Hydraulics and design of wells

Movement of Groundwater

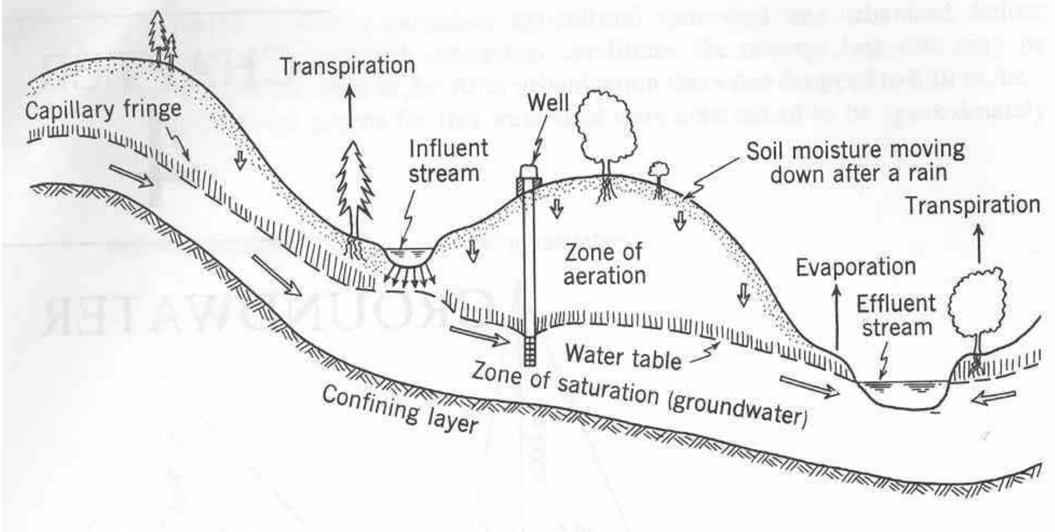


FIGURE 4.1
Schematic diagram illustrating the occurrence of groundwater.

Darcy's Law

$$q = \frac{Q}{A} = KJ \quad \textbf{(1)}$$

$q =$ Specific discharge (m/s)

$Q =$ Flowrate (m^3/s)

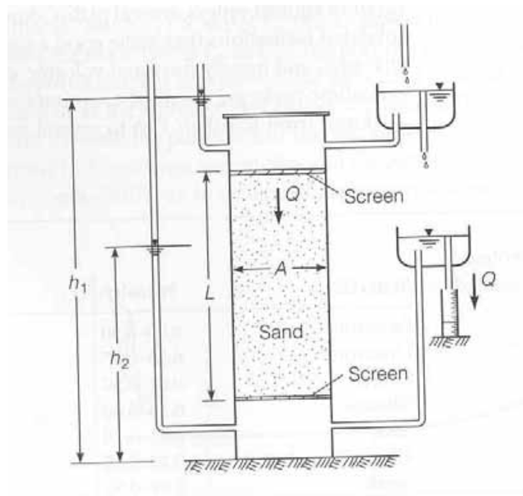
$A =$ Cross-sectional area of the aquifer for Q

$K =$ Saturated Hydraulic Conductivity or Coefficient of Permeability

$J =$ Slope of the energy line

Determination of Permeability in the Laboratory

$$Q = KA\frac{h_1 - h_2}{L} \qquad (2)$$



1. Constant Head Permeameter
2. Falling Head Permeameter

Determination of Permeability in the field

Hydraulics of Wells

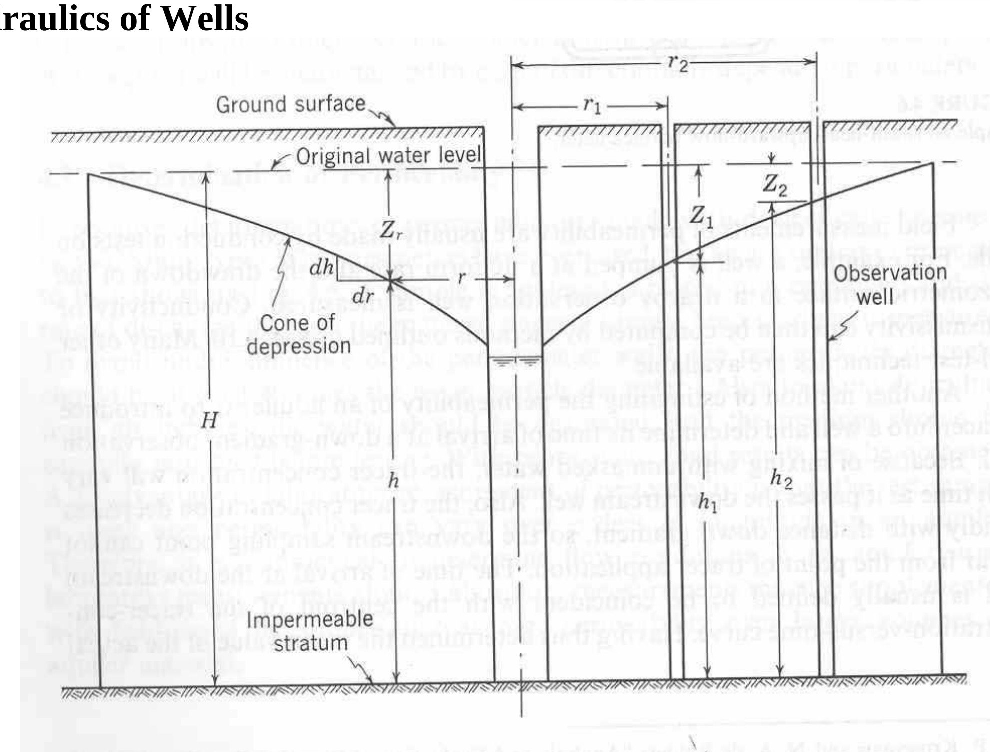


FIGURE 4.7
Definition sketch for a well-discharge equation for water-table conditions.

$$Q = 2\pi r h K \frac{dh}{dr} \tag{3}$$

Integrating w.r.t. r from r_1 to r_2

$$Q = \frac{\pi K (h_2^2 - h_1^2)}{\ln(r_2 / r_1)} = \frac{1.36 K (h_2^2 - h_1^2)}{\log(r_2 / r_1)} \tag{4}$$

Moreover,

$h_2^2 - h_1^2 = (h_2 - h_1)(h_2 + h_1)$ and if $Z << H$

$T \cong K \dfrac{h_2 + h_1}{2} \cong KH$ then

$$Q = \frac{2\pi KH (h_2 - h_1)}{\ln(r_2 / r_1)} = \frac{2\pi T (h_2 - h_1)}{\ln(r_2 / r_1)} = \frac{2.72 T (h_2 - h_1)}{\log(r_2 / r_1)} \tag{5}$$

This equation is called as Thiem Equation

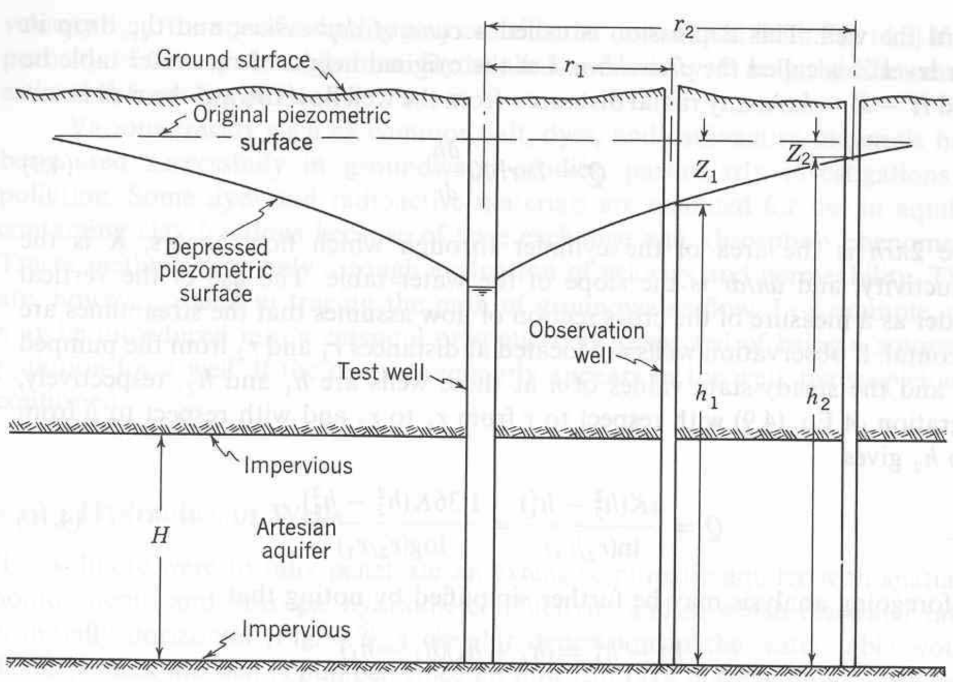


FIGURE 4.8
Definition sketch for a well-discharge equation for confined conditions.

Limitations of Thiem Equation

1. Fully penetrated well
2. $Z << H$
3. Steady discharge

Theis Formula

$$Z_r = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}du \qquad (6)$$

$Z_r =$ Drawdown in an observation well at a distance r from the pumping well.

Q and T must be expressed in similar units, and

$$u = \frac{r^2 S}{4Tt} \qquad (7)$$

$t =$ Time in days since pumping began

$S =$ Storage coefficient of the aquifer (dimensionless): Volume of water available from a unit column of aquifer when water table or piezometric surface is lowered a unit distance.

The integral is expressed as the well function of u and calculated from the following series;

$$\int_u^\infty \frac{e^{-u}}{u}du = W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \qquad (8)$$

Problem: Determine the well function values for a range of values of u from 1 to 0.00001.

Aquifer analysis

Theis formula is used to determine T an S from the pumping tests in the field.

Graphical methods to determine T and S from the pumping tests

1. Theis Method
   a. Plot “Type Curve”, i.e. Graph between W(u) and u on logarithmic scale.
   b. Plot r^2/t versus Z on the same scale as the Type Curve. From Theis formula it is evident that if Q is constant, Z is proportional to W(u) and from the expression for u it is clear that r^2/t is proportional to u. So the graph between r^2/t and Z should be similar to the Type Curve.
   c. Superimpose both the curves with coordinate axes parallel, and read the coordinates of a common point from the curves. These coordinates are then used to solve for T and S.

Example: A 12-in diameter well is pumped at a uniform rate of 1.1 cfs while observations of drawdown are made in a well 100 ft distant. Values of t and Z observed together with values of r^2/t are given in what follows. Find T and S for the aquifer, and estimate the drawdown in the observation well at the end of 1 year of pumping.

t, hr	1	2	3	4	5
Z, ft	0.6	1.4	2.4	2.9	3.3
r^2/t, ft^2/d	2.4×10^5	1.2×10^5	8×10^4	6×10^4	4.8×10^4

t, hr	6	8	10	12	18	24
Z, ft	4.0	5.2	6.2	7.5	9.1	10.5
r^2/t, ft^2/d	4.0×10^4	3.0×10^4	2.4×10^4	2.0×10^4	1.3×10^4	1×10^4

Solution:

The relation between r^2/t and Z and between W(u) and u are plotted on separate sheets with the same logarithmic scale and superimposed.

The coordinates of the match point:
Type curve: u = 0.40 W(u) = 0.70

Data curve: Z = 3.4 ft $r^2/t = 5.3\times10^4$ ft^2/d

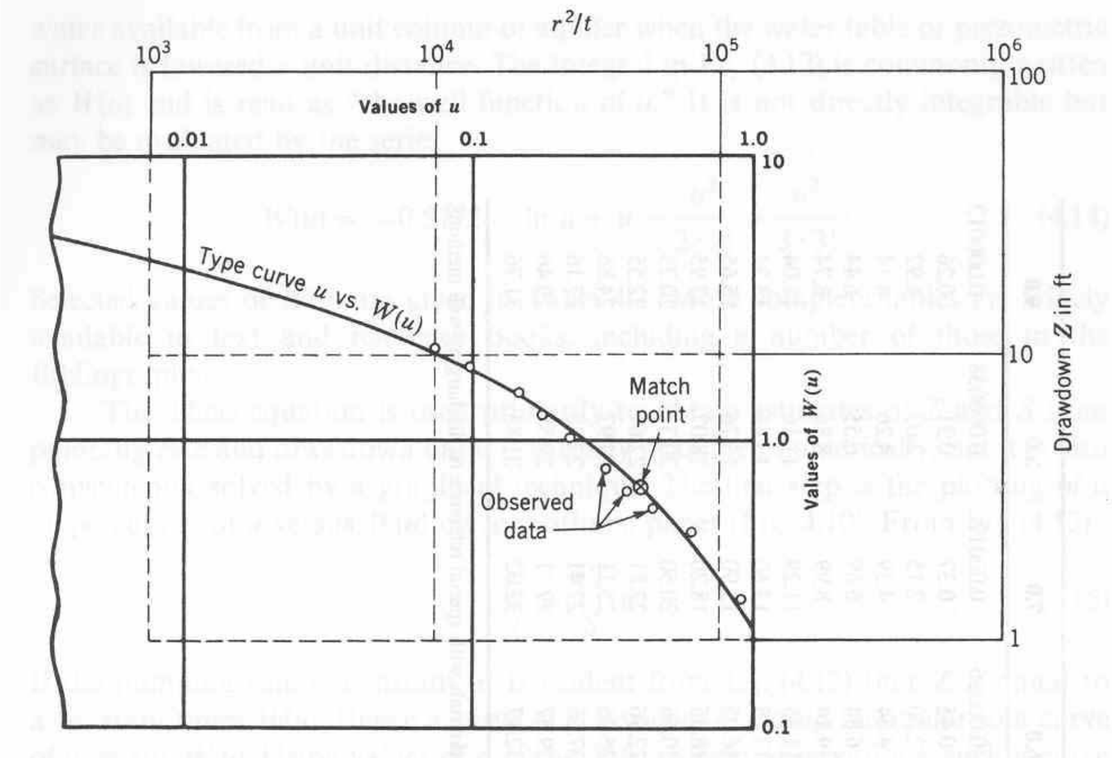


FIGURE 4.10
Graphical solution of a well problem by the Theis method.

From Eq.(6), $$T = \frac{QW(u)}{4\pi Z} = \frac{1.1 \times 86{,}400 \times 0.7}{4\pi \times 3.4} = 1560\,\frac{ft^2}{day}$$

From Eq.(7), $$S = \frac{4uT}{r^2/t} = \frac{4 \times 0.4 \times 1560}{5.3 \times 10^4} = 0.047$$

At the end of one year

$$u = \frac{r^2 S}{4Tt} = \frac{(100)^2 0.047}{4 \times 1560 \times 365} = 2.06 \times 10^{-4}$$

Hence W(u) = 7.92 (From the table or Eq. 8)
And, from Eq. (6)

$$Z = \frac{Q}{4\pi T} W(u) = \frac{1.1 \times 86{,}400 \times 7.92}{4\pi \times 1560} = 38.4\ ft$$

Cooper-Jacob Method

When u is small, Eq.(8) may be written as

$$W(u) = -0.5772 - \ln u \tag{9}$$

And we know from Eq.(7) that u becomes small when t is large. Therefore, for large values of t, Theis formula may be given as;

$$T = \frac{2.3Q}{4\pi\Delta Z}\log\left(\frac{t_2}{t_1}\right) \tag{10}$$

where ΔZ is the change in drawdown between t_1 and t_2. This equation is called Cooper-Jacob equation.

If ΔZ is taken for one complete log cycle, $\log(t_2/t_1) = 1$ and T can be computed from Eq.(10). Using Eq.(9) in Eq.(6) and setting Z to zero, we get,

$$S = \frac{2.25Tt_0}{r^2} \tag{11}$$

Example: Using the Cooper-Jacob method, find the transmissivity and storage coefficient for the data of the previous example.

Solution:

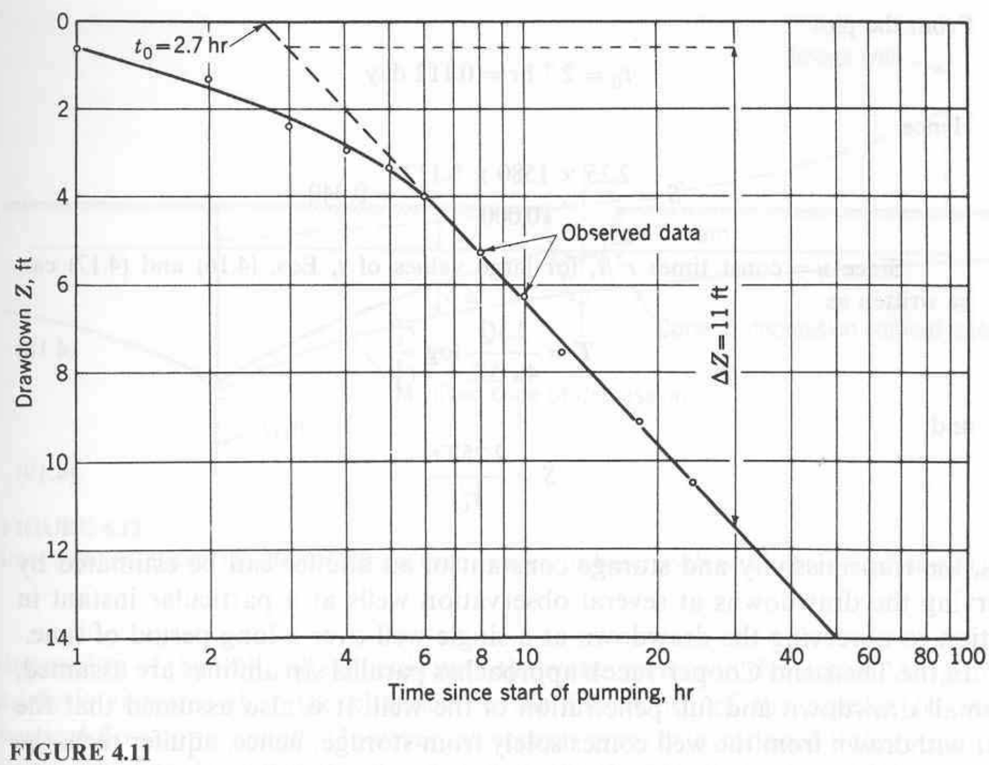


FIGURE 4.11
Graphical solution of a well problem by the Cooper–Jacob method.

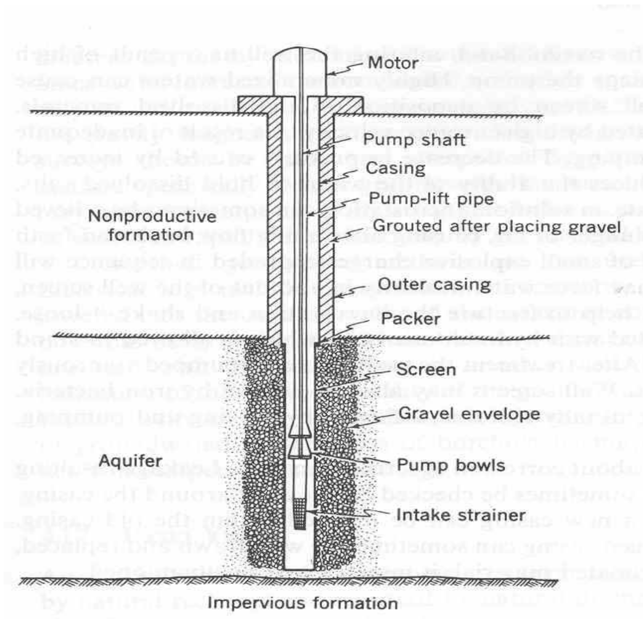
From the figure, we can see between t = 3 hr and t = 30 hr, $\Delta Z = 11$ ft.,
Hence,

$$T = \frac{2.3 \times 1.1 \times 86{,}400}{4\pi \times 11} = 1580\ ft^2 / day$$

From the plot, $t_0 = 2.7\,hr = 0.112\,day$, Hence,

$$S = \frac{2.25 \times 1580 \times 0.112}{(100)^2} = 0.04$$

Design of wells



Purposes of wells:

1. Water supply
2. Dewatering construction sites
3. Artificial- recharge
4. Wastewater disposal, and
5. Groundwater contamination cleanup

Well construction methods

1. Driven or jetted well point
2. Auger bore hole
3. Drilled bore hole
4. Rotary drilling

Casing design

For a water supply well, the diameter of the casing is generally selected as 50 mm larger than the bowl of the pump to be installed in the well.

Table 1

Pumping rate (L/min)	Optimum casing diameter (mm)	Minimum casing diameter (mm)
< 400	150 mm ID	125 mm ID
300-660	200 mm ID	150 mm ID
570-1,500	250 mm ID	200 mm ID
1,300-2,500	300 mm ID	250 mm ID
2,300-3,400	350 mm OD	300 mm ID
3,200-4,900	400 mm OD	350 mm OD
4,500-6,800	500 mm OD	400 mm OD
6,100-11,000	600 mm OD	500 mm OD

Depending on the method used for grouting, the hole is drilled 50-150 mm larger than the outside diameter of the casing. After the casing is set in position, the annular space between the casing and hole is filled with grout.

Screen design

For maximum yield from a confined aquifer, 70-80 percent of the aquifer thickness is screened.

For an unconfined aquifer, typically the lower one-third of the aquifer is screened and the upper two-thirds of the aquifer is reserved for drawdown.

The entrance velocity can be given as;

$$v_s = \frac{Q}{c\pi d_s L_s P}$$

$Q =$ Pumping rate, $c =$ Clogging coefficient (approximated as 0.5 assuming that 50% of the open area of a screen is blocked by aquifer material, $d_s =$ Screen diameter, $L_s =$ Screen length, $P =$ Fraction of open area in the screen (manufacturers specifications)

Table 2

Pumping rate (L/min)	Screen diameter (mm)
< 190	50
190-475	100
475-1,330	150
1,330-3,040	200
3,040-5,320	250
5,320-9,500	300
9,500-13,300	350
13,300-19,000	400
19,000-26,000	450
26,600-34,200	500

Table 3

Hydraulic conductivity of aquifer (m/d)	Optimal screen entrance velocity (m/min)
> 250	3.7
250	3.4
200	3.0
160	2.7
120	2.4
100	2.1
80	1.8
60	1.5
40	1.2
20	0.9
< 20	0.6

The well screen opening size is selected to ensure that formation sand does not enter the well during normal operation. The opening size is based on a grain size analysis of a sample from the aquifer formation. The following figure is a plot of an aquifer grain size analysis. The uniformity coefficient (C_u) is defined as the diameter with 60 percent larger than (d_{60}) divided by the diameter with 10 percent larger than (d_{10}) or

$$C_u = \frac{d_{60}}{d_{10}}$$

Table 4

Aquifer properties	Screen slot size
$C_u < 3$, $d_{10} > 0.25mm$	$d_{40} - d_{60}$
$3 \le C_u \le 5$, $d_{10} > 0.25mm$	$d_{40} - d_{70}$
$C_u > 5$, $d_{10} > 0.25mm$	$d_{50} - d_{70}$

Gravel-pack design

For very fine aquifer material, gravel pack of 8 to 23 cm thickness is used.

Table 5

C_u of aquifer material	Gravel pack criteria	Screen slot size
< 2.5	(a) C_u between 1 and 2.5, with the 50% size not greater than six times the 50% size of the aquifer (preferable) (b) If (a) is not available, C_u between 2.5 and 5, with 50% size not greater than nine times the 50% size of the aquifer (alternative)	5% to 10% passing size of the gravel pack
2.5-5	(a) C_u between 1 and 2.5, with the 50% size not greater than nine times the 50% size of the formation (preferable) (b) If (a) is not available, C_u between 2.5 and 5, with 50% size not greater than 12 times the 50% size of the aquifer (alternative)	5% to 10% passing size of the gravel pack
> 5	Multiply the 30% passing size of the aquifer by 6 and 9 and locate the points on the grain-size distribution graph on the same horizontal line. Through these points draw two parallel lines representing materials with $C_u \le 2.5$. Select gravel pack material that falls between the two lines.	5% to 10% passing size of the gravel pack

Sand trap

A section of well casing below the screened intake having a length of 2 to 6 m and diameter typically the same as the screen is provided to trap the sand coming into the well.

Example: A water-supply well is to be installed in a 25-m thick unconfined aquifer which has a hydraulic conductivity of 30 m/d. A grain-size analysis of the aquifer indicates a uniformity coefficient of 2.7 and a 50-percentile grain size of 1 mm. If the well is to be pumped at 20 L/s, design the well screen and the gravel pack.

Solution

According to Table 5,

Since C_u of the aquifer matrix = 2.7, the gravel pack should have a C_u between 1 and 2.5 and a 50% size not greater than nine times the 50-percentile size d_{90} of the aquifer. Hence, d_{50} for the gravel pack $\leq 9 \times 1 mm$.

Let us use $5 \times 9\,mm$ gravel, i.e. containing particle sizes between 5 and 9 mm. Assuming that the particle sizes are uniformly distributed between 5 mm and 9 mm, then

$d_{10} = 5 + 0.1(9-5) = 5.4\,mm$ and $d_{60} = 5 + 0.6(9-5) = 7.4\,mm$, So $C_u = 7.4/5.4 = 1.4$ which is between 1 and 2.5, hence it is acceptable. The thickness can be taken as 10 cm.

Screen slot size should be 5% to 10% passing size of the gravel pack, so select a slot size of 5mm.

The value of P can be obtained from manufacturers manuals, typically, P=0.1.

Since the screen length in unconfined aquifers should be between 0.3 and 0.5 times the aquifer thickness. Select $L_s = 0.5 \times H = 0.5 \times 25 = 12.5\,m$ to maximize the pumping per unit drawdown.

For a pumping rate of 20L/s(1,200L/min), optimum casing diameter can be chosen from Table 1 as 250mm and Table 2 indicates a minimum screen diameter of 150mm. Taking the screen and casing diameter as 250mm and assuming c=0.5, the screen entrance velocity is computed as

$$v_s = \frac{Q}{c\pi d_s L_s P} = \frac{20 \times (3600 \times 24)/1000}{0.5 \times \pi \times (250/1000) \times 12.5 \times 0.1} = 3520\,m/day = 2.4\,m/\min$$

According to Table 3, this entrance velocity is too high for an aquifer of K=30m/day. The desirable screen would produce an entrance velocity of 1.05m/min. approximately. Hence, taking $v_s = 1.05\,m/\min$, the corresponding screen diameter can be calculated as;

$$d_s = \frac{Q}{c\pi v_s L_s P} = \frac{1728}{0.5\pi \times (1.05 \times 60 \times 24) \times 12.5 \times 0.1} = 0.583\,m \approx 600\,mm$$

Hence the summary of the design;

Screen length = 12.5 m

Screen diameter= 600 mm

Screen slot size = 5 mm

Screen should be surrounded by $5 \times 9\,mm$ gravel pack with a thickness of about 10 cm.

Diameter of the well casing = 600 mm.