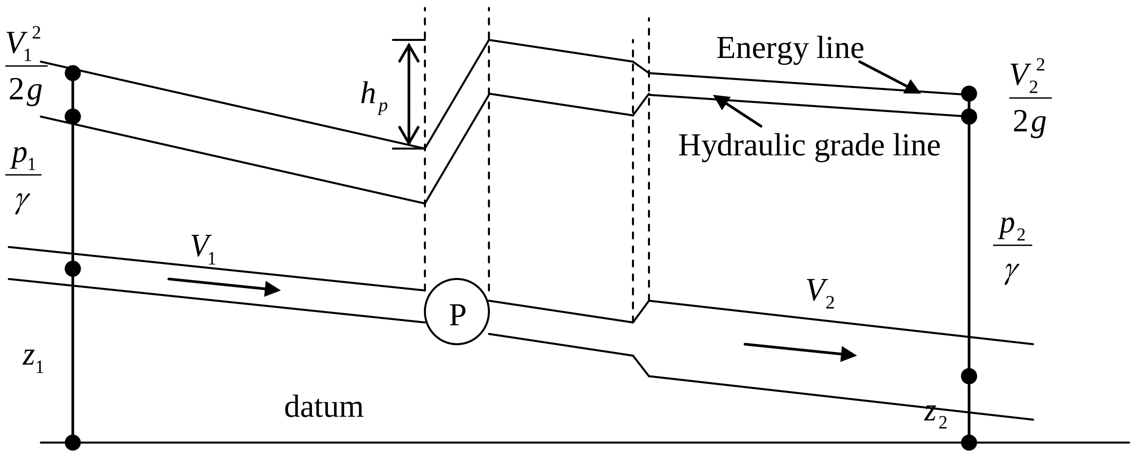
Analysis of Pipe Networks

Applications: Water supply system, Pressure sewerage system, Water treatment plant
Theory:

1. Continuity equation $$A_1V_1 = A_2V_2$$
2. Energy equation



$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$h_L = f\,\frac{L}{D}\frac{V^2}{2g}$$

Friction factor f is a function of Reynolds number (R_N) and relative roughness (k_s/D). It can be found from Moody's diagram or Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{R_N\sqrt{f}}\right)$$

Jain's approximation :

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{5.74}{R_N^{0.9}}\right)$$

Table 1. Equivalent sand roughness, k_s in new pipes, (Moody, 1944)

Material	**Equivalent sand roughness, k_s (mm)**
Riveted steel	0.9-9.0
Concrete	0.3-3.0
Ductile and cast iron	0.26
Galvanized iron	0.15
Asphalt-dipped ductile/cast iron	0.12
Commercial steel or wrought iron	0.046
Copper or brass tubing	0.0015
Glass, plastic (PVC)	≈ 0

Example: Water from a treatment plant is pumped into a distribution system at a rate of 4.38 m^3/s, a pressure of 480kPa, and a temperature of 20°C. The diameter of the pipe is 750mm and is made of ductile iron.

(a) Calculate the friction factor by Colebrook equation

(b) Estimate the pressure 200m downstream of the treatment plant if the pipeline remains horizontal.

Solution:

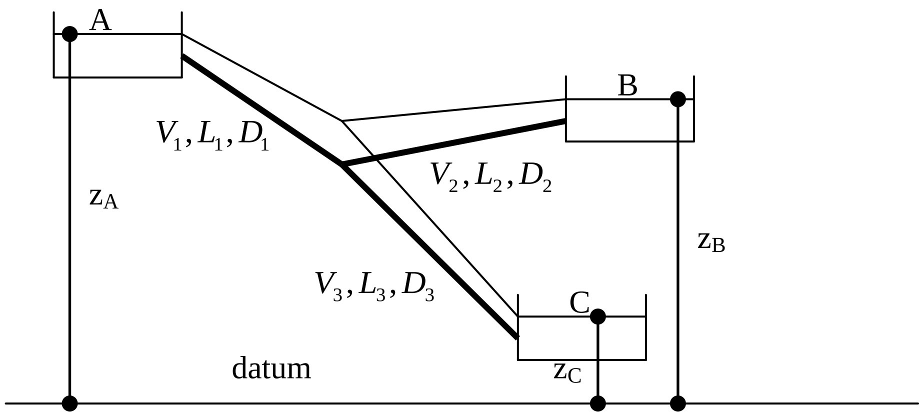
$f = 0.016$ $\qquad p_2 = 270$ kPa

Designing a pipe for a given flowrate and head loss

1. Assume f
2. Calculate diameter from rearranged Darcy-Weisbach equation: $$D = \left(\frac{8LQ^2}{h_L g \pi^2} f \right)^{1/5}$$
3. Calculate Reynolds number, $R_N = \frac{VD}{\nu}$
4. Calculate relative roughness, k_s / D
5. Use R_N and k_s/D to calculate f from Colebrook equation.
6. Using new value of f repeat the procedure until new f agrees with the old one.

Example: A galvanized iron service pipe from a water main is required to deliver 200 L/s during a fire. If the length of the service pipe is 35m and the head loss in the pipe is not to exceed 50m, calculate the minimum pipe diameter that can be used.
Solution: D= 136 mm

Three-reservoir problem



Unknowns: V_1, V_2, V_3

Equations: One continuity and two energy equations

Other friction formulas:

Hazen-Williams formula (SI units) $V = 0.85 C_H R^{0.63} S^{0.54}$

Manning's formula (SI units) $V = \frac{1}{n} R^{2/3} S^{1/2}$

Table 2. Pipe roughness coefficients

Pipe material	C_H	n
Ductile and cast iron:		
New, unlined	130	0.013
Old, unlined	80	-
Cement lined and seal coated	120	0.013
Steel:		
Welded and seamless	120	-
Riveted	110	0.015
Concrete	110	0.015
Vitrified clay	-	0.013
Polyvinyl chloride (PVC)	-	0.009

The head loss can be expressed as:

$$h_L = KQ^x$$

Manning's formula: $x = 2$ Hazen-Williams formula: $x = 1.85$

Darcy-Weisbach equation: $x = 1.75$ (smooth pipe) to $x = 2$ (rough pipe)

Design of water distribution system

A municipal water distribution system is used to deliver water to the consumer. Water is withdrawn from along the pipes in a pipe network system, for computational purposes all demands on the system are assumed to occur at the junction nodes.

Pressure is the main concern in a water distribution system. At no time should the water pressure in the system be so low that contaminated groundwater could enter the system at points of leakage.

The total water demand at each node is estimated from residential, industrial and commercial water demands at that node. The fire flow is added to account for emergency water demand.

Water Demand

Total water demand at a node is equal to the average water demand per capita multiplied by the population served by that node.

The population may be estimated by any of the available models of population growth. These models use the previously available population data for future projections.
For example, according to Arithmetic model:

$$P(t) = P_0 + kt$$

P(t) is the population at time *t* and P_0 is the reference population.
Similarly a higher order polynomial function can be obtained as follows:

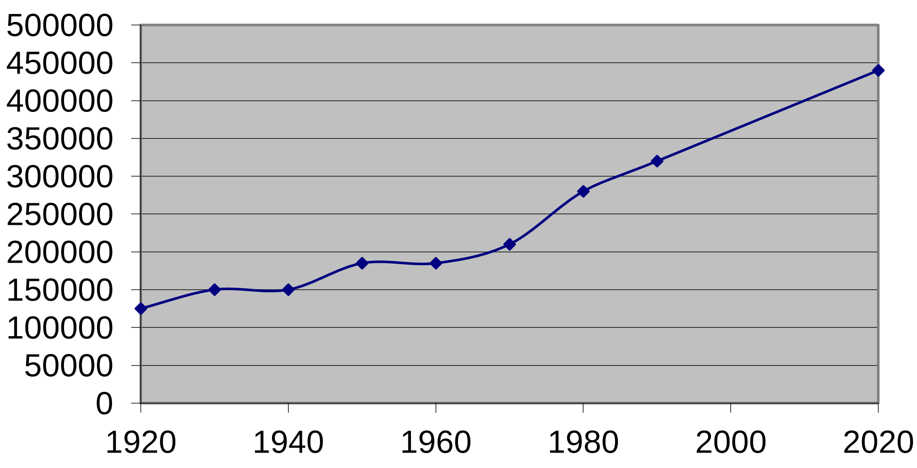
$$P(t) = P_0 + at + bt^2 + ct^3 + \cdots$$

and the population can be estimated at any time *t*.

Example: You are in the process of designing a water-supply system for a town, and the design life of your system is to end in the year 2020. The population in the town has been measured every 10 years since 1920 and is given below. Estimate the population in the town using graphical extension and arithmetic growth projection.

Year	Population
1920	125,000
1930	150,000
1940	150,000
1950	185,000
1960	185,000
1970	210,000
1980	280,000
1990	320,000

Solution:



From the graph, if the line is extended to year 2020, we get, the population as:

$$P = 440{,}000$$

By regression on the data for 1970 to 1990, we get the following expression:

$$P(t) = -60000 + 5500t$$

where, t is the time in years starting from year 1920. So, for year 2020, t= 100, and we get the population,

$$P = 490{,}000$$

Variations in demand

Maximum daily demand, Maximum hourly demand

Table 3. Typical demand factors

Condition	**Range of demand factors**	**Typical value**
Daily average in maximum month	1.1-1.5	1.2
Daily average in maximum week	1.2-1.6	1.4
Maximum daily demand	1.5-3.0	1.8
Maximum hourly demand	2.0-4.0	3.25
Minimum hourly demand	0.2-0.6	0.3

Fire demand

Insurance Services Office, Inc. (ISO, 1980) formula:

$$NFF_i = C_i O_i (X + P)_i$$

where, NFF_i is needed fire flow at location *i*, C_i is the construction factor based on size and type of construction of the building, O_i is the occupancy factor reflecting the kinds of material stored in the building (value range from 0.75 to 1.25), and $(X + P)_i$ is the sum of the exposure factor and communication factor that reflects the proximity and exposure of other buildings (value range from 1.0 to 1.75).

$$C_i\,(\mathrm{L/min}) = 220F\sqrt{A_i}$$

A_i is the effective floor area in square meters, typically equal to the area of the largest floor in the building plus 50% of the area of all other floors, F is a coefficient based on the class of construction. The maximum value of C_i and typical F values are given below:

Class of Construction	Description	F	Max. C_i(L/min)
1	Frame	1.5	30,000
2	Joisted masonry	1.0	30,000
3	Noncombustible	0.8	23,000
4	Masonry, noncombustible	0.8	23,000
5	Modified fire resistive	0.6	23,000

6	Fire resistive	0.6	23,000

Occupancy factors:

Combustibility class	Examples	O_i
C-1 Noncombustible	Steel or concrete products storage	0.75
C-2 Limited combustible	Apartments, mosques, offices	0.85
C-3 Combustible	Department stores, supermarkets	1.0
C-4 Free-burning	Auditoriums, warehouses	1.15
C-5 Rapid burning	Paint shops, upholstering shops	1.25

Average value of $(X+P)_i$ is 1.4. The NFF should be rounded to the nearest 1000L/min if less than 9000 L/min and to the nearest 2000L/min if greater than 9000L/min.

Required fire flow durations:

Required fire flow (L/min)	Duration (h)
<9000	2
11000-13000	3
15000-17000	4
19000-21000	5
23000-26000	6
26000-30000	7
30000-34000	8
34000-38000	9
38000-45000	10

Example: Estimate the flowrate and volume of water required to provide adequate fire protection to a 10-story noncombustible building with an effective floor area of 8000m^2.

Solution: NFF=17000L/min   Volume= 4080m^3.

Design periods and capacities in water supply systems:

Component	Design period(year)	Design capacity
Sources of supply: River Wellfield Reservoir	 Indefinite 10-25 25-50	 Maximum daily demand Maximum daily demand Average annual demand
Pumps*: Low-lift High-lift	 10 10	 Maximum daily demand Maximum hourly demand
Water treatment plant	10-15	Maximum daily demand
Service reservoir	20-25	Working storage + fire demand + emergency storage
Pipes	25-50	Greater of maximum daily demand + fire flow and maximum hourly demand

*: One reserve unit should be kept for the pumps.

Example: A metropolitan area has a population of 130000 people with an average daily demand of 600 L/d/person. If the needed fire flow is 20000 L/min, estimate, (a) the design capacities for the wellfield and the water-treatment plant, (b) the duration that the fire flow must be sustained and the volume of water that must be kept in the service reservoir in case of a fire and (c) the design capacity of the main supply pipeline to the distribution system.

Solution: (a) 1.62 m^3/s (b) 5940 m^3 (c) 2.92 m^3/s

Operating criteria for water supply systems

Primary functions of a water-supply system

1. Meet the demand while maintaining acceptable pressures
2. Supply water for fire without affecting the water supply to the rest of the system
3. Provide sufficient level of redundancy to serve during emergency conditions

Minimum acceptable pressure:

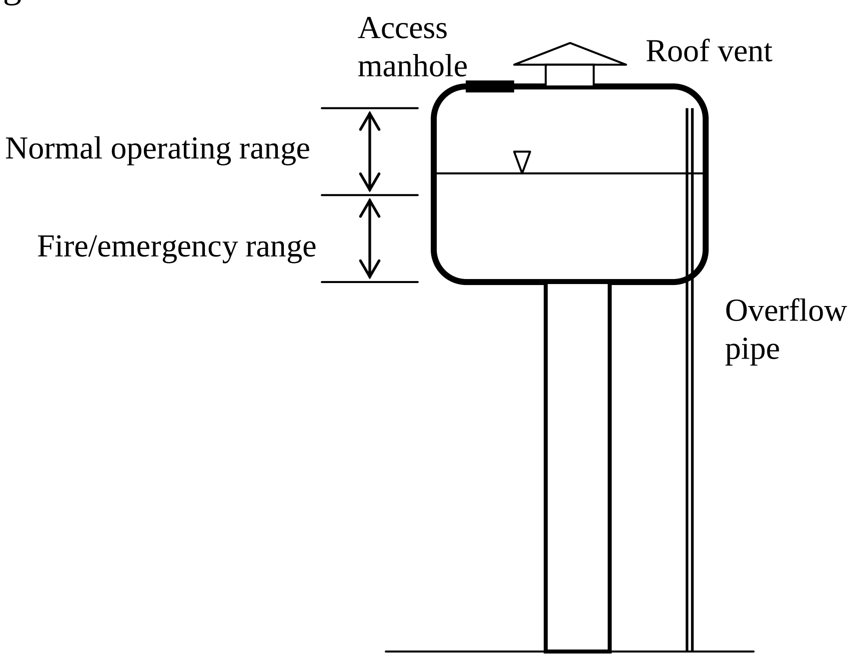
Under normal conditions = 240 to 410 kPa

During fire or emergency = >140 kPa

Maximum pressure (not strict) : 650kPa.

Storage facilities

1. 20% to 25% of the maximum daily demand volume
2. Fire demand
3. Emergency storage (minimum storage equal to average daily system denmand)
4. Minimum height of water in the elevated storage tank based on minimum piezometric head in the service area
5. Normal operating range 4.5 to 6.0 m.



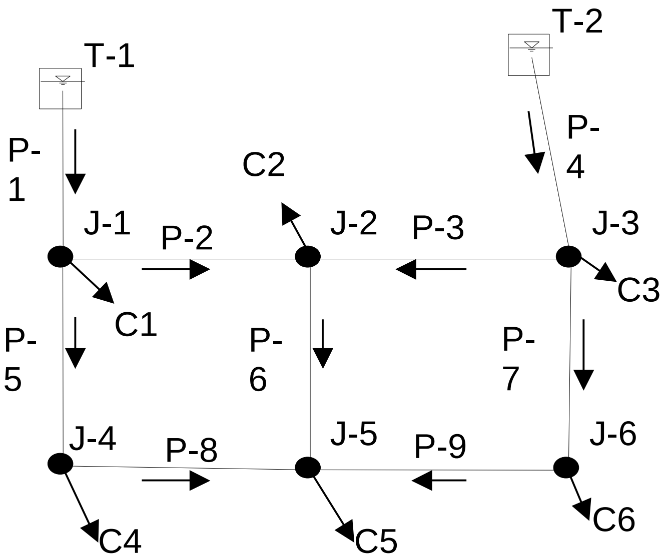
Example: A service reservoir is to be designed for a water-supply system serving 250,000 people with an average demand of 600L/d/capita and a needed fire flow of 37000 L/min. Estimate the required volume of service storage.

Solution: 237600m^3.

Example: A water-supply system is to be designed in an area where the minimum allowable pressure in the distribution system is 300kPa. A hydraulic analysis of the distribution network under average daily demand conditions indicates that the head loss between the low-pressure service location, which has a pipeline elevation of 5.4m and the location of the elevated storage tank is 10m. Under maximum hourly demand conditions, the head loss between the low-pressure service location and the elevated storage tank is 12m. Determine the normal operating range for the water stored in the elevated tank.

Solution: From 48m to 46 m.

Pipe Network Systems



$$N_{eq} = N_j + N_p + N_f - 1$$

N_{eq}= Number of equations required
N_j= number of junction nodes
N_l= number of loops
N_f= number of Fixed Grade Nodes (e.g. elevated reservoirs)

In the above figure

$$N_{eq} = 6 + 2 + 2 - 1 = 9$$

Conditions to satisfy:

1. The algebraic sum of the pressure drops around any closed loop must be zero
2. The flow entering a junction must equal the flow leaving it.

Hardy Cross Method

$$Q = Q_a + \Delta$$

$Q =$ Actual flow rate

$Q_a =$ Assumed flow rate

$\Delta =$ Correction

$$\sum K(Q_a + \Delta)^x = 0$$

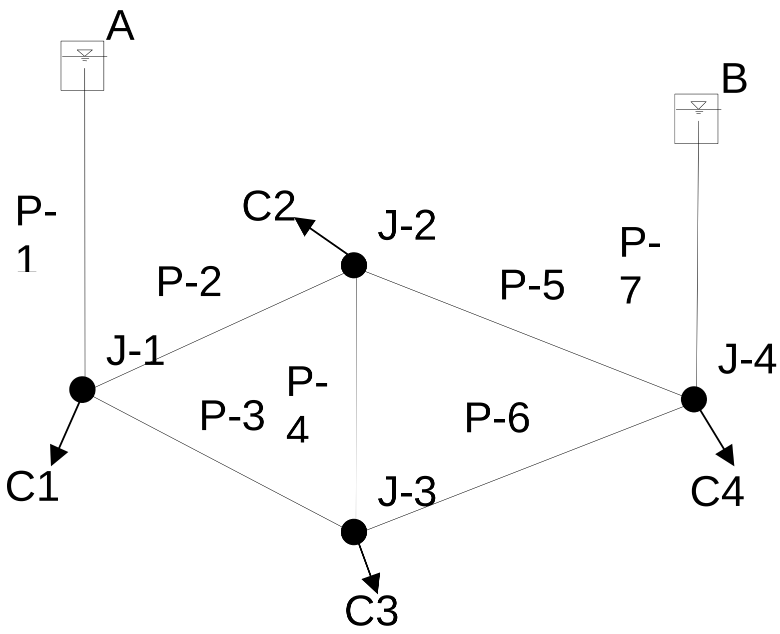
$$\sum KQ_a^x + \sum xK\Delta Q_a^{x-1} + \sum \frac{x-1}{2} xK\Delta^2 Q_a^{x-2} + \cdots = 0$$

For small values of correction

$$\sum KQ_a^x + \Delta \sum KxQ_a^{x-1} = 0$$

$$\Delta = -\frac{\sum KQ_a^x}{\sum \left| xKQ_a^{x-1} \right|}$$

Example: Determine the discharge in each line and the pressure at each junction node for the pipe network shown below using Hardy Cross method. The pipe and junction data is given in the table below.



Line	Nodes	Length(m)	Diameter(cm)	*f*	*K*(s^2/m^5)
1	A-1	300	30	0.015	153.0
2	1-2	250	20	0.019	1226.5
3	1-3	200	20	0.019	981.2
4	2-3	220	15	0.02	4787.6
5	2-4	180	20	0.019	883.1
6	3-4	250	20	0.019	1226.5
7	4-B	270	25	0.017	388.4

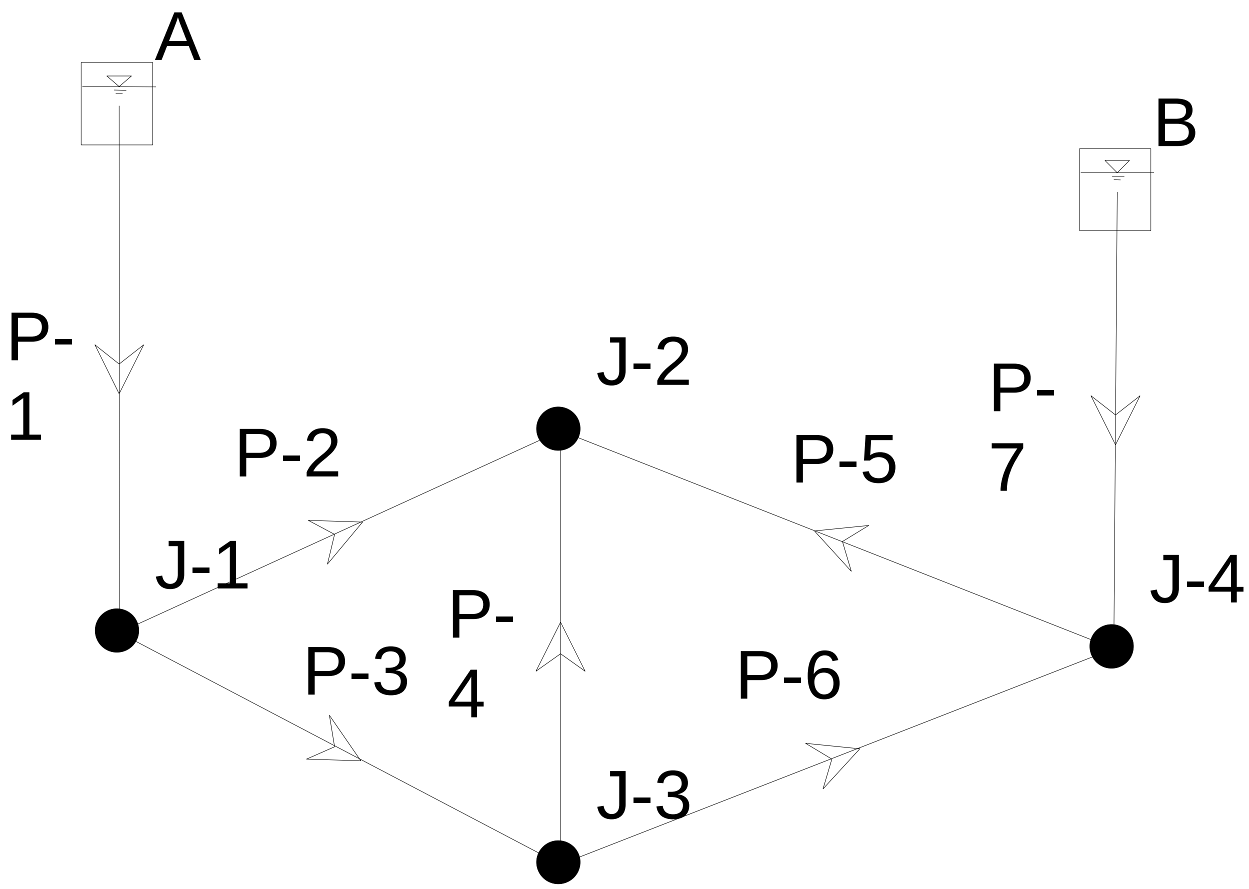
Node	Elevation(m)	Demand(L/s)
A	126	-
B	123	-
1	96	56
2	99	112
3	93	28
4	90	85

From Darcy-Weisbach formula, $h_L = KQ^2$, where

$$K = \frac{8fL}{\pi^2 g D^5}$$

HGL elevation at the end of pipe = HGL elevation at the beginning-Head loss

$$p = \gamma(HGL\,El. - G.El.)$$



Pipe	Discharge(L/s)	Head loss (m)
1	167.9	5.61
2	58.4	4.78
3	53.5	3.22
4	16.4	1.55
5	37.2	1.42
6	9.1	0.13
7	113.1	5.97

Node	Elevation(m)	HGL (m)	*p*(kPa)
A	126		
1	96	120.39	239.3
2	99	115.61	163.0
3	93	117.17	237.1
4	90	117.03	265.2
B	123		

Linear Method

Hardy Cross method requires an initial estimate of the flow that should be close to the final solution.

Linear method is a very stable method in which all the equations are solved simultaneously.

Each nonlinear head loss term is linearized using first two terms in Taylor series:

$$f(Q) = f(q) + \frac{\partial f}{\partial q}(Q - q)$$

q= flow rate from previous iteration

Q= unknown flow rate.

So,

$$KQ^n = Kq^n + nKq^{n-1}(Q - q) = DQ + D'$$

where

$$D = nKq^{n-1} \qquad D' = (1-n)Kq^n$$

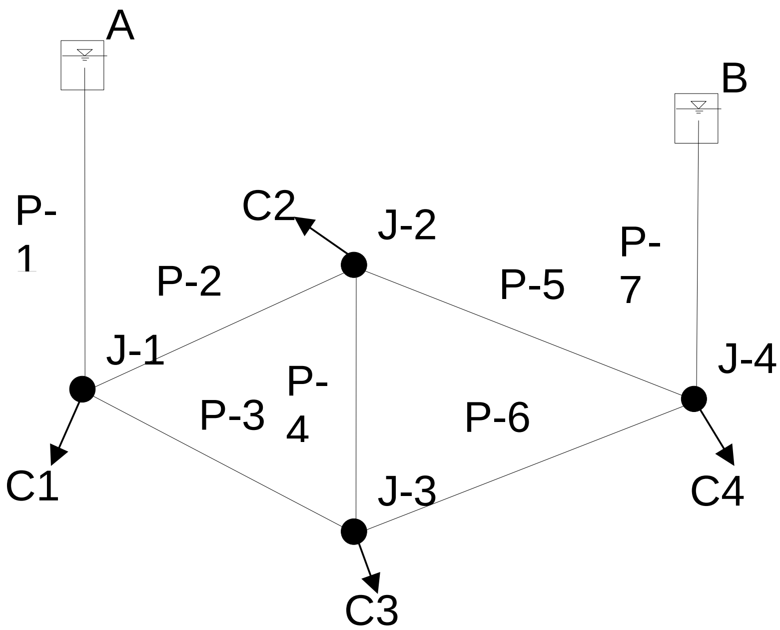
For loops with pumps, nonlinear pump characteristic equation (LHS) is replaced with linear equation (RHS):

$$AQ^2 + BQ + H = Aq^2 + Bq + H + (2Aq + B)(Q - q) = EQ + E'$$

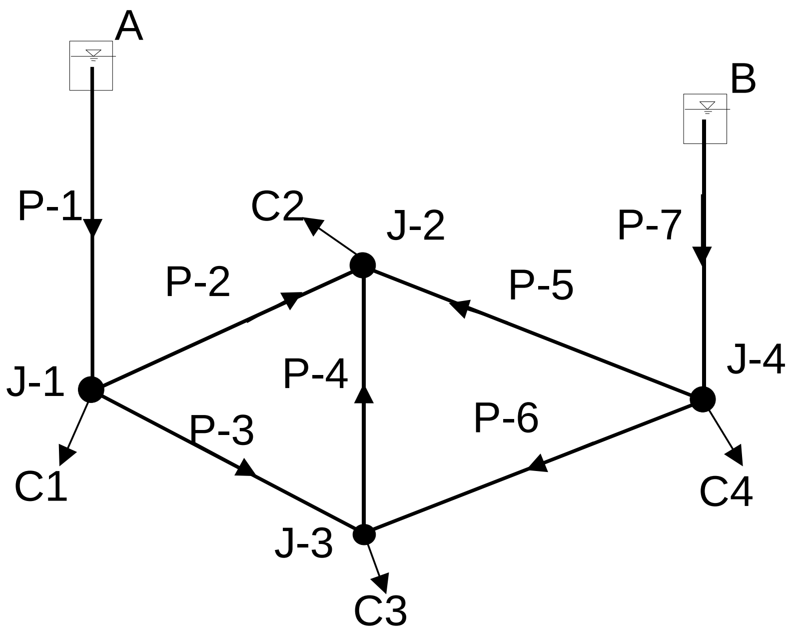
where

$$E = 2Aq + B \qquad E' = H - Aq^2$$

Example: Write down the equations to solve the following network by linear method.



Solution: First of all we assume the direction of flow in each pipe.



Continuity equations:

$$Q_1 - Q_2 - Q_3 - C_1 = 0$$
$$Q_2 + Q_4 + Q_5 - C_2 = 0$$
$$Q_3 - Q_4 + Q_6 - C_3 = 0$$
$$-Q_5 - Q_6 + Q_7 - C_4 = 0$$

Loop equations (clockwise positive):

$$K_2 Q_2^2 - K_3 Q_3^2 - K_4 Q_4^2 = 0$$
$$K_4 Q_4^2 - K_5 Q_5^2 + K_6 Q_6^2 = 0$$
$$-K_1 Q_1^2 - K_2 Q_2^2 + K_5 Q_5^2 + K_7 Q_7^2 = z_B - z_A$$

The continuity equations are already linear, so linearizing the loop equations we get:

$$D_2 Q_2 - D_3 Q_3 - D_4 Q_4 = -D_2' + D_3' + D_4' = C_5$$
$$D_4 Q_4 - D_5 Q_5 + D_6 Q_6 = -D_4' + D_5' - D_6' = C_6$$
$$-D_1 Q_1 - D_2 Q_2 + D_5 Q_5 + D_7 Q_7 = z_B - z_A + D_1' + D_2' - D_5' - D_7' = C_7$$

In matrix form the above equations can be written as follows:

$$[A][Q] = [C]$$

Where $[A]$ is coefficient matrix and $[C]$ is a column matrix of constants.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & D_2 & -D_3 & -D_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_4 & -D_5 & D_6 & 0 \\ -D_1 & -D_2 & 0 & 0 & D_5 & 0 & D_7 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{bmatrix}$$

Solving for the flow rates:

$$[Q] = [A]^{-1}[C]$$

For the initial iteration a velocity of 1m/s may be assumed in each pipe. The computed flow rates (Q) become estimated flow rates (q) for the next iteration. The iterative process is continued until the equations converge to a solution. In other words when the value of

$$\frac{\sum_{i=1}^{N} |Q_i - q_i|}{\sum_{i=1}^{N} |Q_i|}$$

is within a specified limit.