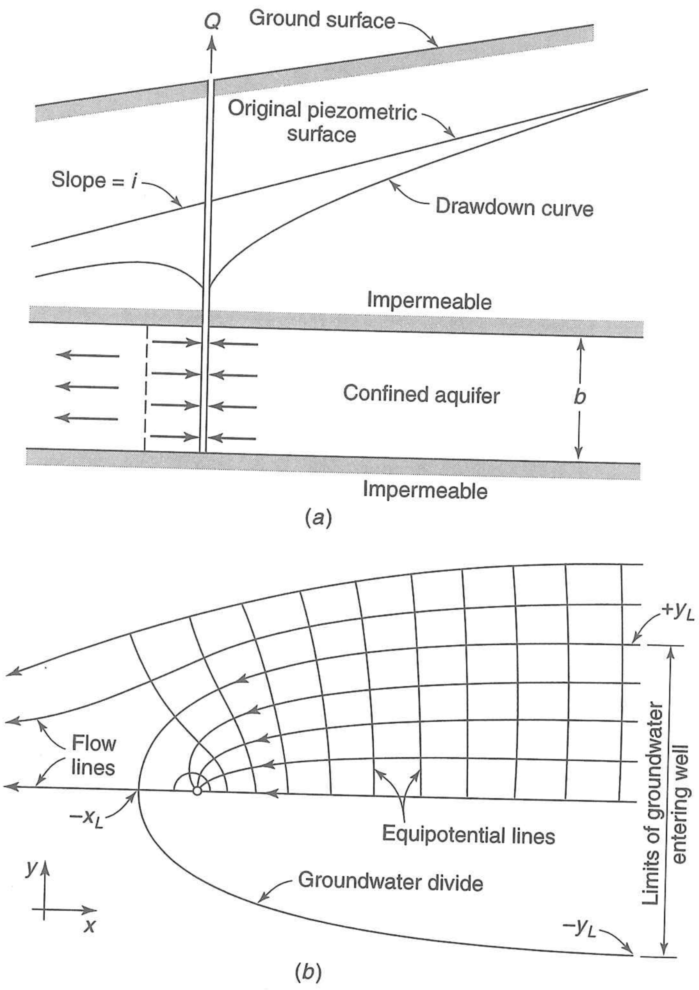


Figure 4.3.1. Flow to a well penetrating a confined aquifer having a sloping plane piezometric surface. (*a*) Vertical section. (*b*) Plan view

EXAMPLE 4.3.1

A fully penetrating production well with a radius of 0.5 m pumps at the rate of 15 L/s from a 35-m thick confined aquifer with a hydraulic conductivity of 20 m/day. If the distance and the observed piezometric head drop between two observation wells were 1000 m and 3 m, respectively, before the production well was installed, determine the longitudinal and transverse limits of groundwater entering the well.

SOLUTION

First determine the slope of the piezometric surface under natural conditions (i.e., before the production well was installed):

$$i = \frac{\Delta h}{\Delta x} = \frac{3\text{ m}}{1000\text{ m}} = 0.003$$

It is assumed that the observation wells were aligned with the groundwater flow direction. Then, using Equations 4.3.3 and 4.3.4, compute the limits of groundwater entering the well on a horizontal plane (i.e., plan view) for $Q = 15$ L/s = 1296 m^3/day:

$$y_L = \pm\frac{Q}{2Kbi} = \pm\frac{1296\text{ m}^3\text{/day}}{2(20\text{ m/day})(35\text{ m})\times 0.003} = \pm 308\text{ m}$$

$$x_L = -\frac{Q}{2\pi Kbi} = -\frac{1296\text{ m}^3\text{/day}}{2\pi(20\text{ m/day})(35\text{ m})\times 0.003} = -98.2\text{ m}$$

A practical result is that contaminant sources farther than 98.2 m downstream of the well or ± 308 m in the transverse direction do not impact the well. ■

4.4 UNSTEADY RADIAL FLOW IN A CONFINED AQUIFER

4.4.1 Nonequilibrium Well Pumping Equation

When a well penetrating an extensive confined aquifer is pumped at a constant rate, the influence of the discharge extends outward with time. The rate of decline of head times the storage coefficient summed over the area of influence equals the discharge. Because the water must come from a reduction of storage within the aquifer, the head will continue to decline as long as the aquifer is effectively infinite; therefore, unsteady, or transient, flow exists. The rate of decline, however, decreases continuously as the area of influence expands.

The applicable differential equation (see Equation 3.9.12) in plane polar coordinates is

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{4.4.1}$$

where h is head, r is radial distance from the pumped well, S is the storage coefficient, T is transmissivity, and t is the time since beginning of pumping. Theis[59] obtained a solution for Equation 4.4.1 based on the analogy between groundwater flow and heat conduction. By assuming that the well is replaced by a mathematical sink of constant strength and imposing the boundary conditions $h = h_0$ for $t = 0$, and $h \rightarrow h_0$ as $r \rightarrow \infty$ for $t \geq 0$, the solution

$$\begin{aligned} s &= \frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-u}du}{u} = \frac{Q}{4\pi T}W(u) \\ &= \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots\right] \end{aligned} \tag{4.4.2}$$

is obtained, where s is drawdown, Q is the constant well discharge, and

$$u = \frac{r^2 S}{4Tt} \tag{4.4.3}$$

Equation 4.4.2 is known as the *nonequilibrium,* or *Theis, equation.* The integral is a function of the lower limit u and is known as an *exponential integral*. It can be expanded as a convergent series as shown in Equation 4.4.2 and is termed the well function, $W(u)$.

Alternatively, using U.S. customary units (gallon-day-foot system) where s is in ft, Q is in gpm, T is in gpd/ft, u is in ft, and t is in days, we have

$$s = \frac{114.6Q}{T}W(u) \tag{4.4.4a}$$

$$u = \frac{1.87r^2 S}{Tt} \quad (t \text{ in days}) \tag{4.4.4b}$$

$$u = \frac{2693r^2 S}{Tt} \quad (t \text{ in minutes}) \tag{4.4.4c}$$

The nonequilibrium equation permits determination of the formation constants S and T by means of pumping tests of wells. The equation is widely applied in practice and is preferred over the equilibrium equation because (1) a value for S can be determined, (2) only one observation well is required, (3) a shorter period of pumping is generally necessary, and (4) no assumption of steady-state flow conditions is required.

The assumptions inherent in Equation 4.4.2 should be emphasized because they are often overlooked in applying the nonequilibrium equation and thereby can lead to erroneous results. The assumptions include:

1. The aquifer is homogeneous, isotropic, of uniform thickness, and of infinite areal extent.
2. Before pumping, the piezometric surface is horizontal.
3. The well is pumped at a constant discharge rate.
4. The pumped well penetrates the entire aquifer, and flow is everywhere horizontal within the aquifer to the well.
5. The well diameter is infinitesimal so that storage within the well can be neglected.
6. Water removed from storage is discharged instantaneously with decline of head.

Seldom, if ever, are these assumptions strictly satisfied, but recognition of them can create an awareness of the approximations involved for employing the nonequilibrium equation under field conditions. Average values of S and T can be obtained in the vicinity of a pumped well by measuring in one or more observation wells the change in drawdown with time under the influence of a constant pumping rate. Because of the mathematical difficulties encountered in applying Equation 4.4.2, or its equivalent, Equation 4.4.4, several investigators have developed simpler approximate solutions that can be readily applied for field purposes. Three methods, by Theis,[59] Cooper and Jacob,[9] and Chow,[7] are described in the following sections with the necessary tables and/or graphs. An illustrative example accompanies each method.

4.4.2 Theis Method of Solution

Equation 4.4.2 may be simplified to

$$s = \left(\frac{Q}{4\pi T}\right)W(u) \tag{4.4.5}$$

where $W(u)$, termed the *well function,* is a convenient symbolic form of the exponential integral. Rewriting Equation 4.4.3 as

$$\frac{r^2}{t} = \left(\frac{4T}{S}\right)u \tag{4.4.6}$$

we can see that the relation between $W(u)$ and u must be similar to that between s and r^2/t because the terms in parentheses in the two equations are constants. Given this similarity, Theis[59] suggested an approximate solution for S and T based on a graphic method of superposition.

A plot on logarithmic paper of $W(u)$ versus u, known as a *type curve,* is prepared. Table 4.4.1 gives values of $W(u)$ for a wide range of u. Values of drawdowns are plotted against values of r^2/t on logarithmic paper of the same size and scale as for the type curve. The observed time–drawdown data are superimposed on the type curve, keeping the coordinate axes of the two curves parallel, and adjusted until a position is found by trial whereby most of the plotted points of the observed data fall on a segment of the type curve. Any convenient point is then selected, and the coordinates of this match point are recorded. With values of $W(u)$, u, s, and r^2/t thus determined, S and T can be obtained from Equations 4.4.5 and 4.4.6.*

In areas where several wells exist near a well being test-pumped, simultaneous readings of s in the wells enable distance–drawdown data to be fitted to a type curve in a manner identical to that for time–drawdown data.

*The computation of r^2/t values can be avoided by plotting s versus t rather than r^2/t. In this case, the type curve must be turned over to obtain coincidence and a match point, but results are identical.

Table 4.4.1 Values of $W(u)$ for Values of u

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

EXAMPLE 4.4.1

Drawdown was measured during a pumping test at frequent intervals in an observation well 200 ft from a well that was pumped at a constant rate of 500 gpm. The data for this pump test is listed in the table. These measurements show that the water level is still dropping after 4,000 minutes of pumping; therefore, analysis of the test data requires use of the Theis nonequilibrium procedure. Determine T and S for this aquifer.

Pump test data	
Time (min)	Drawdown (ft)
1	0.05
2	0.22
3	0.40
4	0.56
5	0.70
7	0.94
10	1.2
20	1.8
40	2.5
100	3.4
300	4.5
1,000	5.6
4,000	7.0

SOLUTION

Step 1. Plot the time–drawdown data on log–log graph paper. The drawdown is plotted on the vertical axis and the time since pumping started on the horizontal axis (not shown).

Step 2. Superimpose this plot on the type curve sheet of the same size and scale as the time-drawdown plot, so that the plotted points match the type curve. The axes of both graphs must be kept parallel.

Step 3. Select a match point, which can be any point in the overlap area of the curve sheets. It is usually most convenient to select a match point where the coordinates on the type curve are known in advance (e.g., $W(u) = 1$ and $1/u = 1$ or $W(u) = 1$ and $1/u = 10$, etc.). Then determine the value of s and t for this match point:

$$W(u) = 1 \qquad s = 1 \text{ ft} \qquad 1/u = 1 \qquad t = 2 \text{ min}$$

Step 4. Determine T

$$T = \frac{114.6\,Q}{s} W(u)$$
$$= \frac{114.6 \times 500}{1} \times 1 = 57300 \text{ gpd / ft}$$

Step 5. Determine S

$$S = \frac{Tt}{\frac{1}{u} \times 2693 r^2}$$
$$= \frac{57300 \times 2}{1 \times 2693 \times 200^2}$$
$$= 1.06 \times 10^{-3}$$

■

EXAMPLE 4.4.2

A well penetrating a confined aquifer is pumped at a uniform rate of 2,500 m^3/day. Drawdowns during the pumping period are measured in an observation well 60 m away; observations of t and s are listed in Table 4.4.2. Using the Theis method determine T and S for this confined aquifer.

SOLUTION

Values of r^2/t in m^2/min are computed and appear in the right column of Table 4.4.2. Values of s and r^2/t are plotted on logarithmic paper. Values of $W(u)$ and u from Table 4.4.1 are plotted on another sheet of logarithmic paper of the same size and scale, and a curve is drawn through the points. The two sheets are superposed and shifted with coordinate axes parallel until the observational points coincide with the curve, as shown in Figure 4.4.1. A convenient match point is selected with $W(u) = 1.00$ and $u = 1 \times 10^{-2}$, so that $s = 0.18$ m and $r^2/t = 150$ m^2/min = 216,000 m^2/day. Thus, from Equation 4.4.5,

$$T = \frac{Q}{4\pi s} W(u) = \frac{2500(1.00)}{4\pi(0.18)} = 1110 \text{ m}^2 \text{ / day}$$

and from Equation 4.4.6,

$$S = \frac{4Tu}{r^2/t} = \frac{4(1110)(1 \times 10^{-2})}{216{,}000} = 0.000206$$

■

Table 4.4.2 Pumping Test Data

(r = 60 m)

t, min	s, m	r^2/t, m^2/min	t, min	s, m	r^2/t, m^2/min
0	0	∞	18	0.67	200
1	0.20	3,600	24	0.72	150
1.5	0.27	2,400	30	0.76	120
2	0.30	1,800	40	0.81	90
2.5	0.34	1,440	50	0.85	72
3	0.37	1,200	60	0.90	60
4	0.41	900	80	0.93	45
5	0.45	720	100	0.96	36
6	0.48	600	120	1.00	30
8	0.53	450	150	1.04	24
10	0.57	360	180	1.07	20
12	0.60	300	210	1.10	17
14	0.63	257	240	1.12	15

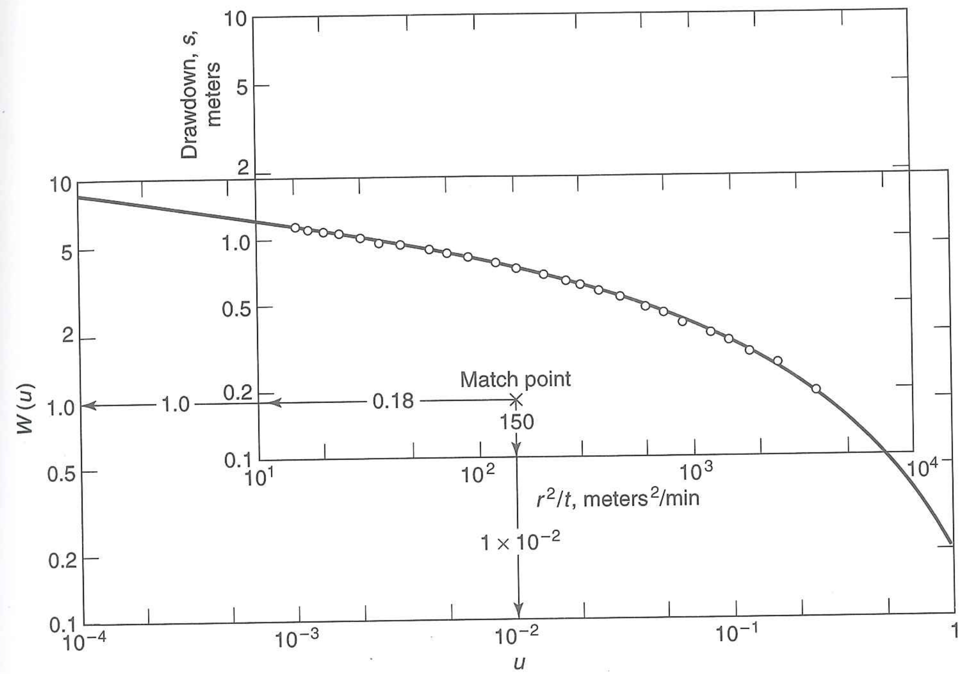


Figure 4.4.1. Theis method of superposition for solution of the nonequilibrium equation.

4.4.3 Cooper–Jacob Method of Solution

It was noted by Cooper and Jacob[9] that for small values of r and large values of t, u is small, so that the series terms in Equation 4.4.4 become negligible after the first two terms. As a result, the drawdown can be expressed by the asymptote

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln\frac{r^2 S}{4Tt}\right) \tag{4.4.7}$$

Rewriting and changing to decimal logarithms reduce this to

$$s = \frac{2.30Q}{4\pi T}\log\frac{2.25Tt}{r^2 S} \tag{4.4.8}$$

Therefore, a plot of drawdown s versus the logarithm of t forms a straight line. Projecting this line to $s = 0$, where $t = t_0$ (see Figure 4.4.2), we have

$$0 = \frac{2.30Q}{4\pi T}\log\frac{2.25Tt_0}{r^2 S} \tag{4.4.9}$$

and it follows that for the above to hold true, log (1) = 0 and

$$\frac{2.25Tt_0}{r^2 S} = 1 \tag{4.4.10}$$

resulting in

$$S = \frac{2.25Tt_0}{r^2} \tag{4.4.11}$$

A value for T can be obtained by noting that if $t/t_0 = 10$, then $\log t/t_0 = 1$; therefore, replacing s by Δs, where Δs is the drawdown difference per log cycle of t, Equation 4.4.8 becomes[33]

$$T = \frac{2.30Q}{4\pi\Delta s} \tag{4.4.12}$$

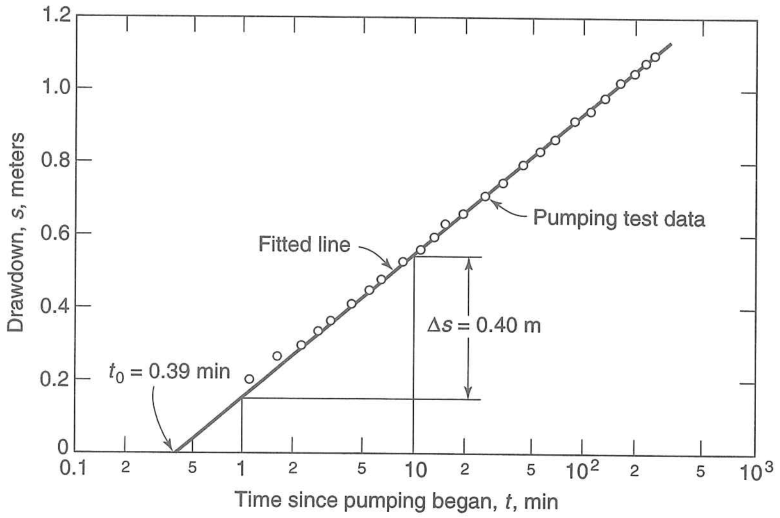


Figure 4.4.2. Cooper–Jacob method for solution of the nonequilibrium equation.

Thus, the procedure is first to solve for T with Equation 4.4.12 and then to solve for S with Equation 4.4.11. The straight-line approximation for this method should be restricted to small values of u ($u < 0.01$) to avoid large errors.

EXAMPLE 4.4.3

Rework Example 4.4.2 using the Cooper–Jacob method.

SOLUTION

From the pumping test data in Table 4.4.2, s and t are plotted on semilogarithmic paper, as shown in Figure 4.4.2. A straight line is fitted through the points, and $\Delta s = 0.40$ m and $t_0 = 0.39$ min $= 2.70 \times 10^{-4}$ day are read. Thus,

$$T = \frac{2.30(2500)}{4\pi(0.40)} = 1144 \text{ m}^2/\text{day}$$

and

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25(1144)(2.70 \times 10^{-4})}{(60)^2} = 0.000193$$

■

EXAMPLE 4.4.4

Using the Cooper–Jacob approximation, compute the rate of piezometric drawdown around a pumping well with respect to time. If the well is pumping at a constant rate of 55 gpm from a sandy confined aquifer with $T = 3{,}600$ ft²/day and $S = 10^{-4}$, what is the time to reach near–steady-state conditions 200 ft from the pumping well? Assume that near–steady-state conditions are achieved when the drawdown rate falls below 0.5 in/hr (based on accuracy of groundwater level measurements with the available equipment). How does the answer change if the transmissivity of the aquifer is 1,200 ft²/day?

SOLUTION

First, we must compute the critical time after which the Cooper–Jacob method becomes valid (i.e., $u < 0.01$) at 200 ft:

$$t \geq \frac{r^2 S}{4Tu} = \frac{(200 \text{ ft})^2 (1 \times 10^{-4})}{4(3600 \text{ ft}^2/\text{day})(0.01)} \rightarrow t \geq 40 \text{ min}$$

The drawdown is approximated by

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln\frac{r^2 S}{4Tt}\right)$$

which can be rearranged to

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln\frac{r^2 S}{4T} + \ln t\right)$$

Taking the derivative of drawdown with respect to time yields

$$\frac{ds}{dt} = \frac{Q}{4\pi T}\frac{1}{t}$$

This relationship implies that according to the Cooper–Jacob approximation, the rate of drawdown is independent of radial distance and is inversely proportional with time. The change in drawdown with respect to time and the time are, respectively,

$$\frac{ds}{dt} = \frac{Q}{4\pi T}\frac{1}{t} = 0.5 \text{ in/hr}$$

$$\frac{10587 \text{ ft}^3/\text{day}}{4\pi\left(3600 \text{ ft}^2/\text{day}\right)}\frac{1}{t} = \frac{0.5}{12} \text{ ft/hr}$$

$$t = 5.6 \text{ hr}$$

Note that the Cooper–Jacob approximation is satisfied so that the near–steady-state conditions at 200 ft will be reached after 5.6 hrs of pumping at this location. If the transmissivity were 1200 ft^2/day, the approximation would be valid when $t \geq 120$ min and the drawdown rate at 200 ft would be negligible after 16.8 hours of pumping. Thus, it would take longer to reach steady conditions with a lower transmissivity. ■

4.4.4 Chow Method of Solution

Chow[7] developed a method of solution with the advantages of avoiding curve fitting and being unrestricted in its application. Again, measurements of drawdown in an observation well near a pumped well are made. The observational data are plotted on semilogarithmic paper in the same manner as for the Cooper–Jacob method. On the plotted curve, choose an arbitrary point and note the coordinates, t and s. Next, draw a tangent to the curve at the chosen point and determine the drawdown difference Δs, in feet, per log cycle of time. Compute $F(u)$ from

$$F(u) = \frac{s}{\Delta s} \tag{4.4.13}$$

and find the corresponding values of $W(u)$ and u from Figure 4.4.3.* Finally, compute the formation constant T by Equation 4.4.5 and S by Equation 4.4.6.

EXAMPLE 4.4.5

Repeat Example 4.4.2 using the Chow method.

SOLUTION

In Figure 4.4.4 data are plotted from Table 4.4.2 and point A is selected on the curve where $t = 6$ min $= 4.2 \times 10^{-3}$ day and $s = 0.47$ m. A tangent is constructed as shown; the drawdown difference per log cycle of time is $\Delta s = 0.38$ m. Then $F(u) = 0.47/0.38 = 1.24$, and from Figure 4.4.3, $W(u) = 2.75$ and $u = 0.038$. Hence,

$$T = \frac{Q}{4\pi s}W(u) = \frac{2500}{4\pi(0.47)}2.75 = 1160 \text{ m}^2/\text{day}$$

and

$$S = \frac{4Ttu}{r^2} = \frac{4(1160)\left(4.2\times10^{-3}\right)(0.038)}{(60)^2} = 0.000206$$ ■

*For $F(u) > 2.0$, $W(u) = 2.30F(u)$, and u is obtained from Table 4.4.1.

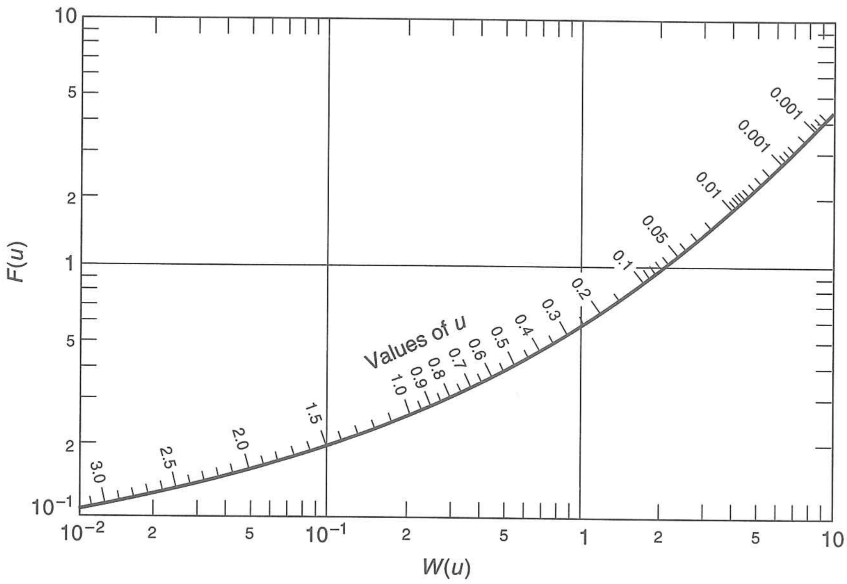


Figure 4.4.3. Relation among $F(u)$, $W(u)$, and u (after Chow[7]).

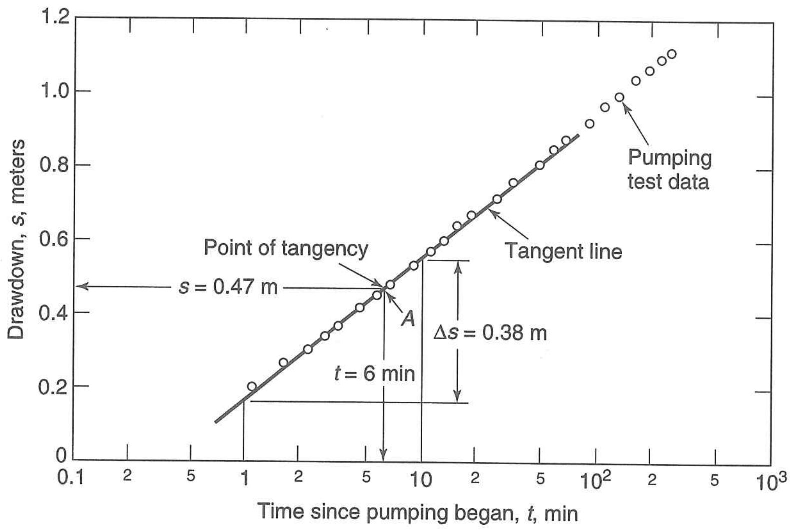


Figure 4.4.4. Chow method for solution of the nonequilibrium equation.

4.4.5 Recovery Test

At the end of a pumping test, when the pump is stopped, the water levels in pumping and observation wells will begin to rise. This is referred to as the *recovery* of groundwater levels, while measurements of drawdown below the original static water level (prior to pumping) during the recovery period are known as *residual drawdowns.* A schematic diagram of change in water level with time during and after pumping is shown in Figure 4.4.5.

It is good practice to measure residual drawdowns because analysis of the data enables transmissivity to be calculated, thereby providing an independent check on pumping test results. Also, costs are nominal in relation to the conduct of a pumping test.* Furthermore, the rate of

*In addition, it should be noted that measurement of the recovery within a pumped well will provide an estimate of transmissivity even without an observation well.

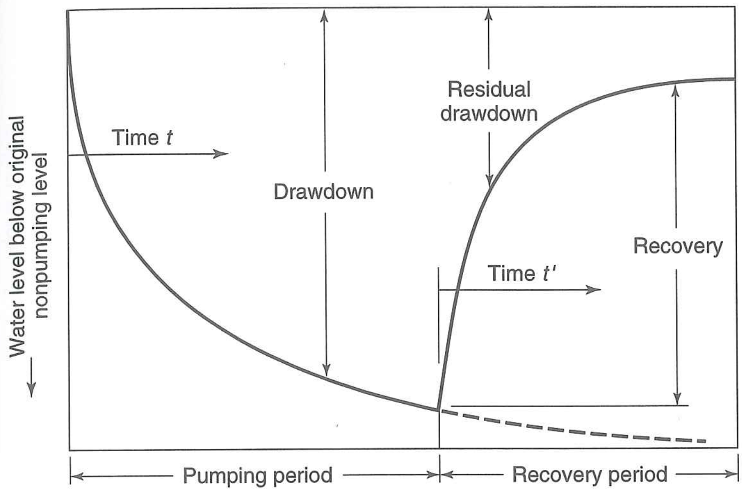


Figure 4.4.5. Drawdown and recovery curves in an observation well near a pumping well.

recharge Q to the well during recovery is assumed constant and equal to the mean pumping rate, whereas pumping rates often vary and are difficult to control accurately in the field.

If a well is pumped for a known period of time and then shut down, the drawdown thereafter will be identically the same as if the discharge had been continued and a hypothetical recharge well with the same flow were superposed on the discharging well at the instant the discharge is shut down. From this principle, Theis[59] showed that the residual drawdown s' can be given as

$$s' = \frac{Q}{4\pi T}\left[W(u) - W(u')\right] \tag{4.4.14}$$

where

$$u = \frac{r^2 S}{4Tt} \quad \text{and} \quad u' = \frac{r^2 S}{4Tt'} \tag{4.4.15}$$

and t and t' are defined in Figure 4.4.5. For r small and t' large, the well functions can be approximated by the first two terms of Equation 4.4.2 so that Equation 4.4.14 can be written as

$$s' = \frac{2.30Q}{4\pi T}\log\frac{t}{t'} \tag{4.4.16}$$

Thus, a plot of residual drawdown s' versus the logarithm of t/t' forms a straight line. The slope of the line equals $2.30Q/4\pi T$ so that for $\Delta s'$, the residual drawdown per log cycle of t/t', the transmissivity becomes

$$T = \frac{2.30Q}{4\pi\Delta s'} \tag{4.4.17}$$

No comparable value of S can be determined by this recovery test method.

EXAMPLE 4.4.6

A well pumping at a uniform rate of 2,500 m^3/day was shut down after 240 min; thereafter, measurements of s' and t' tabulated in Table 4.4.3 were made in an observation well. Determine the transmissivity.

SOLUTION

Values of t/t' are computed, as shown in Table 4.4.3, and then plotted versus s' on semilogarithmic paper (see Figure 4.4.6). A straight line is fitted through the points and $\Delta s' = 0.40$ m is determined; then,

$$T = \frac{2.30Q}{4\pi\Delta s'} = \frac{2.30(2500)}{4\pi(0.40)} = 1140 \text{ m}^2/\text{day}$$

Table 4.4.3 Recovery Test Data (pump shut down at $t = 240$ min)

t', min	t, min	t/t'	s', m
1	241	241	0.89
2	242	121	0.81
3	243	81	0.76
5	245	49	0.68
7	247	35	0.64
10	250	25	0.56
15	255	17	0.49
20	260	13	0.55
30	270	9	0.38
40	280	7	0.34
60	300	5	0.28
80	320	4	0.24
100	340	3.4	0.21
140	380	2.7	0.17
180	420	2.3	0.14

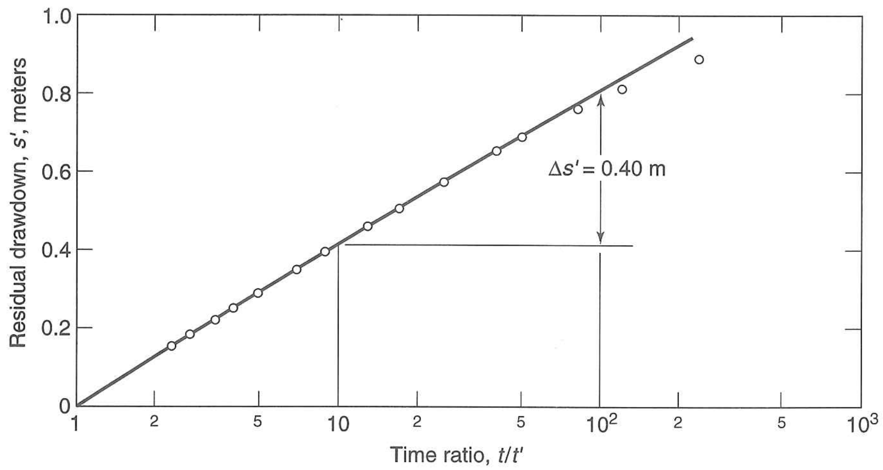


Figure 4.4.6. Recovery test method for solution of the nonequilibrium equation.

4.5 UNSTEADY RADIAL FLOW IN AN UNCONFINED AQUIFER

The previous solution methods for the nonequilibrium equation applied to pumping tests in confined aquifers can also be applied to unconfined aquifers provided that the basic assumptions are satisfied. In general, if the drawdown is small in relation to the saturated thickness, good approximations are possible.[53]

Where drawdowns are significant, the assumption that water released from storage is discharged instantaneously with decline of head is frequently violated in unconfined aquifers. Pumping test data reveal that as a water table is lowered, gravity drainage of water from the unsaturated zone proceeds at a variable rate, known as *delayed yield*.[4, 14, 42] In a series of contributions, Boulton[4–6] developed special type curves for analyzing pumping test data of unconfined aquifers and for taking account of delayed yield.[65] These time–drawdown curves of delayed yield are shown in Figure 4.5.1. The interpretation of any one curve can be considered in three time segments. In the first segment, measured in seconds to a few minutes, water is released essentially instantaneously from storage by compaction of the aquifer and by expan-

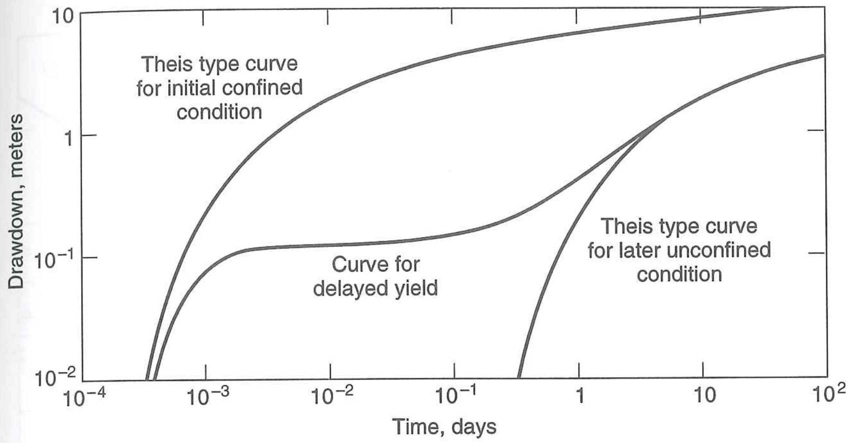


Figure 4.5.1. Type curves of drawdown versus time illustrating the effect of delayed yield for pumping tests in unconfined aquifers (after U.S. Bureau of Reclamation[61]).

sion of entrapped air. This portion of the curve can be fitted by a type curve with a storage coefficient equivalent to that of a confined aquifer. The second segment displays a flattening in slope caused by gravity drainage replenishment from the pore space above the cone of depression. Finally, in the third segment an equilibrium is approached between gravity drainage and the rate of decline of the water table. This condition occurs after several minutes to several days and can be fitted by a type curve with a storage coefficient for an unconfined aquifer.

From a water production standpoint, the storage coefficient obtained from the third segment of the curve in Figure 4.5.1, which is the specific yield, is the most reliable and hence most important. For simplicity a pumping test should be continued sufficiently long enough to define the third segment of the curve; then, by applying one of the solution methods previously described for the nonequilibrium equation, a value for S can be obtained.

The minimum length of pumping test to achieve an accurate estimate of S in an unconfined aquifer depends on the transmissivity of the aquifer. One approach, based on an empirical study of various alluvial aquifer materials, is given by the graphs in Figure 4.5.2. The *delay index* t_d is estimated in Figure 4.5.2a from the composition of aquifer material. Then knowing the distance r between pumping and observation wells, and estimating S and T, an approximation to the minimum pumping time t_{min} can be calculated from Figure 4.5.2b.

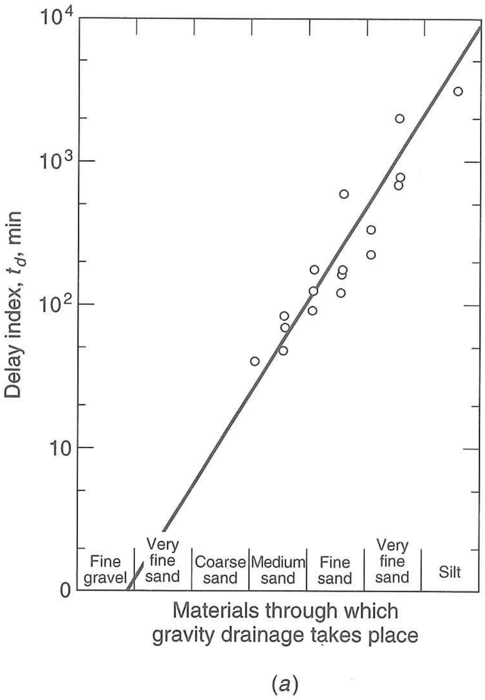
Another approach is simply to ensure that the pumping test duration exceeds the following suggested guidelines[61].

Predominant aquifer material	Minimum pumping time, hours
Silt and clay	170
Fine sand	30
Medium sand and coarser materials	4

Prickett[47] developed a type curve solution for water table conditions based upon Boulton[5]. The following equation for drawdown in an unconfined aquifer with fully penetrating wells and a constant discharge condition was presented later by Neuman:[43]

$$s = \frac{Q}{4\pi T} W(u_a, u_y, \eta) \tag{4.5.1}$$

where $u_a \doteq \dfrac{r^2 S}{4Tt}$ (applicable for early drawdown data) (4.5.2)



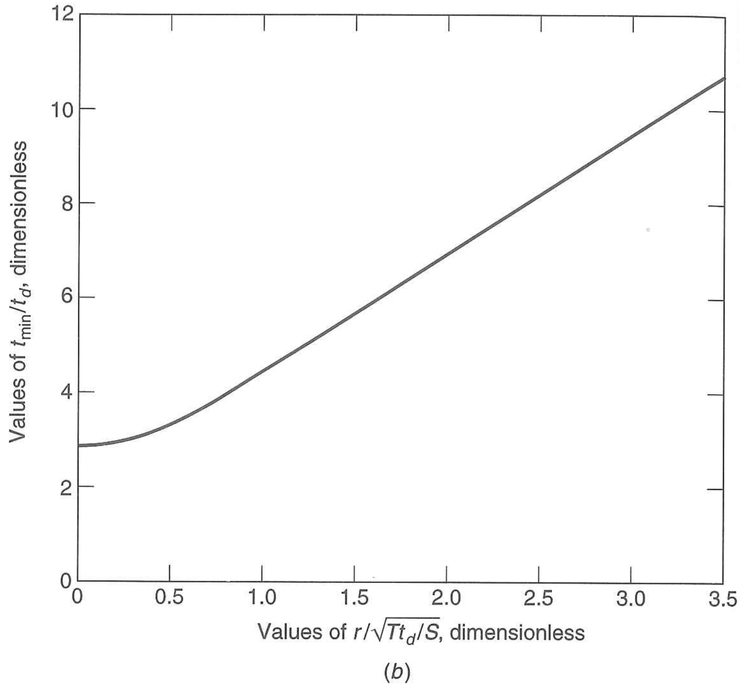


Figure 4.5.2. Empirical method for estimating the minimum length of a pumping test in an unconfined aquifer (after Prickett[47]). (*a*) Empirical relation of delay index to character of materials through which gravity drainage occurs (*b*) Curve for estimating the minimum time t_{min} at which effects of delayed gravity drainage cease to influence drawdown of a pumping well in an unconfined aquifer.

$$u_y = \frac{r^2 S_y}{4Tt} \quad \text{(applicable for later drawdown data)} \tag{4.5.3}$$

$$\eta = \frac{r^2 K_z}{b^2 K_h} \tag{4.5.4}$$

$W(u_a, u_y, \eta)$ is the *unconfined well function* (Figure 4.5.3); K_h and K_v are the horizontal and vertical hydraulic conductivities for an isotropic aquifer $K_v = K_z$ and $\eta = r^2/b^2$; and b is the initial saturated thickness of the unconfined aquifer.

EXAMPLE 4.5.1

(adapted from U.S. Department of the Interior)[62].

A well pumping at 144.4 ft^3/min fully penetrates an unconfined aquifer with a saturated thickness of 25 ft. Determine the transmissivity, storativity, specific yield, and horizontal and vertical hydraulic conductivities using the tabulated time–drawdown data in Table 4.5.1 for an observation well located 73 ft away.

SOLUTION

Time–drawdown data (Table 4.5.1) are plotted in Figure 4.5.4, which shows the typical three phases of drawdown for unconfined aquifers. The early drawdown versus time data fit best on the type-a curves for $\eta = 0.06$. The selected match point in Figure 4.5.4 has the following coordinates: ($t = 0.17$ min, $s = 0.57$ ft) and ($1/u_a = 1.0$, $W(u_a, u_y, \eta) = 1.0$). Using Equation 4.5.1 with a discharge of $Q = 144.4$ ft^3/min, we find the transmissivity to be

$$T = \frac{Q}{4\pi s} W(u_a, u_y, \eta) = \frac{(144.4 \text{ ft}^3/\text{min})}{4\pi(0.57 \text{ ft})}(1.0) = 20.16 \text{ ft}^2/\text{min} \cong 29{,}900 \text{ ft}^2/\text{day}$$

Next, the storativity value is computed using Equation 4.5.2:

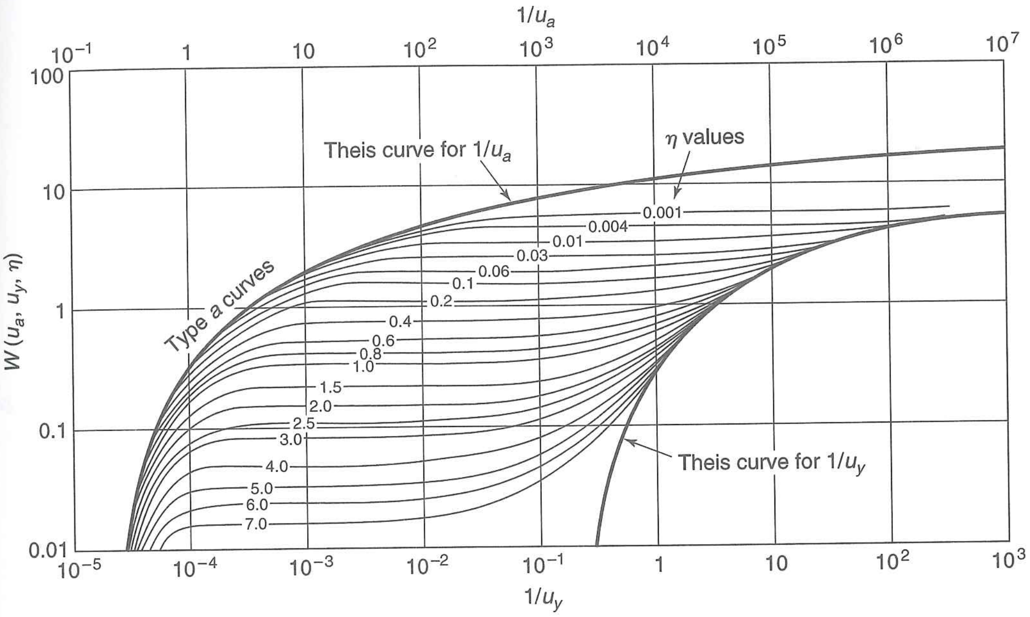


Figure 4.5.3. Theoretical curves of $W(u_a,u_y,\eta)$ versus $1/u_a$ and $1/u_y$ for an unconfined aquifer (after Neuman[43]).

Table 4.5.1 Time–Drawdown Data for Example 4.5.1.

t (min)	*s*, feet	*t* (min)	*s*, feet	*t* (min)	*s*, feet	*t* (min)	*s*, feet
0.165	0.12	1.68	0.82	10	1.02	200	1.52
0.25	0.195	1.85	0.84	12	1.03	250	1.59
0.34	0.255	2	0.86	15	1.04	300	1.65
0.42	0.33	2.15	0.87	18	1.05	350	1.7
0.5	0.39	2.35	0.9	20	1.06	400	1.75
0.58	0.43	2.5	0.91	25	1.08	500	1.85
0.66	0.49	2.65	0.92	30	1.13	600	1.95
0.75	0.53	2.8	0.93	35	1.15	700	2.01
0.83	0.57	3	0.94	40	1.17	800	2.09
0.92	0.61	3.5	0.95	50	1.19	900	2.15
1	0.64	4	0.97	60	1.22	1,000	2.2
1.08	0.67	4.5	0.975	70	1.25	1,200	2.27
1.16	0.7	5	0.98	80	1.28	1,500	2.35
1.24	0.72	6	0.99	90	1.29	2,000	2.49
1.33	0.74	7	1	100	1.31	2,500	2.59
1.42	0.76	8	1.01	120	1.36	3,000	2.66
1.5	0.78	9	1.015	150	1.45		

$$S = \frac{4Tu_a t}{r^2} = \frac{4\left(20.16\ \text{ft}^2/\text{min}\right)(1.0)(0.17\ \text{min})}{(73\ \text{ft})^2} = 0.00257$$

Moving the data curve to the right on the type curve to the best late-time match (for $\eta = 0.06$) where $s = 0.57$ ft (see the match point on Figure 4.5.5) yields ($t = 13$ min, $s = 0.57$ ft) and ($1/u_y = 0.1$, $W(u_y, \eta) = 1$). Inserting the appropriate values in Equation 4.5.1 does not change the transmissivity estimate, but using Equation 4.5.3 yields

$$S_y = \frac{4Tu_y t}{r^2} = \frac{4\left(20.16\ \text{ft}^2/\text{min}\right)(0.1)(13\ \text{min})}{(73\ \text{ft})^2} = 0.02$$

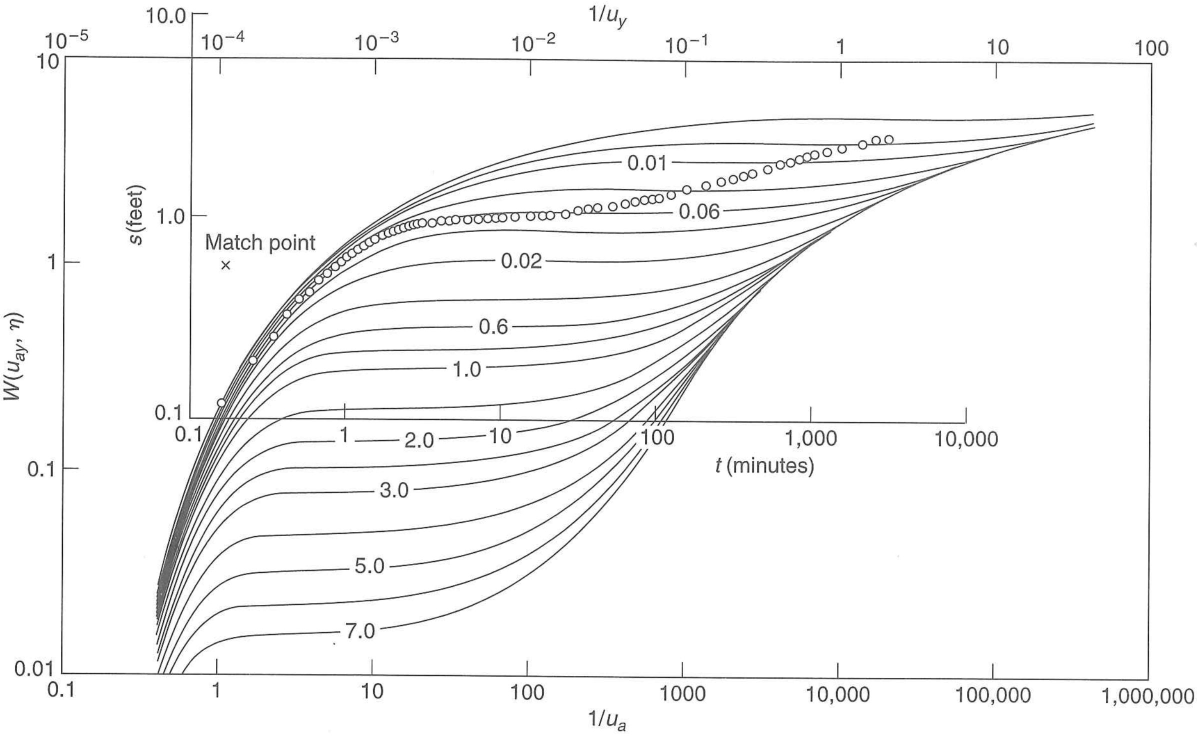


Figure 4.5.4. Type-a curve matching for Example 4.5.1.

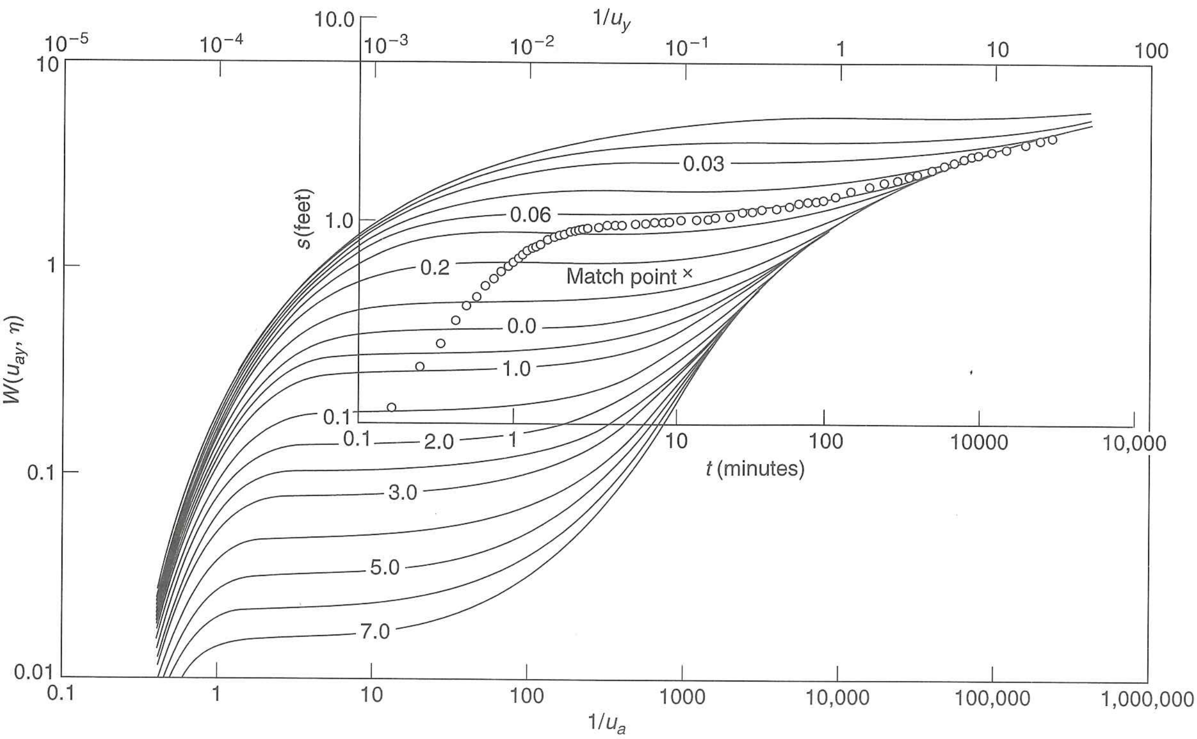


Figure 4.5.5. Type-y curve matching for Example 4.5.1.

The horizontal hydraulic conductivity, K_r or K_h, is computed using

$$K_h = \frac{T}{b} = \frac{20.16\ \text{ft}^2/\text{min}}{25\ \text{ft}^2} = 0.806\ \text{ft/min or } 1160\ \text{ft/day}$$

and the vertical hydraulic conductivity, K_z or K_v, is computed using Equation 4.5.4:

$$K_v = \frac{\eta b^2 K_h}{r^2} = \frac{(0.06)(25\ \text{ft})^2(1160\ \text{ft/day})}{(73\ \text{ft})^2} = 8.2\ \text{ft/day}$$

■

4.6 UNSTEADY RADIAL FLOW IN A LEAKY AQUIFER

When a leaky aquifer, as shown in Figure 4.6.1, is pumped, water is withdrawn both from the aquifer and from the saturated portion of the overlying aquitard, or semipervious layer. Lowering the piezometric head in the aquifer by pumping creates a hydraulic gradient within the aquitard; consequently, groundwater migrates vertically downward into the aquifer. The quantity of water moving downward is proportional to the difference between the water table and the piezometric head[8, 29, 58]

Steady-state flow is possible to a well in a leaky aquifer because of the recharge through the semipervious layer. The equilibrium will be established when the discharge rate of the pump equals the recharge rate of vertical flow into the aquifer, provided that the water table remains constant. Solutions for this special steady-state situation are available,[25, 33] but a more general analysis for unsteady flow follows.

When pumping starts from a well in a leaky aquifer, drawdown of the piezometric surface can be given by[19, 21, 25]

$$s = \frac{Q}{4\pi T} W(u, r/B) \tag{4.6.1}$$

where s, Q, and r are defined in Figure 4.6.1, and again

$$u = \frac{r^2 S}{4Tt} \tag{4.6.2}$$

The quantity r/B is given by

$$\frac{r}{B} = \frac{r}{\sqrt{T/(K'/b')}} \tag{4.6.3}$$

where T is the transmissivity of the leaky aquifer, K' is the vertical hydraulic conductivity of the aquitard, and b' is the thickness of the saturated semipervious layer (see Figure 4.6.1). Values of the function $W(u, r/B)$ were tabulated by Hantush.[19] It can be noted that Equation 4.6.1 has the form of the Theis equation (see Equation 4.4.5); in fact, for a confined aquifer, $K' \rightarrow 0$, so that $B \rightarrow \infty$ and $r/B \rightarrow 0$, thereby reducing Equation 4.6.1 to the Theis equation.

Employing this analogy and the Theis method of solution, Walton[64] prepared a family of type curves for $W(u, r/B)$ as presented in Figure 4.6.2. Here values of $W(u, r/B)$ are plotted against $1/u$ for various values of r/B. On another sheet of logarithmic paper of the same scale, s versus t is plotted. Superimposing the two sheets while keeping the coordinate axes parallel, a position is found where most of the data points fall on one of the type curves. Selecting any convenient match point, values of $W(u, r/B)$, $1/u$, s, and t are noted. T is then found from

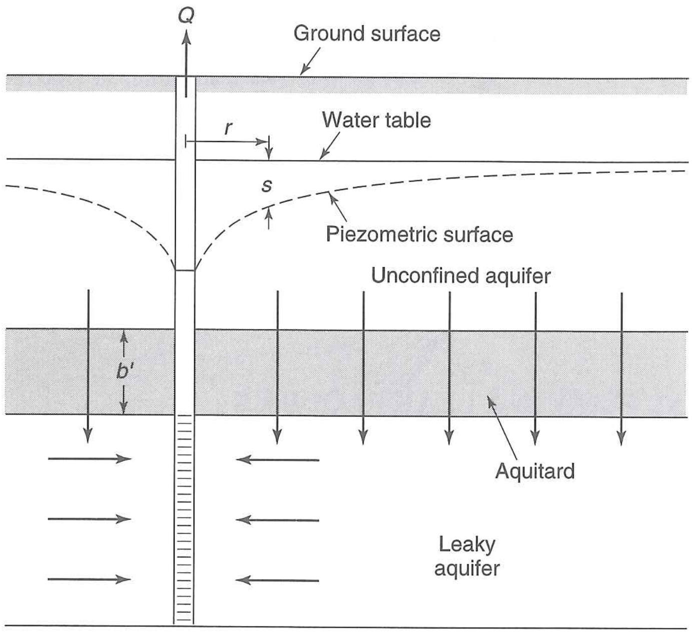


Figure 4.6.1. Well pumping from a leaky aquifer.

Equation 4.6.1 and S from Equation 4.6.2. Finally, from the value of r/B belonging to the type curve of best fit, it is possible to calculate K'/b' from Equation 4.6.3; and if b' is known from field conditions, K' can be evaluated.

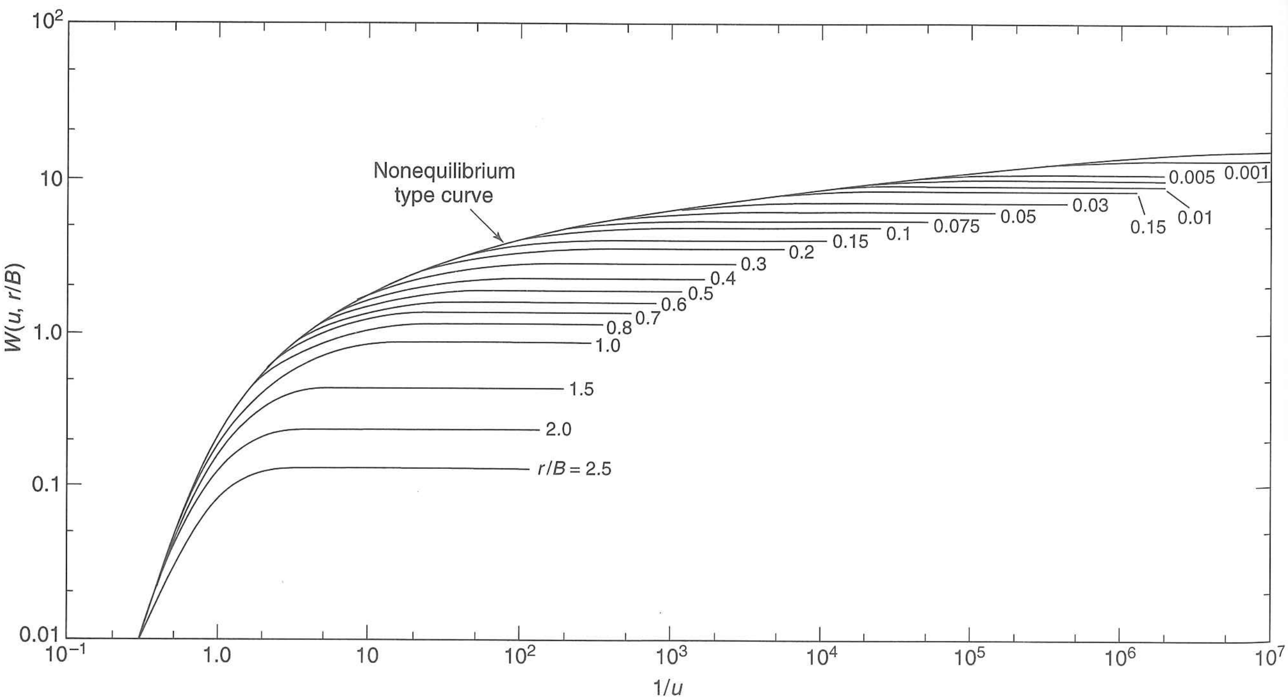


Figure 4.6.2. Type curves for analysis of pumping test data to evaluate storage coefficient and transmissivity of leaky aquifers (after Walton[64]).

EXAMPLE 4.6.1

(adapted from U.S. Department of the Interior)[62].

A well pumping at 600 ft^3/min fully penetrates a confined aquifer overlain by a leaky confining layer of 14-ft thickness. Using the tabulated time–drawdown data for an observation well 40 ft away from the pumping well, estimate the transmissivity and storage coefficient of the confined aquifer, and the permeability of the aquitard. Assume that the confining layer does not release water from storage.

Time (min)	Drawdown (ft)	Time (min)	Drawdown (ft)
0	0.00	80	12.02
2	5.65	90	12.26
4	6.96	100	12.33
6	7.72	110	12.37
8	8.00	120	12.41
10	8.71	150	12.69
15	9.47	180	12.85
20	9.99	210	13.09
25	10.35	240	13.13
30	10.70	270	13.25
40	11.14	300	13.33
50	11.46	360	13.37
60	11.62	420	13.41
70	11.86		

SOLUTION

The time–drawdown field data were superimposed on the family type curves for leaky aquifers (Figure 4.6.3). Comparison shows that the best fit occurs for r/B = 0.03. The coordinates of the match point selected are

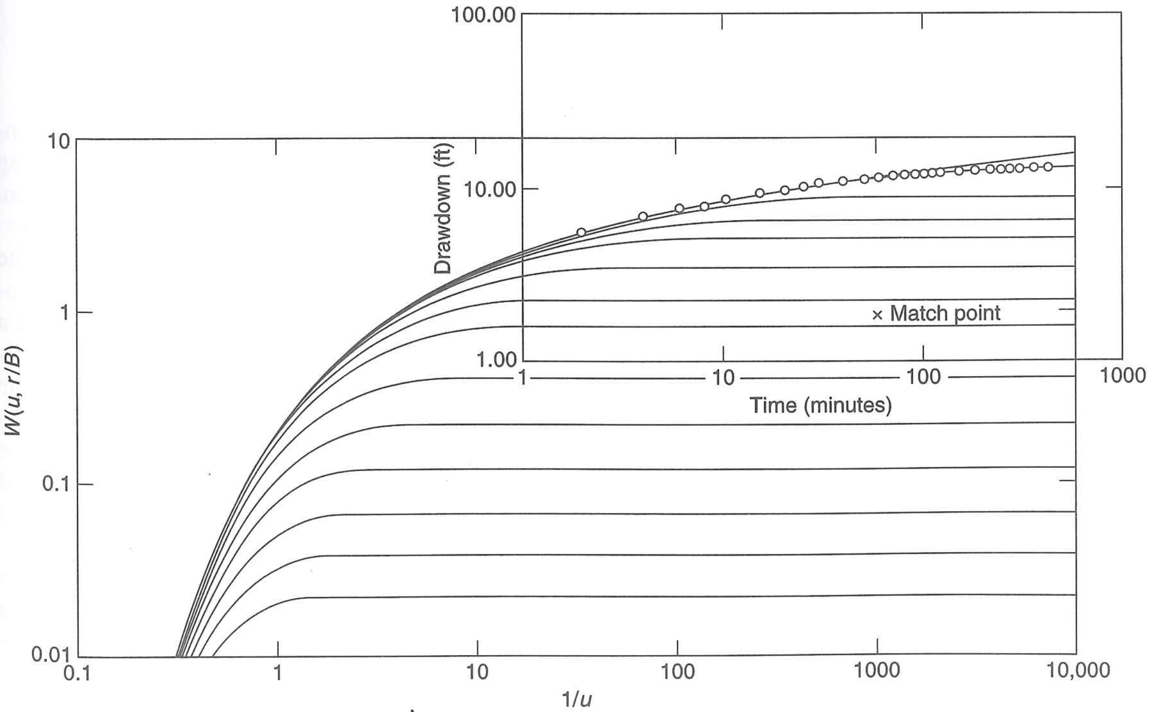


Figure 4.6.3. Leaky type curve matching for Example 4.6.1.

$$\frac{1}{u} = 1000, \qquad W\left(u, \frac{r}{B}\right) = 1.0$$

$$t = 59 \text{ min}, \qquad s = 1.93 \text{ ft}$$

Next we must perform the following unit conversions in order to obtain the transmissivity in units of ft^2/day and hydraulic conductivity of the aquitard in units of ft/day for $Q = 600$ ft^3/min = 864,000 ft^3/day and $t = 59$ min = 0.041 days. The transmissivity and storage coefficient of the confined aquifer are computed using Equations 4.6.1 and 4.6.2 rearranged respectively as

$$T = \frac{Q}{4\pi s} W(u, r/B) = \frac{864{,}000 \text{ ft}^3/\text{day}}{4\pi(1.93 \text{ ft})}(1.0) = 35{,}624 \text{ ft}^2/\text{day}$$

$$S = \frac{4Ttu}{r^2} = \frac{4(35{,}624 \text{ ft}^2/\text{day})(0.041 \text{ days})(0.001)}{(40 \text{ ft})^2} = 0.00365$$

The hydraulic conductivity of the aquitard is computed by rearranging Equation 4.6.3

$$K' = \frac{Tb'(r/B)^2}{r^2} = \frac{(35{,}624 \text{ ft}^2/\text{day})(14 \text{ ft})(0.03)^2}{(40 \text{ ft})^2} = 0.28 \text{ ft/day}$$

■

4.7 WELL FLOW NEAR AQUIFER BOUNDARIES

Where a well is pumped near an aquifer boundary, the assumption that the aquifer is of infinite areal extent no longer holds. Analysis of this situation involves the principle of superposition by which the drawdown of two or more wells is the sum of the drawdowns of each individual well. By introducing imaginary (or *image*) wells, an aquifer of finite extent can be transformed into an infinite aquifer so that the solution methods previously described can be applied.

4.7.1 Well Flow Near a Stream

An example of the usefulness of the method of images is the situation of a well near a perennial stream.[16, 22, 30] It is desired to obtain the head at any point under the influence of pumping at a constant rate Q and to determine what fraction of the pumpage is derived from the stream. Sectional views are shown in Figure 4.7.1 of the real system and an equivalent imaginary system. Note in Figure 4.7.1b that an imaginary recharge well* has been placed directly opposite and at the same distance from the stream as the real well. This image well operates simultaneously and at the same rate as the real well so that the buildup (increase of head around a recharge well) and drawdown of head along the line of the stream exactly cancel. This furnishes a constant head along the stream, which is equivalent to the constant elevation of the stream forming the aquifer boundary. Thus, in the plan view of the resulting flow net, illustrated by Figure 4.7.2, a single equipotential line is coincident with the axis of the stream. The resultant asymmetrical drawdown of the real well is given at any point by the algebraic sum of the drawdown of the real well and the buildup of the recharge well, as if these wells were located in an infinite aquifer.

Examples of *hydraulically equivalent aquifer systems* bounded by streams with various configurations are shown in Figure 4.7.3. Note that combinations of both image recharge and pumping wells are required.[33] For the single stream in Figure 4.7.3*a*, the steady-state drawdown at any point (x,y) is given by

*A *recharge well* is a well through which water is added to an aquifer; hence, it is the reverse of a pumping well.

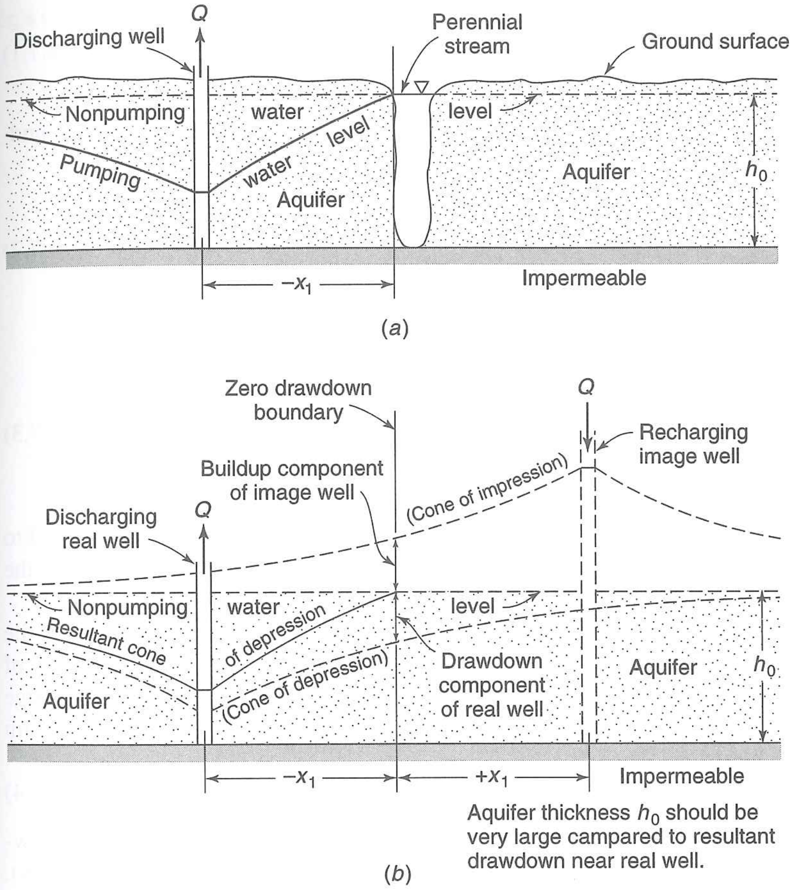


Figure 4.7.1. Sectional views. (*a*) Discharging well near a perennial stream (*b*) Equivalent hydraulic system in an aquifer of infinite areal extent (after Ferris et al.[15])

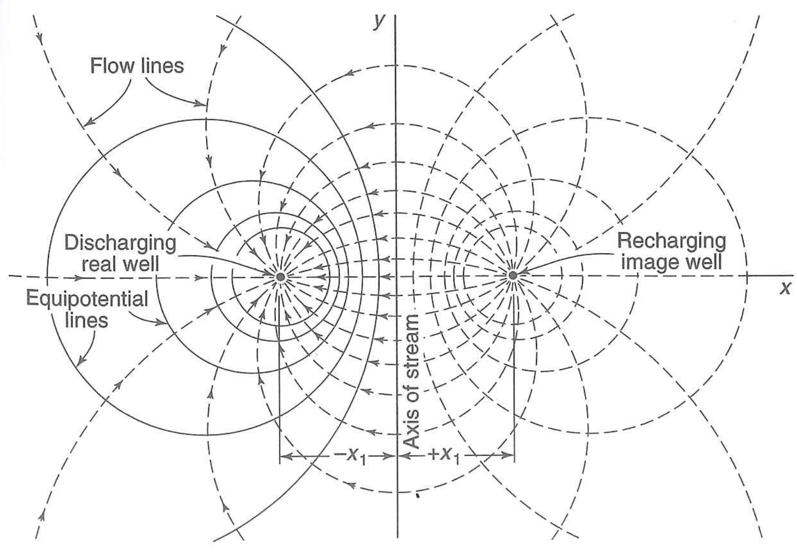


Figure 4.7.2. Flow net for discharging real well and recharging image well (after Ferris, et al.[15]).

$$s = \frac{Q}{4\pi T}\ln\frac{(x+x_w)^2+(y-y_w)^2}{(x-x_w)^2+(y-y_w)^2} \tag{4.7.1}$$

where (x_w, y_w) are the coordinates of the pumped well. Similarly, for the right-angle boundaries of Figure 4.7.3*b*,

$$s = \frac{Q}{4\pi T}\ln\frac{\left[(x-x_w)^2+(y+y_w)^2\right]\left[(x+x_w)^2+(y-y_w)^2\right]}{\left[(x-x_w)^2+(y-y_w)^2\right]\left[(x+x_w)^2+(y+y_w)^2\right]} \tag{4.7.2}$$

And for the strip aquifer bounded by two straight parallel streams (see Figure 4.7.3*c*),

$$s = \frac{Q}{4\pi T}\ln\frac{\cosh\dfrac{\pi(y-y_w)}{2a}+\cosh\dfrac{\pi(x+x_w)}{2a}}{\cosh\dfrac{\pi(y-y_w)}{2a}-\cosh\dfrac{\pi(x+x_w)}{2a}} \tag{4.7.3}$$

and the angles are expressed in radians. Actually, in Figure 4.7.3*c*, the image wells extend to infinity; however, in practice it is only necessary to include pairs of image wells closest to the real well because others have a negligible influence on the drawdown.

The water level in the wells will draw down initially only under the influence of the pumped well. After a time the effects of the recharge boundary will cause the time rate of drawdown to decrease and eventually reach equilibrium conditions. This occurs when recharge equals the pumping rate, as illustrated in Figure 4.7.4. The total drawdown for equilibrium conditions can be expressed as

$$s_r = s_p - s_i \tag{4.7.4}$$

in which s_r is the drawdown in an observation well near a recharge boundary, s_p is the drawdown due to the pumped well, and s_i is the buildup due to the image well (recharge boundary). The drawdown equation can be written as

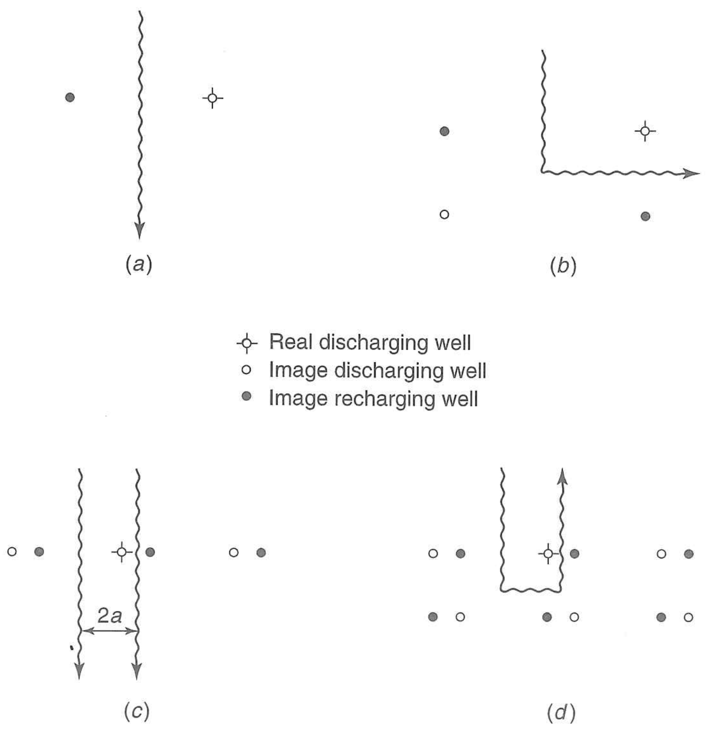


Figure 4.7.3. Image well systems for aquifers bounded by streams of various geometries. (*a*) Unidirectional stream (*b*) Rectangular stream (*c*) Two parallel streams (*d*) U-shaped stream Theoretically, image wells in (*c*) and (*d*) extend left and right to infinity.

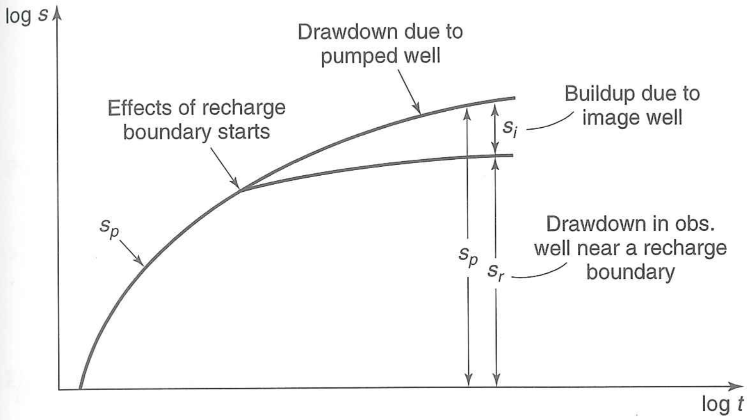


Figure 4.7.4. Recharge boundary effects on time–drawdown curve.

$$s_r = \frac{Q}{4\pi T}\left[W(u_p) - W(u_i)\right] \tag{4.7.5}$$

where Q is the constant pumping rate [L^3/T], T is the transmissivity [L^2/T], $W(u_p)$ and $W(u_i)$ are dimensionless, u_p and u_i are

$$u_p = \frac{r_p^2 S}{4Tt_p} \quad \text{and} \quad u_i = \frac{r_i^2 S}{4Tt_i}$$

in which r_i and r_p are in [L] and t is time in [T].

The drawdown in U. S. customary units (the gallon-day-foot system) can be expressed as

$$s_r = \frac{114.6Q}{T}\left[W(u_p) - W(u_i)\right] \tag{4.7.6}$$

where $u_p = \dfrac{1.87r_p^2 S}{Tt_p}$ and $u_i = \dfrac{1.87r_i^2 S}{Tt_i}$.

For large values of time, t, the well functions can be expressed as

$$W(u_p) = -0.5772 - \ln u_p \tag{4.7.7}$$

and

$$W(u_i) = -0.5772 - \ln u_i \tag{4.7.8}$$

This allows Equation 4.7.5 to be simplified to

$$s_r = \frac{Q}{4\pi T}\left[-\ln u_p + \ln u_i\right] \tag{4.7.9}$$

and Equation 4.7.6 is simplified to

$$s_r = \frac{114.6Q}{T}\left[-\ln u_p + \ln u_i\right] \tag{4.7.10}$$

Now using the gallon-day-foot system with time in minutes

$$u_p = \frac{2693r_p^2 S}{Tt} \tag{4.7.11}$$

and

$$u_i = \frac{2693r_i^2 S}{Tt} \tag{4.7.12}$$

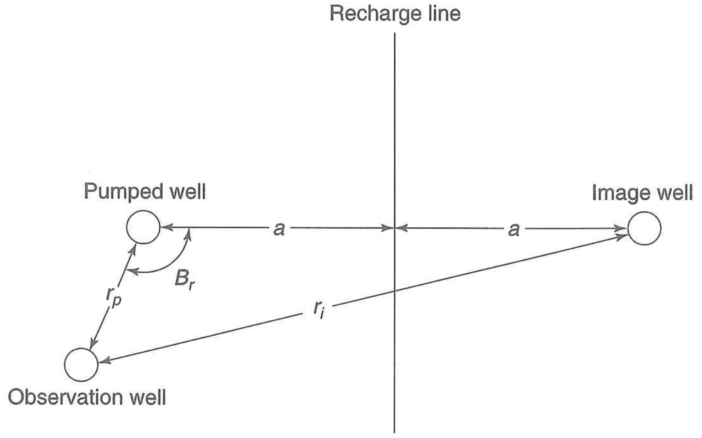


Figure 4.7.5. Definition of terms for Equation 4.7.15.

The drawdown from Equation 4.7.10 is expressed as

$$s_r = \frac{114.6Q}{T}\left[-\ln\left(\frac{2693r_p^2S}{Tt}\right)+\ln\left(\frac{2693r_i^2S}{Tt}\right)\right] \tag{4.7.13}$$

which simplifies to

$$s_r = \frac{528}{T}Q\log\left(\frac{r_i}{r_p}\right) \tag{4.7.14}$$

Rorabaugh[50] expressed this equation in terms of the distances between the pumped well and the line of recharges as

$$s_r = \frac{528Q\log\sqrt{\left(4a^2+r_p^2-4ar_p\cos B_r\right)}/r_p}{T} \tag{4.7.15}$$

where a is the distance from the pumped well to the recharge boundary in ft, and B_r is the angle between a line connecting the pumped and image wells and a line connecting the pumped and observation wells. Refer to Figure 4.7.5 for an explanation of terms.

EXAMPLE 4.7.1

A 0.5-m diameter well (200 m from a river) is pumping at an unknown rate from a confined aquifer (see Figure 4.7.6). The aquifer properties are $T = 432$ m^2/day and $S = 4.0 \times 10^{-4}$. After eight hours of pumping, the drawdown in the observation well (60 m from the river) is 0.8 m. Compute the rate of pumping and the drawdown in the pumped well. What is the effect of the river on drawdown in the observation well and in the pumped well?

SOLUTION

The following information is given in the above statement: $r_w = 0.25$ m, $T = 432$ m^2/day $= 5.0 \times 10^{-3}$ m^2/s, $S = 4 \times 10^{-5}$, $t = 8$ hr $= 28{,}800$ s, and $s = 0.8$ m. A recharging image well is placed at the same distance from the river as the pumped well as shown in Figure 4.7.6b.

Equation 4.7.5 is used to compute the discharge from the pumped well knowing the above information:

$$s = \frac{Q}{4\pi T}W(u_p) - \frac{Q}{4\pi T}W(u_i)$$

$$u_p = \frac{r_p^2S}{4Tt} = \frac{(140)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 1.36\times10^{-2}$$

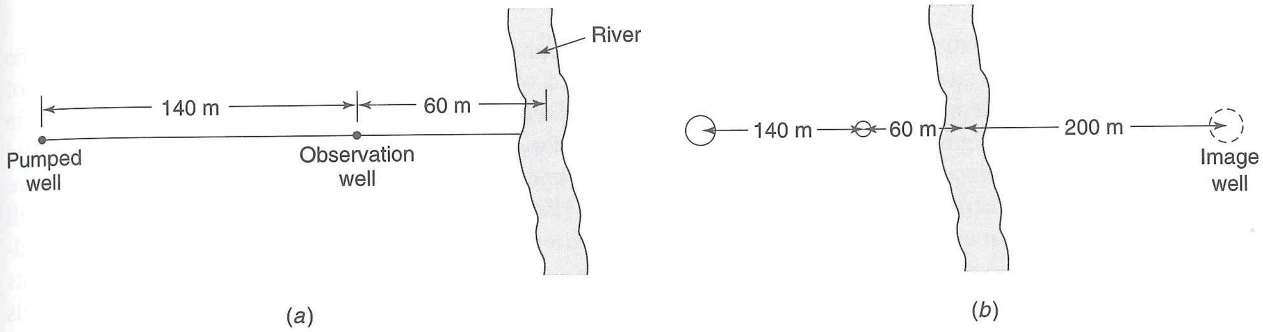


Figure 4.7.6. Example 4.7.1 system. (*a*) Well locations (*b*) Image well location

$$u_i = \frac{r_i^2 S}{4Tt} = \frac{(260)^2 (4\times10^{-4})}{4(5\times10^{-3})(28800)} = 4.69\times10^{-2}$$

$$W(u_p) = 3.79 \text{ for } u_p = 1.36\times10^{-2} \text{ and } W(u_i) = 2.54 \text{ for } u_i = 4.69\times10^{-2}$$

Thus the discharge is computed using

$$0.8 = \frac{Q}{4\pi(5\times10^{-3})}(3.79) - \frac{Q}{4\pi(5\times10^{-3})}(2.54)$$

so that $Q = 0.04$ m^3/s.

The drawdown in the pumped well is computed using equation 4.7.5:

$$u_w = \frac{r_w^2 S}{4Tt} = \frac{(0.25)^2 (4\times10^{-4})}{4(5\times10^{-3})(28800)} = 4.34\times10^{-8}$$

$$u_i = \frac{(400)^2 (4\times10^{-4})}{4(5\times10^{-3})(28800)} = 0.111$$

$$W(u_w) = 16.38 \text{ for } u_w = 4.39\times10^{-8} \text{ and } W(u_i) = 1.75 \text{ for } u_i = 0.111$$

Thus the drawdown is

$$s_w = \frac{0.04}{4\pi(5\times10^{-3})}(16.38) - \frac{0.04}{4\pi(5\times10^{-3})}(1.75) = 9.31 \text{ m}$$

The effect of the river on the wells is to decrease the drawdown, so the reduced drawdown in the observation well is

$$s_{\text{river}} = -\frac{Q}{4\pi T} W(u_i) = -\frac{0.04}{4\pi(5\times10^{-3})}(2.54) = -1$$

Similarly, in the pumped well, the reduced drawdown is

$$s_{\text{river}} = -\frac{0.04}{4\pi(5\times10^{-3})}(1.75) = -1.11 \text{ m}$$

■

4.7.2 Well Flow Near an Impermeable Boundary

In the case of an impermeable or *barrier boundary,* water cannot flow across the boundary; no water is being contributed to the pumped well from the impervious formation. The cone of depression that would exist for a pumped well in an aquifer of infinite areal extent is shown in Figure 4.7.7*b*. Because of a barrier boundary, the cone of depression shown is no longer valid since there can be no flow across the boundary. Placing an image well, discharging in nature across the barrier boundary, creates the effect of no flow across the boundary. The image well must be placed perpendicular to the barrier boundary and the same distance from the boundary as the real well. The resulting real cone of depression is the summation of the components of both the real and image well depression cones as shown in the Figure 4.7.7*b*. Water levels in wells will decline at an initial rate due only to the influence of the pumped well. As pumping continues the barrier boundary effects will begin as simulated by the image well affecting the real well. When the effects of the barrier boundary are realized, the time rate of drawdown will increase (Figure 4.7.8). When this occurs, the total rate of withdrawal from the aquifer is equal to that of the pumped well plus that of the discharging image well causing the cone of depression of the real well to be deflected downward.

The total drawdown in the real well can be expressed as

$$s_b = s_p + s_i \tag{4.7.16}$$

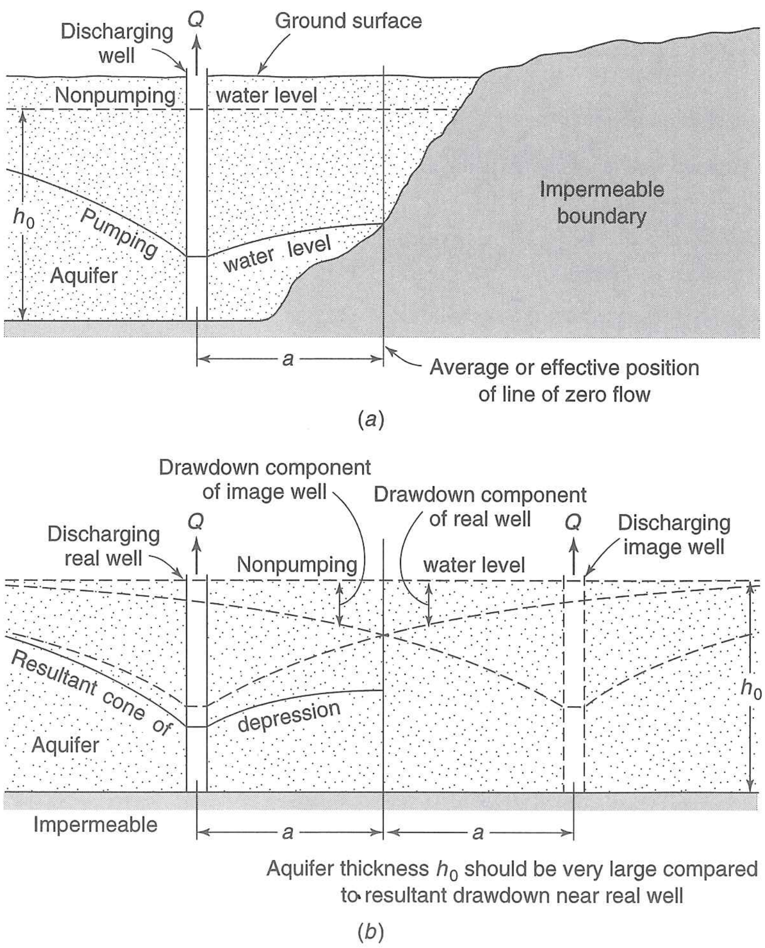


Figure 4.7.7. Sectional views. (*a*) Discharging well near an impermeable boundary (*b*) Equivalent hydraulic system in an aquifer of infinite areal extent (after Ferris et al.[15])

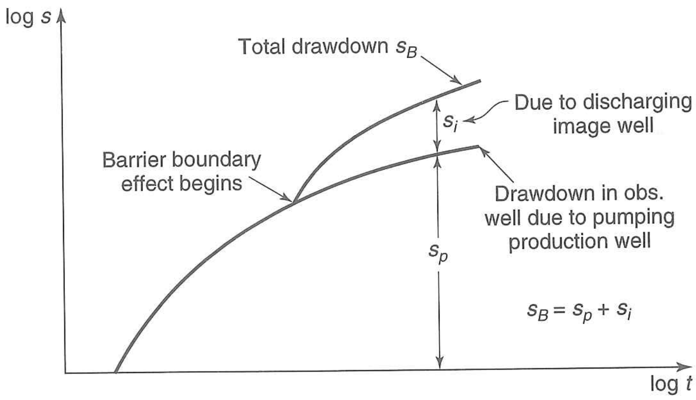


Figure 4.7.8. Barrier boundary effects on time–drawdown curve.

in which s_b is the total drawdown, s_p is the drawdown in an observation well due to pumping of the production well, and s_i is the drawdown due to the discharging image well (barrier boundary). The total drawdown can be expressed as

$$s_b = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i) \tag{4.7.17}$$

where Q is the constant pumping rate [L^3/T], T is the transmissivity [L^2/T], $W(u_p)$ and $W(u_i)$ are dimensionless, and u_p and u_i are

$$u_p = \frac{r_p^2 S}{4Tt_p} \quad \text{and} \quad u_i = \frac{r_i^2 S}{4Tt_i}$$

in which r_i and r_p are in [L] and t is time in [T].

Drawdown Equation 4.7.16 can also be expressed in U.S. customary units (gal-day-foot) system where S is in ft, Q is in gpm, T is in gpd/ft, r is in ft, and t is in days:

$$\begin{aligned} s_b &= \frac{114.6Q}{T} W(u_p) + \frac{114.6Q}{T} W(u_i) \\ &= \frac{114.6Q}{T}\left[W(u_p) + W(u_i)\right] \end{aligned} \tag{4.7.18}$$

where $u_p = \dfrac{1.87 r_p^2 S}{Tt_p}$ and $u_i = \dfrac{1.87 r_i^2 S}{Tt_i}$

Now suppose that we choose drawdowns at times t_p and t_i such that $s_p = s_i$, then $W(u_p) = W(u_i)$ and $u_p = u_i$. Then

$$\frac{r_i^2 S}{Tt_i} = \frac{r_p^2 S}{Tt_p}$$

which reduces to

$$\frac{r_i^2}{t_i} = \frac{r_p^2}{t_p} \tag{4.7.19}$$

Equation 4.7.19 defines the *law of times* which states that for a given aquifer, the times of occurrence of equal drawdown vary directly as the squares of distances from an observation well to a production well and an image well of equal discharge. The law of times can be used to determine the distance from an image well to an observation well, using

$$r_i = r_p\sqrt{\frac{t_i}{t_p}} \tag{4.7.20}$$

in which r_i is the distance from the image well to the observation well in ft, r_p is the distance from the pumped well to the observation well in ft, t_p is the time after pumping started and before the barrier boundary is effective, and t_i is the time (after pumping started and after the barrier boundary becomes effective) where $s_p = s_i$.

EXAMPLE 4.7.2

A well is pumping near a barrier boundary (see Figure 4.7.9) at a rate of 0.03 m^3/s from a confined aquifer 20 m thick. The hydraulic conductivity of the aquifer is 27.65 m/day and its storativity is 3×10^{-5}. Determine the drawdown in the observation well after 10 hours of continuous pumping. What is the fraction of the drawdown attributable to the barrier boundary?

SOLUTION

The following information is given in the above problem statement: $Q = 0.03$ m^3/s, $b = 20$ m, $K = 27.65$ m/day $= 3.2 \times 10^{-4}$ m/s, $S = 3 \times 10^{-5}$, $t = 10$ hrs $= 36{,}000$ s. An image well is placed across the boundary at the same distance from the boundary as the pumped well (as shown in Figure 4.7.9b). The drawdown in the observation well is due to the real well and the imaginary well (which accounts for the barrier boundary). Hence, using Equation 4.7.17

$$s = \frac{Q}{4\pi T}W(u_p) + \frac{Q}{4\pi T}W(u_i)$$

$$u_p = \frac{r_p^2 S}{4Tt} = \frac{(240)^2(3\times10^{-5})}{4(20)(3.2\times10^{-4})(36{,}000)} = 1.88\times10^{-3}$$

Next compute the distance from the observation well to the image well: $r_i^2 = 600^2 + 240^2 - 2(600)(300)\cos 30° = 16{,}8185$ m^2 so $r_i = 410$ m. Using r_i, compute

$$u_i = \frac{168185(3\times10^{-5})}{4(20)(3.2\times10^{-4})(36{,}000)} = 5.47\times10^{-3}$$

The well functions are now computed or obtained from Table 4.4.1 as $W(u_p) = 5.72$ for $u_p = 1.88 \times 10^{-3}$ and $W(u_i) = 4.64$ for $u_i = 5.47 \times 10^{-3}$.

The drawdown at the observation well is computed as

$$s = \frac{0.03}{4\pi(20)(3.2\times10^{-4})} = (5.72 + 4.64) = 3.86 \text{ m}.$$

The drawdown attributable to the barrier boundary is computed as

$$s_i = \frac{Q}{4\pi T}W(u_i) = \frac{0.03}{4\pi(20)(3.2\times10^{-4})}(4.64) = 1.73 \text{ m}$$

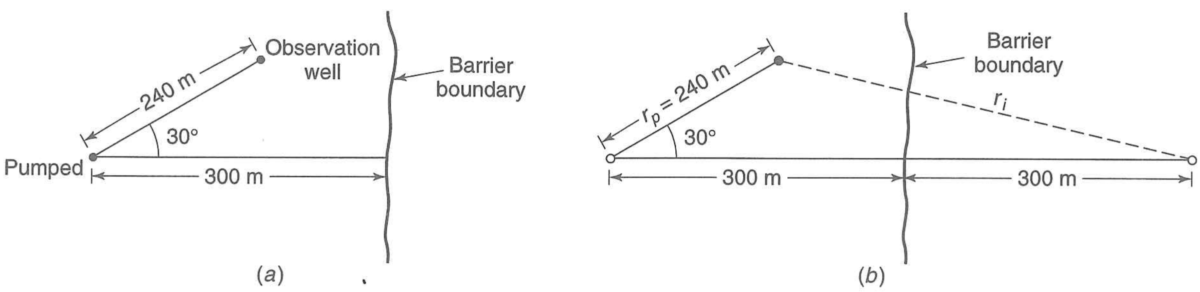


Figure 4.7.9. Example 4.7.2 system. (*a*) Well locations (*b*) Image well location

and the fraction of drawdown attributable to the impermeable boundary is

$$\frac{s_i}{s} = \frac{1.73}{3.86} = 0.45\,(45\%).$$

4.7.3 Well Flow Near Other Boundaries

In addition to the previous two examples, the method of images can be applied to a large number of groundwater boundary problems. As before, actual boundaries are replaced by an equivalent hydraulic system, which includes imaginary wells and permits solutions to be obtained from equations applicable only to extensive aquifers. Two boundary conditions to suggest the adaptability of the method are shown in Figure 4.7.10. Figure 4.7.10*a* shows a discharging well in an aquifer bounded on two sides by impermeable barriers. The image discharge wells I_1 and I_2 provide the required flow but, in addition, a third image well I_3 is necessary to balance drawdowns along the extensions of the boundaries. The resulting system of four discharging wells in an extensive aquifer represents hydraulically the flow system for the physical boundary conditions. Finally, Figure 4.7.10*b* presents the situation of a well near an impermeable boundary and a perennial stream. The image wells required follow from the previous illustrations.

For a wedge-shaped aquifer, such as a valley bounded by two converging impermeable barriers, the drawdown at any location within the aquifer can be calculated by the same method of images.[54] Consider the aquifer formed by two barriers intersecting at an angle of 45 degrees shown in Figure 4.7.11. Seven image pumping wells plus the single real pumping well form a circle with its center at the wedge apex; the radius equals the distance from the apex to the real pumping well.[15] The drawdown at any point between the two barriers can then be calculated by summing the individual drawdowns. In general, it can be shown that the number of image wells n required for a wedge angle θ is given by

$$n = \frac{360^\circ}{\theta} - 1 \tag{4.7.21}$$

where θ is an aliquot part of 360 degrees.

If the arrangement of two boundaries is such that they are parallel to each other, analysis by the image-well theory requires the use of an image-well system extending to infinity.[15] Each successively added secondary image well produces a residual effect at the opposite boundary.

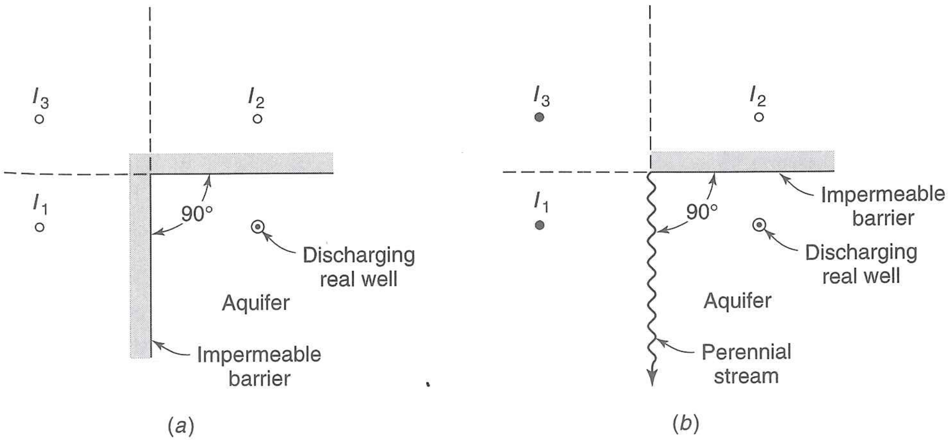


Figure 4.7.10. Image well systems for a discharging well near aquifer boundaries. (*a*) Aquifer bounded by two impermeable barriers intersecting at right angles (*b*) Aquifer bounded by an impermeable barrier intersected at right angles by a perennial stream. Open circles are discharging image wells; filled circles are recharging image wells (after Ferris et al.[15]).

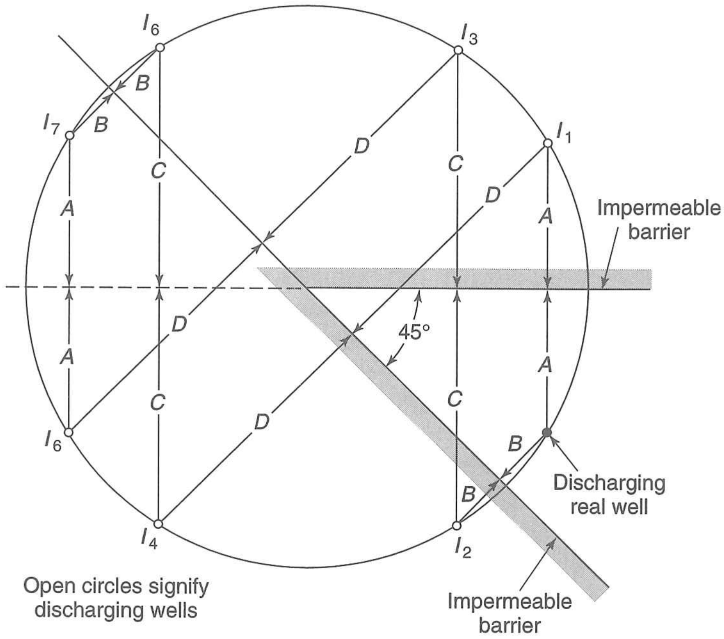


Figure 4.7.11. Image well system for a discharging well in an aquifer bounded by two impermeable barriers intersecting at an angle of 45 degrees (after Ferris et al.[15]).

It is only necessary to add pairs of image wells until the next pair has negligible influence on the sum of all image-well effects out to the point. The use of spreadsheets makes the computation much easier. Image-well systems for several parallel-boundary situations are shown in Figures 4.7.12 and 4.7.13.

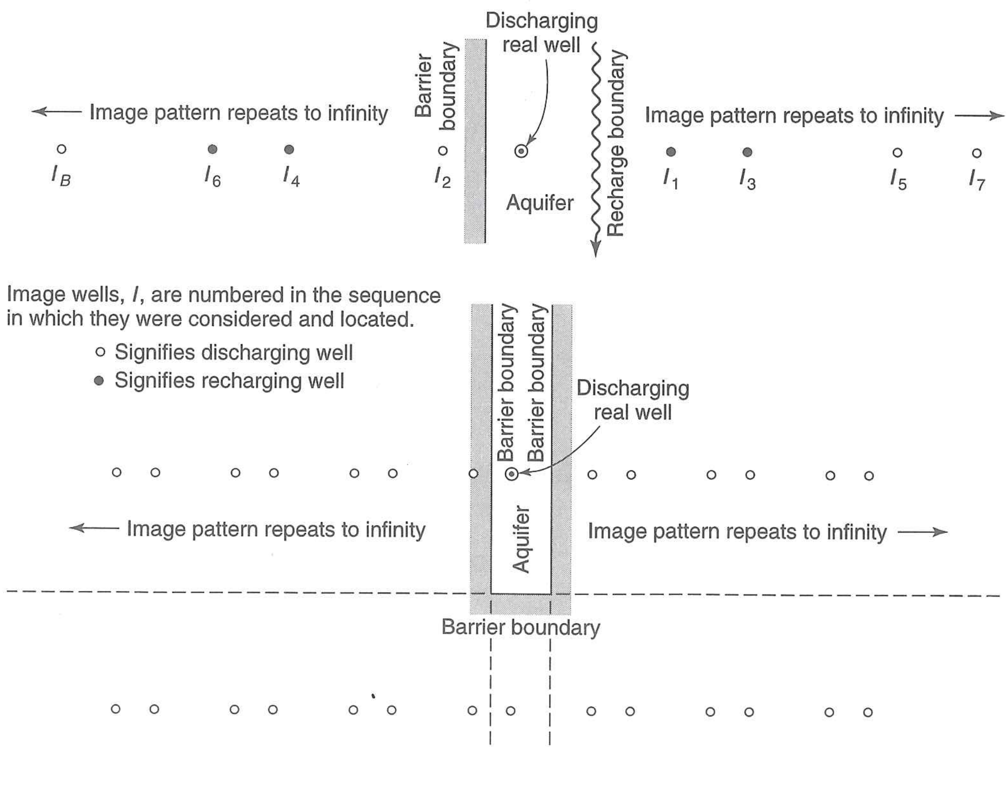


Figure 4.7.12. Plans of image-well systems for selected parallel-boundary situations (after Ferris et al.[15]).

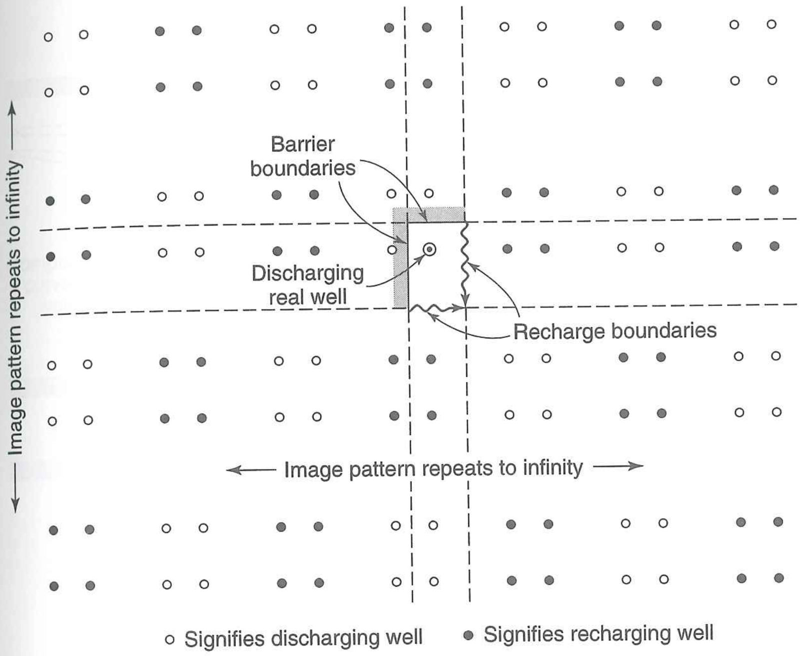


Figure 4.7.13. Plans of image-well systems for a rectangular aquifer (after Ferris et al.[15]).

The above equations involve T for confined aquifers. To adapt them to unconfined aquifers, s should be replaced by $s'' = s - s^2/2h_0$ where h_0 is the initial saturated aquifer thickness. Storage coefficients cannot be calculated from steady-state boundary equations.

Procedures for analyzing unsteady well flows near aquifer boundaries, involving graphic solutions, are available.[15, 21]

4.7.4 Location of Aquifer Boundary

Permeable aquifer boundaries such as streams would normally be visible near a pumping well; however, impermeable subsurface boundaries such as faults or dikes may not be apparent. Where this situation is encountered, the location and orientation of such a barrier can be defined by careful analysis of pumping test data.[15, 61] In the Cooper–Jacob method (see Equation 4.4.8) the slope of the straight line on semilogarithmic paper depends only on the pumping rate and the transmissivity. If an impermeable boundary is present, the rate of drawdown in an observation well will double under the influence of an image pumping well (see Figure 4.7.14*a*).* To determine the location of the image well, straight lines are fitted through the two legs of the data. An arbitrary drawdown s_A is selected and a time t_r for this to occur under the influence of the real well is measured (see Figure 4.7.14*a*). Similarly, a time t_i for the same drawdown to be produced by the image well is defined. Then, knowing the distance r_r between the real well and the observation well, the distance r_i to the image well (see Figure 4.7.14*b*) can be found from the law of times (Equation 4.7.20). The distance r_i defines only the radius of a circle on which the image well lies. It requires measurements in two more observation wells in order to define uniquely by intersection of three arcs the location of the image well (see Figure 4.7.14*b*). The boundary then lies at the midpoint of and perpendicular to a line connecting the real and image wells.

*It should be noted that if the boundary is a stream recharging the aquifer, an image recharge well is introduced. This produces a slope of equal but opposite sign on the drawdown curve, resulting in a horizontal asymptote.

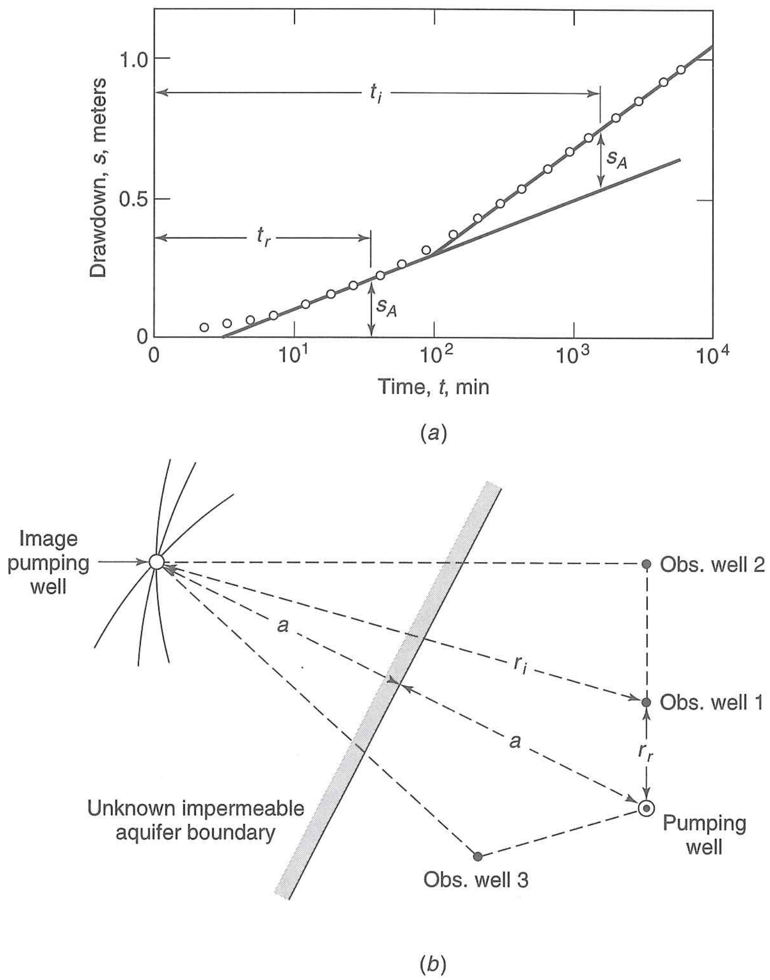


Figure 4.7.14. Diagrams illustrating procedure to locate an unknown impermeable aquifer boundary near a pumping well. (*a*) Cooper–Jacob drawdown curve showing effect of an impermeable boundary (*b*) Field situation required to locate an unknown impermeable aquifer boundary

4.8 MULTIPLE WELL SYSTEMS

Where the cones of depression of two nearby pumping wells overlap, one well is said to *interfere* with another because of the increased drawdown and pumping lift created. For a group of wells forming a well field, the drawdown can be determined at any point if the well discharges are known, or vice versa. From the principle of superposition, the drawdown at any point in the area of influence caused by the discharge of several wells is equal to the sum of the drawdowns caused by each well individually. Thus,

$$s_T = s_1 + s_2 + s_3 + \ldots + s_n \tag{4.8.1}$$

where s_T is the total drawdown at a given point and $s_1, s_2, s_3, \ldots, s_n$ are the drawdowns at the point caused by the discharge of wells 1, 2, 3, ..., n, respectively. The summation of drawdowns may be illustrated in a simple way by the well line of Figure 4.8.1; the individual and composite drawdown curves are given for $Q_1 = Q_2 = Q_3$. Clearly, the number of wells and the geometry of the well field are important in determining drawdowns. Solutions can be based on the equilibrium or nonequilibrium equation. Equations of well discharge for particular well patterns have been developed.[41, 49]

In general, wells in a well field designed for water supply should be spaced as far apart as possible so their areas of influence will produce a minimum of interference with each other. On

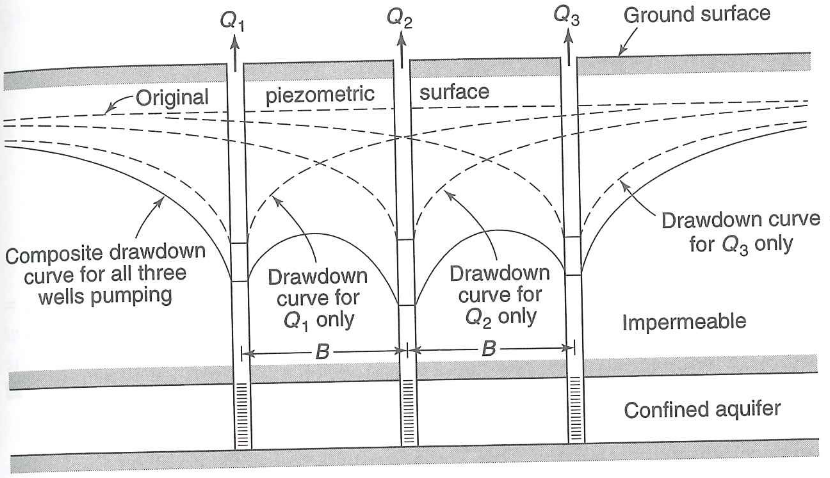
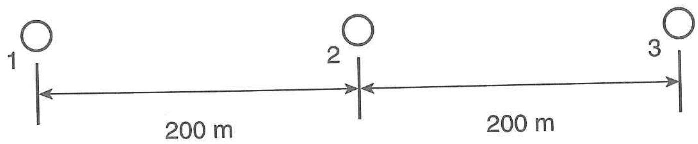


Figure 4.8.1. Individual and composite drawdown curves for three wells in a line.

the other hand, economic factors such as cost of land or pipelines may lead to a least-cost well layout that includes some interference.[21] For drainage wells designed to control water table elevations, it may be desirable to space wells so that interference increases the drainage effect.

EXAMPLE 4.8.1

Three pumping wells located along a straight line are spaced at 200 m apart. What should be the steady-state pumping rate from each well so that the near steady-state drawdown in each well will not exceed 2 m? The transmissivity of the confined aquifer, which all the wells fully penetrate, is 2400 m^2/day and all the wells are 40 cm in diameter. The thickness of the aquifer is 40 m and the radius of influence of each well is 800 m.



SOLUTION

The following information is given in the above problem statement: $s_1 \leq 2$ m, $s_2 \leq 2$ m, and $s_3 \leq 2$ m, $T =$ 2,400 m^2/day = 27.8×10^{-3} m^2/s, $r_w = 0.2$ m, $b = 40$ m, $r_0 = 800$ m, and $r = 200$ m. Let Q be the pumping rate from each well and h_0 be the head before pumping started. For well 1, $s_1 = s_{11} + s_{12} + s_{13}$ where s_{ij} is the drawdown in well i due to pumping in well j. Thus, for the other wells, $s_2 = s_{21} + s_{22} + s_{23}$, and $s_3 = s_{31} + s_{32} + s_{33}$. By symmetry, $s_1 = s_3$. The drawdowns in well 1 due to pumping in wells 1, 2, and 3 are respectively

$$s_{11} = \frac{Q \ln\left(\frac{r_0}{r_w}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{0.2}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 47.48Q$$

$$s_{12} = \frac{Q \ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{200}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 7.94Q$$

$$s_{13} = \frac{Q \ln\left(\frac{r_0}{r_{13}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{400}\right)}{2\pi\left(27.8 \times 10^{-3}\right)} = 3.97Q$$

The drawdowns in wells 1 and 3 are identical so total drawdown in the wells is $s_1 = s_3 = 47.48Q + 7.94Q + 3.97Q = 59.39Q$. The drawdowns in well 2 due to pumping in wells 1, 2, and 3 are respectively

$$s_{21} = \frac{Q\ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{200}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 7.94Q$$

$$s_{22} = s_{11} = 47.48Q$$

$$s_{23} = s_{21} = 7.94Q$$

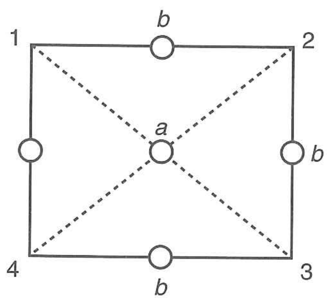
The total drawdown in well 2 is $s_2 = 7.94Q + 47.48Q + 7.94Q = 63.36Q$. The relationships for $s_1 = 59.39Q$ and $s_2 = 63.36Q$ show that for the same discharge from all the wells, more drawdown results at the middle well; therefore, the drawdown in this well governs. So using $s_2 \le 2$ or $63.36Q \le 2$, then the steady-state pumping rate from each well should be $Q \le 3.16 \times 10^{-2}$ m^3/s = 113 m^3/hr. ■

EXAMPLE 4.8.2

It is required to dewater a construction site 80 m by 80 m. The bottom of the construction will be 1.5 m below the initial water surface elevation of 90 m. Four pumps are to be used in 0.5-m diameter wells at the four corners of the site. The transmissivity and the storage coefficient of the aquifer are 1,600 m^2/day and 0.16, respectively. The site needs to be ready after one month of pumping. Determine the required pumping rate.

SOLUTION

To solve this problem, the least drawdown at the site should be greater than 1.5 m. It can be shown that the potential points of interest that may have the least drawdown are the center of the square (point a) and the midpoint on each side of the square (points b). Approximation is made using the Cooper-Jacob method.



At point a (the center of the square), $r = \sqrt{40^2 + 40^2} = 56.6$ m, and

$$u = \frac{r^2 S}{4Tt} = \frac{(56.6\text{ m})^2(0.16)}{4\left(1600\text{ m}^2/\text{day}\right)(30\text{ days})} = 0.00267$$

Since $u < 0.01$, we can use the approximate solution by Cooper-Jacob expressed by Equation 4.4.7:

$$s_a = \frac{Q}{4\pi T}\left(-0.5772 - \ln(u)\right) = \frac{Q}{4\pi\left(1600\text{ m}^2/\text{day}\right)}\left(-0.5772 - \ln(0.00267)\right) = 0.0002661Q$$

Using the principle of superposition and by symmetry, the drawdown caused by the four wells is $s_T = 4 \times s_a = 4 \times 0.0002661Q = 0.0010643Q$ and $s_T = 0.0010643Q = 1.5\text{ m} \rightarrow Q = 1409$ m^3/day.

At any of the four points represented by b, $r_1 = 40$ m for two of the wells and $r_2 = \sqrt{80^2 + 40^2} = 89.44$ m for the remaining two wells. Then

$$u_1 = \frac{r^2 S}{4Tt} = \frac{(40\text{ m})^2(0.16)}{4\left(1600\text{ m}^2/\text{day}\right)(30\text{ days})} = 0.0013333$$

$$u_2 = \frac{r^2 S}{4Tt} = \frac{(89.44\text{ m})^2(0.16)}{4\left(1600\text{ m}^2/\text{day}\right)(30\text{ days})} = 0.006666$$

Since $u_1 < 0.01$ and $u_2 < 0.01$, the Cooper–Jacob method of solution can be used again:

$$s_b = 2\left[\frac{Q}{4\pi T}\left(-0.5772 - \ln(u_1)\right)\right] + 2\left[\frac{Q}{4\pi T}\left(-0.5772 - \ln(u_2)\right)\right]$$

$$= 2\left[\frac{Q}{4\pi\left(1600\ \text{m}^2/\text{day}\right)}\left(-0.5772 - \ln(0.0013333)\right)\right] +$$

$$2\left[\frac{Q}{4\pi\left(1600\ \text{m}^2/\text{day}\right)}\left(-0.5772 - \ln(0.006666)\right)\right]$$

$$= 2 \times 0.0003Q + 2 \times 0.0002205Q$$

$$= 1.041 \times 10^{-3} Q = 1.5\ \text{m} \rightarrow Q = 1441\ \text{m}^3/\text{day}$$

Thus the points represented by b are critical and a discharge of 1,441 m^3/day from each well is required. ■

4.9 PARTIALLY PENETRATING WELLS

A well whose length of water entry is less than the aquifer it penetrates is known as a *partially penetrating well.* Figure 4.9.1 illustrates the situation of a partially penetrating well in a confined aquifer. The flow pattern to such wells differs from the radial horizontal flow assumed to exist around fully penetrating wells. The average length of a flow line into a partially penetrating well exceeds that into a fully penetrating well so a greater resistance to flow is thus encountered. For practical purposes this results in the following relationships between two similar wells, one partially and one fully penetrating the same aquifer: if $Q_p = Q$, then $s_p > s$; and if $s_p = s$, then $Q_p < Q$. Here Q is well discharge, s is drawdown at the well, and the subscript p refers to the partially penetrating well. The effect of partial penetration is negligible on the flow pattern and the drawdown beyond a radial distance larger than 0.5 to 2 times the saturated thickness b, depending on the amount of penetration.

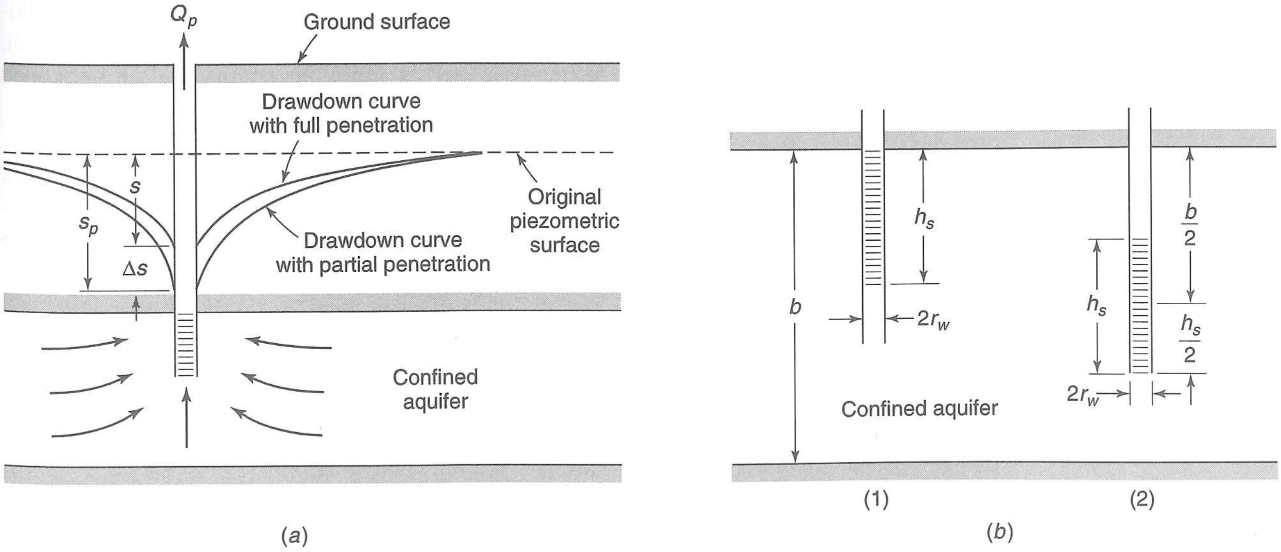


Figure 4.9.1. Partially penetrating wells in a confined aquifer. (*a*) Effect of partially penetrating well on drawdown (*b*) Two configurations of partially penetrating wells

The drawdown s_p at the well face of a partially penetrating well in a confined aquifer (see Figure 4.9.1*a*) can be expressed as

$$s_p = s + \Delta s \tag{4.9.1}$$

where Δs refers to the additional drawdown resulting from the effect of partial penetration. It can be shown for steady-state conditions and the typical situation* in Figure 4.9.1*b*(1),[28]

$$\Delta s = \frac{Q_p}{2\pi T}\frac{1-p}{p}\ln\frac{(1-p)h_s}{r_w} \tag{4.9.2}$$

where T is transmissivity, p is the penetration fraction ($p = h_s/b$), and h_s and r_w are shown in Figure 4.9.1*b*(1). Equation 4.9.2 applies where $p > 0.20$.

For the case of a well screen centered in the thickness of the aquifer [see Figure 4.9.1*b*(2)], the value of Δs is given by

$$\Delta s = \frac{Q_p}{2\pi T}\frac{1-p}{p}\ln\frac{(1-p)h_s}{2r_w} \tag{4.9.3}$$

Equation 4.9.2 can be modified for a well in an unconfined aquifer by defining

$$\Delta s\, 2h_w = \frac{Q_p}{\pi K}\frac{1-p}{p}\ln\frac{(1-p)h_s}{r_w} \tag{4.9.4}$$

where h_w is the saturated thickness at the well with full penetration and the hydraulic conductivity $K = T/h_w$. Then

$$s_p^{\,2} = s^2 + \Delta s\, 2h_w \tag{4.9.5}$$

and similarly for Equation 4.9.3.

Detailed methods for analyzing effects of partial penetration on well flow for steady and unsteady conditions in confined, unconfined, leaky, and anisotropic aquifers have been outlined by Hantush[23, 27] and others.[32, 34, 57] Although evaluating the effects is complicated except for the simplest cases, common field situations often reduce the practical importance of partial penetration.† One occurs where an observation well is located more than 1.5 to 2 times the saturated aquifer thickness from a pumping well; in this situation the effect of partial penetration can be neglected for homogeneous and isotropic aquifers. Another applies to many alluvial aquifers with pronounced anisotropy. Here the vertical flow components become small, thereby enabling a pumping well to be approximated as a fully penetrating well in a confined or leaky aquifer with a saturated thickness equal to the length of the well screen.

EXAMPLE 4.9.1

Compare the four cases that are depicted in Figure 4.9.2 where a 1-m diameter well fully/partially penetrates vertically through a confined aquifer whose thickness is 28 m. Comment on their relative efficiencies by evaluating the specific capacity (Q/s) for each case. Take the radius of influence for all cases as 150 m. The pumping rates and the aquifer transmissivity are the same for all cases.

SOLUTION

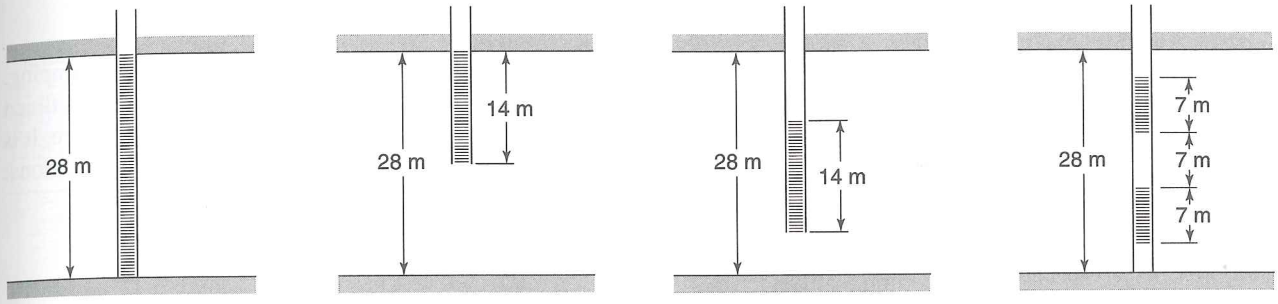
Case (a): The drawdown in this case where the pumping well fully penetrates is computed by rearranging the Thiem equation (4.2.4):

$$s = \frac{Q}{2\pi T}\left(\ln\left(\frac{R}{r_w}\right)\right) = \frac{Q}{2\pi T}\left(\ln\left(\frac{150}{0.5}\right)\right) = 0.9078\left(\frac{Q}{T}\right)$$

---

*The drawdown increment is the same whether partial penetration starts from the top or from the bottom of the aquifer.

† It should be noted that any well with 85 percent or more open or screened hole in the saturated thickness may be considered as fully penetrating.



(*a*) Well fully penetrating

(*b*) Well penetrating the top 14 m of the confined aquifer

(*c*) Well penetrating 14 m of the confined aquifer with the well screen centered

(*d*) Well penetrating 14 m of the confined aquifer with two 7 m screened intervals

Figure 4.9.2. Example 4.9.1 cases

Case (b): The additional drawdown is computed by rearranging Equation 4.9.2:

$$\Delta s = \left[\frac{Q_p}{2\pi T}\right]\left[\frac{1-p}{p}\right]\left[\ln\frac{(1-p)h_s}{r_w}\right] = \left[\frac{Q}{2\pi T}\right]\left[\frac{1-0.5}{0.5}\right]\left[\ln\frac{(1-0.5)(14)}{(0.5)}\right]$$

$$= 0.4200\left(\frac{Q}{T}\right)$$

So the total drawdown becomes s_b (0.9078 + 0.4200)(Q/T) = 1.3278 (Q/T).

Case (c): The additional drawdown in the case of a well screen centered in the thickness of the aquifer is given by Equation 4.9.3:

$$\Delta s = \left[\frac{Q_p}{2\pi T}\right]\left[\frac{1-p}{p}\right]\left[\ln\frac{(1-p)h_s}{2r_w}\right] = \left[\frac{Q}{2\pi T}\right]\left[\frac{1-0.5}{0.5}\right]\left[\ln\frac{(1-0.5)(14)}{2(0.5)}\right]$$

$$= 0.3097\left(\frac{Q}{T}\right)$$

The total drawdown for this case becomes s_c (0.9078 + 0.3097) (Q/T) = 1.2175 (Q/T)

Case (d): This case is equivalent to a screen length of 7 m centered in a 14-m thick aquifer. Again using Equation 4.9.3,

$$\Delta s = \left[\frac{Q_p}{2\pi T}\right]\left[\frac{1-p}{p}\right]\left[\ln\frac{(1-p)h_s}{2r_w}\right] = \left[\frac{Q}{2\pi T}\right]\left[\frac{1-0.5}{0.5}\right]\left[\ln\frac{(1-0.5)(7)}{2(0.5)}\right]$$

$$= 0.1994\left(\frac{Q}{T}\right)$$

The total drawdown for this case becomes s_d (0.9078 + 0.1994) (Q/T) = 1.1072 (Q/T).
Calculate the specific capacity (Q/s) for each case:

$$Q/s_a = 1.1016(T)$$

$$Q/s_b = 0.7531(T)$$

$$Q/s_c = 0.8214(T)$$

$$Q/s_d = 0.9032(T)$$

Or taking case (a), fully penetrating well, as the maximum potential specific capacity, cases (b), (c), and (d) yield 68 percent, 75 percent, and 82 percent of the maximum capacity, respectively. So among the three cases, case (d) is the most effective one. Also, centering the well screen yields higher capacity compared to the case where the same partial penetration starts from the top or from the bottom of the aquifer. ■

4.10 WELL FLOW FOR SPECIAL CONDITIONS

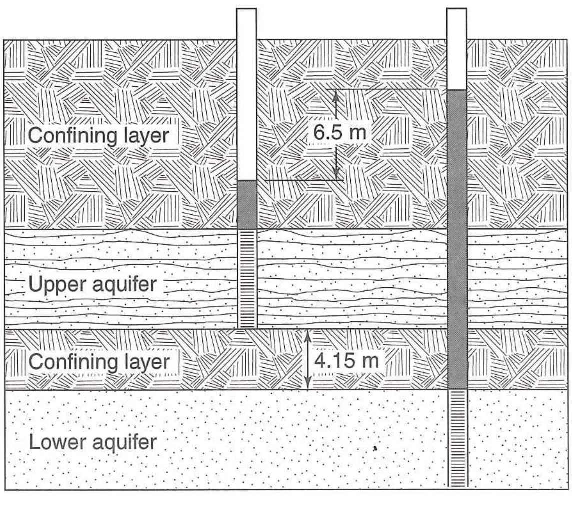
A variety of solutions to well flow problems have been derived for special aquifer, pumping, and well conditions.[23,38] Inasmuch as these are of less general application than those outlined heretofore and involve more extensive mathematical treatment, they will be omitted here. It is worth noting, however, that solutions have been obtained for the following special conditions:

1. Constant well drawdown[2, 21]
2. Varying, cyclic, and intermittent well discharges[20, 36, 39, 52, 55, 56]
3. Sloping aquifers[21]
4. Aquifers of variable thickness[21]
5. Two-layered aquifers[24, 44]
6. Anisotropic aquifers[10, 23, 27, 43, 66]
7. Aquifer conditions varying with depth[40, 51]
8. Large-diameter wells[46, 67]
9. Collector wells (see Chapter 5)[28, 45]
10. Wells with multiple-sectioned well screens[57]

PROBLEMS

4.1.1 A confined aquifer is 18.5 m thick. The potentiometric surface elevations at two observation wells 822 m apart are 25.96 and 24.62 m. If the horizontal hydraulic conductivity of the aquifer is 25 m/day, determine the flow rate per unit width of the aquifer, specific discharge, and average linear velocity of the flow assuming steady unidirectional flow.

4.1.2 Two confined aquifers are separated by an aquitard as shown below. The piezometric head difference between the upper and lower aquifer measured along a vertical line is 6.5 m. If the vertical hydraulic conductivity of confining unit is 0.046 m/day, determine the direction and magnitude of leakage per km^2 between the upper and lower confined aquifers through the confining unit. Also, estimate the travel time of a water particle through the confining layer between the two aquifers. Estimated thickness of the separation is 4.15 m.



4.1.3 Two observation wells are to be constructed to determine the natural slope of the piezometric surface in a confined aquifer. Assuming a steady unidirectional flow, if the accuracies of piezometric head measurements and horizontal distance measurements are ±3 cm and ±5 cm, respectively, determine the percent error in estimating the slope of the piezometric surface as a function of the distance between the two wells. What would be the accuracy of a hydraulic gradient estimate if a piezometric head drop of 3 m is observed between two wells 100 m apart?

4.1.4 Rework Problem 4.1.1 if a ±10° error is also possible in estimating the groundwater flow direction.

4.1.5 Near steady-state conditions, explain how the hydraulic gradient changes in the flow direction in

(a) a confined aquifer;

(b) an unconfined aquifer.

4.1.6 Consider two strata of the same soil material that lie between two channels. The first stratum is confined and the second one is unconfined, and the water surface elevations in the channels are 24 and 16 m above the bottom of the unconfined aquifer. What should be the thickness of the confined aquifer for which

1) the discharge through both strata are equal,
2) the discharge through the confined aquifer is half of that through the unconfined aquifer?

4.1.7 An unconfined aquifer in a stratum of clean sand and gravel has a hydraulic conductivity of 10^{-2} cm/sec. From two observation wells 200 m apart, the observed water table elevations are 11 and 7 m measured from the bottom of the stratum. Determine the discharge per unit width of the aquifer.

4.1.8 An earthen dam is 200 m across (i.e., the distance from the upstream face to the downstream face) and underlain by impermeable bedrock. The average hydraulic conductivity of the mate-

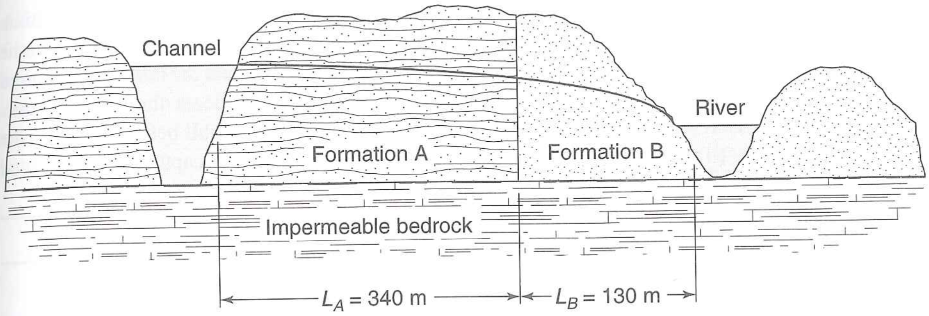


Figure to accompany Problem 4.1.9

rial of which the dam is composed is 0.065 m/day. If the water surface elevations in the reservoir and the tailwaters are 25 and 4.5 m, respectively, estimate the magnitude of leakage from the reservoir to the tailwaters per 100-m width of the dam.

4.1.9 Compute the volume of water that seeps from the channel into the river in the figure above. The water surface elevations in the channel and river with respect to the underlying bedrock are 13 and 10.5 m, respectively. The hydraulic conductivity of formation A is 5.6 m/day and that of formation B is 12.3 m/day.

4.1.10 A canal is constructed parallel to a river 460 m away both fully penetrating an unconfined aquifer of clean sand and gravel as shown in the illustration below. The aquifer has a hydraulic conductivity of K = 18.5 m/day and is subject to an average infiltration of 1.6 m/year. The water surface elevation in the canal is 8.5 m and in the river it is 10 m. If the mound between the canal and the river gets contaminated and the river is to remain free of contamination, (a) determine the daily discharge of groundwater into the canal and into the river per kilometer of both; (b) estimate the travel times from the water divide to the canal and to the river (n_e = 0.35); (c) assuming that the contaminant travels mainly by advection, propose any operational changes to the given layout to prevent the river from being contaminated.

4.2.1 A well that pumps at a constant rate of 0.5 m^3/s fully penetrates a confined aquifer of 34-m thickness. After a long period of pumping, near steady-state conditions, the measured drawdowns at two observation wells 50 and 100 m from the pumping well are 0.9 and 0.4 m, respectively. (a) Calculate the hydraulic conductivity and transmissivity of the aquifer, (b) estimate the radius of influence of the pumping well, and (c) calculate the expected drawdown in the pumping well if the radius of the well is 0.4 m.

4.2.2 A confined aquifer of 10-m thickness and 16.43 m/day hydraulic conductivity is fully penetrated by a pumping well of 0.5 m radius operating at Q = 425 m^3/day. Determine the drawdown under steady-state conditions in the pumping well and 50 m away from the well. Take the radius of influence of the pumping as 300 m.

4.2.3 What percent increase/decrease would occur in the drawdown of the pumping well if the radius of the well is doubled and the pumping rate is kept the same in Problem 4.2.2? Assume the same radius of influence.

4.2.4 What percent increase/decrease would occur in the well flow if the well diameter is doubled and the drawdown in the well is kept constant in Problem 4.2.2? Assume the same radius of influence.

4.2.5 The initial piezometric surface in a confined aquifer of 20-m thickness is 34 m above the bottom. After a long period of pumping, the piezometric surface stabilizes at 29.3 m above the bottom. The hydraulic conductivity of the aquifer is 12.2 m/day. If the radius of the well is 0.5 m and the radius of influence of the pumping is 500 m, what is the steady-state well discharge?

4.2.6 The initial piezometric head in a confined aquifer that has a thickness of 11.6 m is 85.7 m above sea level. A well with a radius of 0.5 m pumps at a constant rate of 1,240 m^3/day. After the cone of depression has achieved equilibrium, the piezometric heads at two observation wells 40 and 95 m from the pumping well are measured as 78.9 and 83.4 m above sea level, respectively. Also, the piezometric head in the pumping well during equilibrium remains at 46.6 m. Determine (a) the aquifer transmissivity, (b) the radius of influence of the pumping, and (c) the total well losses in and around the pumping well.

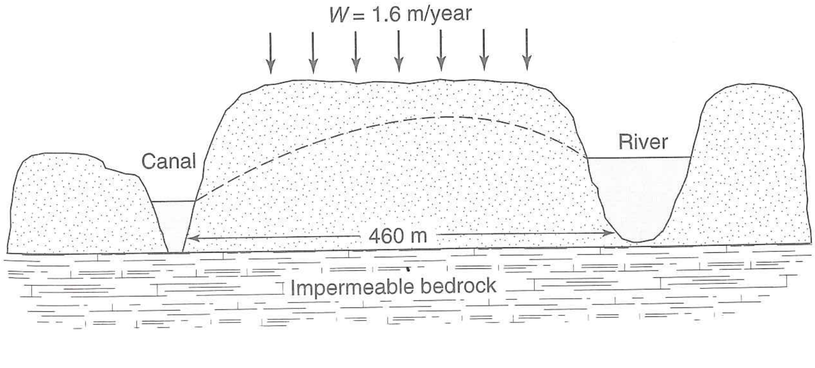


Figure to accompany Problem 4.1.10

4.2.7 A pumping well of 0.75-m radius fully penetrates an unconfined aquifer of 24-m thickness and produces at a rate of 10 L/s. After a long period of pumping, the drawdown in an observation well 30 m from the pumping well is 1.6 m. The drawdown in another observation well 60 m from the pumping well is 1.1 m. Calculate (a) the hydraulic conductivity of the aquifer, (b) the expected drawdown in the pumping well, and (c) the radius of influence of the pumping well.

4.2.8 After a long period of pumping from an unconfined aquifer at a constant rate of 850 m^3/day, the cone of depression reaches equilibrium. The aquifer has an initial saturated thickness of 20 m and a hydraulic conductivity of 8.65 m/day. During the equilibrium, the water levels in an observation well 50 m away and in the pumping well are measured as 18.4 and 9.9 m. Determine (a) the radius of influence of the pumping, (b) the radial distance where the steady state drawdown is 5 cm, (c) the expected drawdown in the pumping well ($r_w = 0.4$ m), and (d) the total well head losses.

4.2.9 Water is pumped at a constant rate of 500 m^3/day from an unconfined aquifer whose thickness is 15 m and hydraulic conductivity is 5.5 m/day. Given that the steady-state drawdown 50 m from the pumping well is 2.5 m, plot the water table profile under steady-state conditions for $r > 1.5H$, where H is the initial saturated thickness of the aquifer.

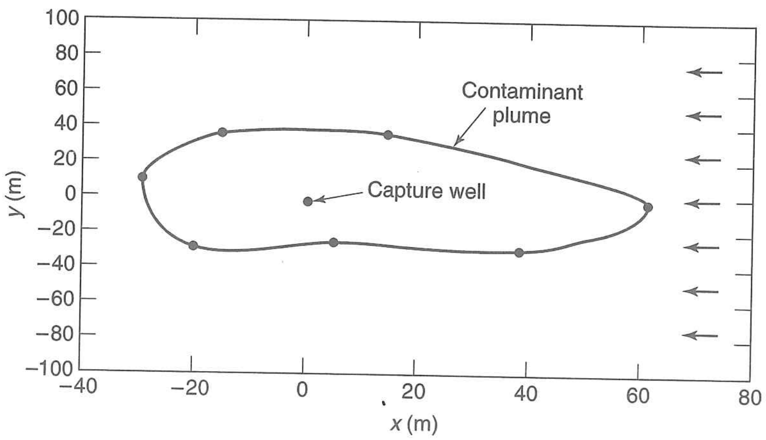
4.2.10 Rework Problem 4.2.5 in the absence of the recharge. (*Hint:* You may have to guess the radius of influence for this problem.)

4.2.11 Rework Problem 4.2.5 with the recharge rate doubled (i.e., W = 0.12 m/day). Approximately what percentage of the extracted water comes from the aquifer itself?

4.2.12 A well with a radius of 0.5 m pumps at the rate of 15 L/s from an unconfined aquifer that is uniformly recharged at a rate of 0.6 m/day. Without pumping from the well, the water table is nearly horizontal and the aquifer thickness is 30 m. The hydraulic conductivity of the aquifer is 20 m/day. Determine the radius of influence of the well and the approximate drawdown at the well location near steady-state conditions.

4.3.1 A confined aquifer of 35-m thickness is contaminated as shown in the plan view below. The regional groundwater gradient is $i = 0.003$ and the confined aquifer has a hydraulic gradient of 20 m/day. A capture well is proposed to clean up the contamination. The coordinates of the limits of contamination with respect to the capture well are tabulated below. Determine the minimum required pumping rate for the cleanup and delineate the capture zone. Assume that the plume remains relatively still before or during the operation of the capture well. Note that the capture well is located at (0 m, 0 m).

x (m)	y (m)
−15	35
−29	9.5
−20	−28.8
5	−26
38	−29
62	−1
14.2	33.4
−15	35



4.4.1 A fully penetrating production well pumps from a confined aquifer at a constant rate of 64 L/s. If the coefficients of transmissivity and storage of the aquifer are 1,240 m^2/day and 4×10^{-4}, respectively, estimate the drawdowns at a distance of 200 m from the pumping well for pumping periods of 8 hours, 30 days, and 6 months using the Theis equation. Also, estimate the radius of influence of the pumping well after 1 hour of pumping.

4.4.2 Rework Problem 4.4.1 using the Cooper–Jacob method of solution.

4.4.3 Given that T = 125 m^2/day, $S = 10^{-4}$, t = 2,693 min, Q = 5,500 m^3/day, and r = 305 m for a confined aquifer, compute the drawdown.

4.4.4 Using the Cooper–Jacob method of solution, plot drawdown versus time for four observation wells 5, 50, 100, and 500 m from a well pumping at a constant rate of 300 m^3/day in a sandy confined aquifer with b = 12 m, K = 25 m/day, and $S = 10^{-4}$. Given that depths to water in wells and piezometers can be measured with an accuracy of about 0.5 cm, how long does it take to reach near-steady conditions? Also, calculate the volume of water accumulated above ground until the near-steady conditions are achieved at those distances.

4.4.5 Drawdown was measured during a pumping test in a confined aquifer at frequent intervals in an observation well 200 ft from a well that was pumped at a constant rate of 500 gpm. The data for this pump test is listed below. Determine T and S for this aquifer.

Pump test data	
Time (min)	Drawdown (ft)
1	0.05
2	0.22
3	0.40
4	0.56
5	0.70
7	0.94
10	1.2
20	1.8
40	2.5
100	3.4
300	4.5
1,000	5.6
4,000	7

4.4.6 For the time–drawdown data listed below for a confined aquifer, calculate T and S using Jacob's approximation. After computing T and S, check to see that the basic assumption of this approximation is satisfied. For the values of T and S that you computed, after how many minutes of pumping would Jacob's approximation be valid? The discharge is $Q = 1{,}500$ gpm and the radius $r = 300$ ft.

Time after pumping started (min)	Drawdown (ft)
1	0.45
2	0.74
3	0.91
4	1.04
6	1.21
8	1.32
10	1.45
30	2.02
40	2.17
50	2.30
60	2.34
80	2.50
100	2.67
200	2.96
400	3.25
600	3.41
800	3.50
1,000	3.60
1,440	3.81

4.4.7 For the time–drawdown data recorded during the recovery phase of a pump test, determine the transmissivity and storativity of the confined aquifer. The pumping rate is 162.9 ft^3/min and the pump is shut off at 800 min.

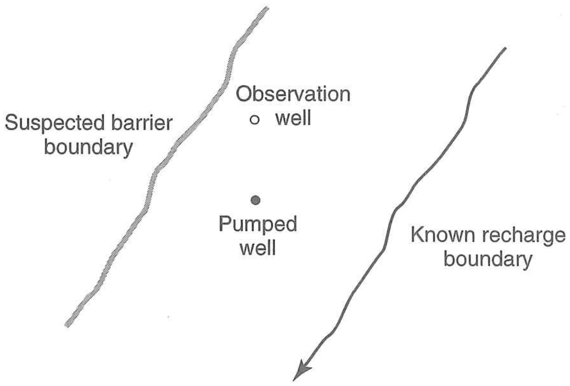
Recovery of test well		Recovery of observation well	
Time (min)	s' (ft)	Time (min)	s' (ft)
800	12.5	800	1.86
803	20	805	1.78
808	5	810	1.64
813	0.5	815	1.53
820	1.5	820	1.45
880	1	825	1.37
940	0.80	830	1.32
995	0.69	840	1.22
1,055	0.59	850	1.15
1,115	0.51	860	1.09
1,175	0.49	870	1.03
1,235	0.46	880	0.97
1,295	0.38	890	0.94
1,360	0.34	900	0.90
1,416	0.33	910	0.87
1,418	0.33	920	0.85
1,527	0.22	980	0.70
1,600	0.22	1,040	0.61
		1,100	0.54
		1,160	0.49
		1,220	0.46
		1,280	0.40
		1,340	0.36
		1,400	0.36
		1,460	0.34
		1,520	0.31
		1,600	0.29

4.4.8 (The data given in this problem also refer to Problems 4.4.9 and 4.7.1 below.) A fully penetrating pumping well in a confined aquifer is located between a suspected barrier boundary and a known recharge boundary as shown in the figure on the next page. Drawdown data from the observation well in the figure during a typical pumping test of the well in question are given in the following table. If the pumping is constant at a rate of 100 gpm and the observation well is 200 ft away from the pumping well, determine the transmissivity and the storage coefficient of the aquifer using the given aquifer test data.

Data from observation well	
Time (min)	Drawdown (ft)
2	0.025
3	0.04

(*continues*)

Data from observation well	
Time (min)	Drawdown (ft)
4	0.06
5	0.07
6	0.085
7	0.095
8	0.105
9	0.115
10	0.12
15	0.16
20	0.18
30	0.22
40	0.25
50	0.27
60	0.285
70	0.3
80	0.33
90	0.35
100	0.37
150	0.45
200	0.54
300	0.58
400	0.64
500	0.69
600	0.74



4.4.9 Determine the aquifer properties in Problem 4.4.8 using the Cooper–Jacob method of solution.

4.5.1 (This problem is adapted from *U.S. Geological Survey Water-Resources Investigations Report* 89-4081 (1989).) The Galena-Platteville Aquifer Test was performed on June 8–15, 1987, with a 6-inch-diameter fully penetrating well pumping under a constant rate (Q) of 19.4 gal/min for 54 hr. Eight partially penetrating observation wells in the Galena-Platteville Aquifer were monitored during the pumping test. The time–drawdown data for observation well PZ-1, which is 108 ft from the pumping well, is given in the following table. The average thickness of the aquifer near this observation well is 84 ft. The test also reported that the Galena-Platteville Aquifer has a heterogeneous and anisotropic nature and consists of two media, fractures and porous rock. The estimated hydraulic conductivity and specific yield of the aquifer are respectively 8 ft/day and 0.049 from the PZ-1 observation well data. Using the given time–drawdown data and the Neuman Type A and Type Y curves for unconfined aquifers, make your own estimate of the aquifer hydraulic conductivity and specific yield.

Time (min)	Drawdown (ft)	Time (min)	Drawdown (ft)
0.9	0.012	100	0.805
1	0.014	230	0.842
1.5	0.032	320	0.86
2	0.052	410	0.877
2.5	0.072	450	0.883
3	0.093	530	0.891
3.5	0.111	630	0.903
4	0.13	810	0.946
4.5	0.148	890	0.969
5	0.165	980	0.98
6	0.198	1070	0.976
7	0.254	1190	0.991
8	0.277	1350	1.006
9	0.299	1525	1.031
10	0.319	1720	1.065
20	0.509	2130	1.085
30	0.687	2250	1.114
40	0.754	2400	1.136
50	0.774	2500	1.149
60	0.783	2600	1.159
70	0.788	2810	1.175
80	0.791	3140	1.204
90	0.8		

4.6.1 (This problem is adapted from the *Ground Water Manual.*[62]) The drawdown versus time data for an observation well 160 feet from the pumping well of the same aquifer as in Example 4.6.1 is given below. Estimate the transmissivity and storage coefficient of the confined aquifer and the permeability of the aquitard. Compare the results with the answers to Example 4.6.1.

Time (min)	Drawdown (ft)
4	2.15
6	2.86
8	3.46
10	3.78
15	4.58
20	5.09

(continues)

Time (min)	Drawdown (ft)
25	5.49
30	5.85
40	6.37
50	6.64
60	6.8
70	6.96
80	7.16
90	7.36
100	7.44
110	7.52
120	7.56
150	7.64
180	7.88
210	7.92
240	7.96
270	7.96
300	7.96
360	7.95
420	7.96

4.7.1 What would the drawdown in the pumping well be in Problem 4.4.8 at the end of pumping at a constant rate of 100 gpm for a continuous period of 180 days? The pumping well with a radius of 1 ft is located 500 ft away from the barrier boundary and 1,000 ft away from the recharge boundary.

4.7.2 (This problem is adapted from the *Ground Water Manual.*[62]) Drawdown versus time data for an observation well 100 ft from the pumping well in a pump test are tabulated below. Identify the type of boundary and determine the radius of the image well from the observation well. What additional information would you need to locate the boundary? (*Hint:* Use the method described at the end of Section 4.7)

Time (min)	Drawdown (ft)
5	0.08
10	0.22
15	0.32
20	0.41
25	0.49
30	0.56
40	0.67
50	0.77
60	0.85
70	0.95
80	1.01
90	1.08
100	1.14
110	1.20
120	1.25
180	1.51
240	1.70
300	1.87
360	1.99
420	2.10
480	2.20
540	2.28
600	2.36
660	2.46
720	2.50
840	2.63
960	2.77

(*continues*)

4.7.3 A production well fully penetrating a nonleaky isotropic artesian aquifer delimited by two barrier boundaries (perpendicular to each other) was continuously pumped at a constant rate of 1,485 gpm for a period of four hours. The drawdowns in the following table were observed at a distance of 300 ft in a fully penetrating observation well. Compute the coefficients of transmissivity and storage of the aquifer and the distances to each image well from the observation well.

t (min)	s (ft)
2	0.80
3	0.92
4	1.06
5	1.17
6	1.23
7	1.32
8	1.37
9	1.43
10	1.48
20	1.88
30	2.11
40	2.34
50	2.52
60	2.70
70	2.83
80	3.00
90	3.17
100	3.30
200	4.21
300	4.43

4.8.1 It is required to dewater a construction site 80 m by 80 m. The bottom of the construction will be 1.5 m below the initial water surface elevation of 90 m. Four pumps are to be used in 0.5-m diameter wells at the four corners of the site. Determine the required pumping rate. The aquifer has $T = 1600$ m^2/day and the wells each have a radius of influence of 600 m.

4.8.2 Reposition the wells in Example 4.8.1 such that they form an equilateral triangle (same spacings). For the same restrictions on the drawdown, will the discharge decrease or increase? If so, by what percent? If not, what difference do you perceive between the two problems?

4.8.3 Rework Problem 4.8.1 as if the site is to be ready after one month of pumping. Assume the storage coefficient of the aquifer is $S = 0.16$.

4.8.4 Two pumping wells 1,000 m away fully penetrate the same confined aquifer. One of the wells pumps at a rate of 1,240 m^3/day. The second well pumps at 850 m^3/day. If the aquifer has a transmissivity of 2,000 m^2/day and a storage coefficient of 4×10^{-4}, when would the wells start interfering with each other?

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