Hydrologic Principles
Part C

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Hydrologic losses-Evaporation and Evapotranspiration (ET)

Water budget method

$$E = -\Delta S + I + P - O - GW$$

E=evaporation
ΔS=change in storage in a specified time period
I = surface inflow
P = precipitation
O = surface outflow
GW= subsurface seepage to groundwater flow

Mass transfer method

$$E = (e_s - e_a)(a + bu)$$

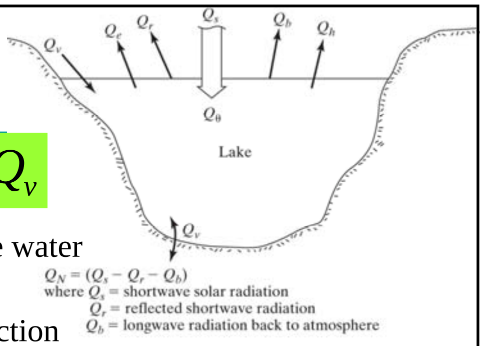
e_s = saturation vapor pressure at the T_s of the water surface

e_a = vapor pressure at some fixed level above the water surface = RH x saturation vapor pressure at T_a of the air

u= wind speed at a fixed level above the water surface

a , b =empirical constants

Energy budget method



$$Q_N - Q_h - Q_e = Q_\theta - Q_v$$

Q_N = net radiation absorbed by the water body

Q_h = sensible heat transfer (conduction and convection to the atmosphere

Q_e = energy used for evaporation

Q_θ =increase in energy in the water body

Q_v =advected energy of inflow and outflow

Units for energy

Langleys per day

1 Langleys (Ly) = 1 cal/cm^2

Energy budget method

$$E = \frac{Q_N + Q_v - Q_\theta}{\rho L_e (1+R)}$$

E = evaporation (cm/day)
L_e = latent heat of vaporization (cal/g)
ρ =mass density of water (gm/cm^3)

$$R = 0.66\left(\frac{T_s - T_a}{e_s - e_a}\right)\left(\frac{p}{100}\right) = \gamma\left(\frac{T_s - T_a}{e_s - e_a}\right)$$

p = atmospheric pressure (mb)
T_a = air temperature (^{o}C)
T_s = water surface temperature (^{o}C)
e_a = vapor pressure of the air (mb)
e_s = vapor pressure at the water surface (mb)
γ = the psychrometric constant 0.66P/1000 (mb/^{o}C)

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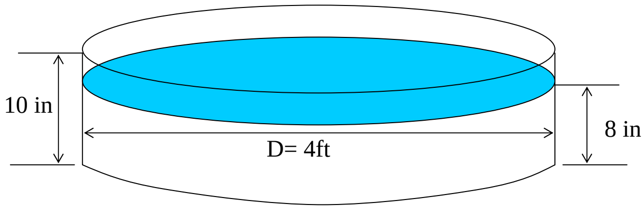
Pan evaporation

Galvanized iron
evaporation pan



$$E = k \times E_p$$

E = evaporation (cm/day)
E_p = pan evaporation (cm/day)
k= pan coefficient (range: from 0.64 to 0.81)

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Table 1–5. Pan Coefficients for Evapotranspiration Estimates

Type of Cover	Pan Coefficient	Reference
St. Augustine grass	0.77	Weaver and Stephens (1963)
Bell peppers	0.85–1.04	
Grass and clover	0.80	Brutsaert (1982, p. 253)
Oak–pine flatwoods (east Texas)	1.20	Englund (1977)
Well-watered grass turf		Shih et al. (1983)
Light wind, high relative humidity	0.85	
Strong wind, low relative humidity	0.35	
Everglades agricultural areas	0.65	
Irrigated grass pasture (central California)	0.76	Hargreaves and Samani (1982)

Combined method-Penman's formula

$$E_h = \frac{\Delta}{\Delta+\gamma} Q_N + \frac{\gamma}{\Delta+\gamma} E_a$$

E_h = flux of latent heat due to evaporation (energy/area-time)=$\rho L_e E$

L_e = latent heat of vaporization at air temperature (energy/mass)

Δ =slope of the saturation vapor pressure versus temperature curve (pressure/degree temperature)

$$\Delta = \frac{de_s}{dT} = \frac{2.7489\times10^8 \times 4278.6}{(T+242.79)^2} \exp\left(-\frac{4278.6}{T+242.79}\right)$$

Q_N = net radiation absorbed (energy/area-time)

γ = psychrometric constant (mb/^{o}C)

Combined method-Penman's formula

$$E_a = \rho L_e (a + bu)(e_{sa} - e_a)$$

E_a = drying power of the air (energy/area-time)
e_{sa} = saturation vapor pressure at temperature of the air
e_a =actual vapor pressure in air RH x e_{sa}
RH= relative humidity (fraction)

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Example 1-7

EVAPORATION USING THE PENMAN EQUATION

An example of Eq. (1–12) is given by Meyer (1944) for Minnesota lakes:

$$E = 0.0106(1 + 0.1u)(e_s - e_a)$$

with E in in./day, u in mph, and vapor pressures in mb. For an air temperature of 90°F (32.2°C), wind speed of 20 mph, relative humidity of 30%, and net radiation flux of 400 ly/day, estimate the evaporation rate using the Penman equation. Assume atmospheric pressure ≈ 1000 mb, so the psychometric constant $\gamma = 0.66$ mb/°C.

We evaluate Δ from Eq. (1–18) at the air temperature of 32.2°C:

$$\Delta = \frac{de_s}{dT} = \frac{2.7489 \times 10^8 \times 4278.6}{(32.2 + 242.79)^2} \exp\left(-\frac{4278.6}{32.2 + 242.79}\right)$$
$$= 2.72 \text{ mb/°C}.$$

For Eq. (1–17) we need the saturation vapor pressure at 32.2°C. From Eq. (1–6), we then have

$$e_{sa} = 2.7489 \times 10^8 \exp[-4278.6/(32.2 + 242.79)] = 48.1 \text{ mb}.$$

Thus, $e_a = e_{sa} = 0.3(48.1) = 14.4$ mb. The latent heat of vaporization at the air temperature of 32.2°C is

$$L_e = 597.3 - 0.57(32.2) = 579 \text{ cal/g}.$$

The density of water will be taken as 1 g/cm^3. We include a change in units from in./day to cm/day while evaluating E_a,

$$E_a = 0.0106(1 + 0.1 \cdot 20)(\text{in./day-mb})2.54(\text{cm/in.})1(\text{g/cm}^3)$$
$$\times 579(\text{cal/g})(48.1 - 14.4)(\text{mb})$$
$$= 1590\text{cal/cm}^2\text{-day} = 1590 \text{ ly/day}.$$

The Penman evaporation energy flux is thus

$$E_h = 2.72/(2.72 + 0.66) \cdot 400 + 0.66/(2.72 + 0.66) \cdot 1590$$
$$= 632 \text{ ly/day}.$$

This can be converted to depth/day by dividing by ρL_e:

$$E = 632/1(579) = 1.09 \text{ cm/day} = 0.43 \text{ in./day of evaporation}.$$

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Hydrologic losses- Infiltration

Horton's equation $f = f_c + (f_0 - f_c)e^{-kt}$

$$F = f_c t + (f_0 - f_c)(1 - e^{-kt})/k$$

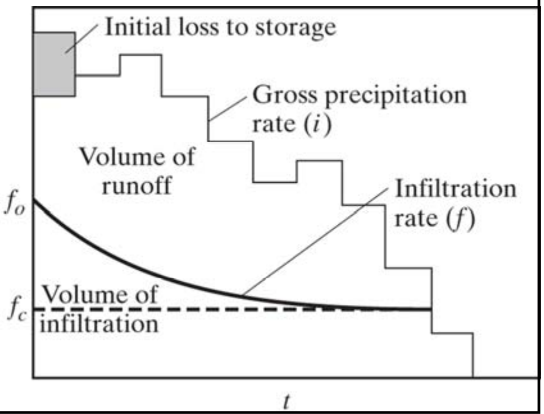
f = infiltration rate at time t (length/time)

f_c = final infiltration rate (length/time)

f_0 = initial infiltration rate (length/time)

k= empirical constant (1/time)

F= cumulative infiltration until time t from the beginning (length)



Example 1-8

HORTON'S INFILTRATION EQUATION

The initial infiltration capacity f_0 of a watershed is estimated as 1.5 in./hr, and the time constant is taken to be 0.35 hr^{-1}. The equilibrium capacity f_c is 0.2 in./hr. Use Horton's equation to find (a) the values of f at t = 10 min, 30 min, 1 hr, 2 hr, and 6 hr, and (b) the total volume of infiltration over the 6-hr period.

Horton's equation [Eq. (1–20)] is

$$f = f_c + (f_0 - f_c)e^{-kt}.$$

Substituting the values for f_0, f_c, and k gives

$$f = 0.2 \text{ in./hr} + 1.3(e^{-0.35t})\text{in./hr}.$$

Solving for each value of t gives the following table:

t (hr)	f (in./hr)
1/6	1.43
1/2	1.29
1	1.12
2	0.85
6	0.36

The volume (in inches over the watershed) can be found by plotting the curve given by the table of values and then finding the area under the curve bounded by t = 0 and t = 6 hr. The plot is shown in Fig. E1–8. The curve is given by the equation below, and to find F', the volume of infiltration and the area under the F curve, we must integrate Horton's equation.

$$F' = f_c t + \frac{(f_0 - f_c)e^{-kt}}{-k}.$$

$$f = 0.2 \text{ in./hr} + 1.3\,(e^{-0.35t})\text{in./hr}.$$

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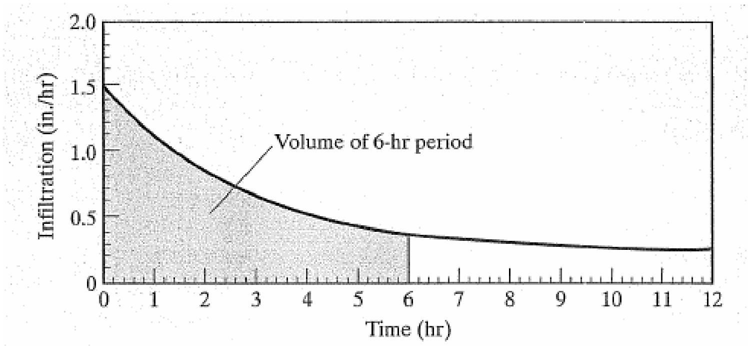

Integrating over the interval $t = 0$ to $t = 6$ hr gives

$$\begin{aligned} \text{Vol} &= \int f\,dt \\ &= \int 0.2 + 1.3(e^{-0.35t})\,dt \\ &= [0.2t + (1.3/-0.35)e^{-0.35t}]_0^6 \\ &= 4.46 \text{ in. over the watershed} \end{aligned}$$

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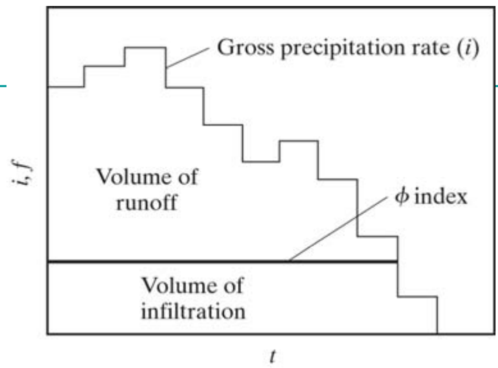
Philip's equation

$$f(t) = \frac{1}{2}\frac{S}{\sqrt{t}} + K \qquad F(t) = S\sqrt{t} + Kt$$

S = sorptivity (length/time$^{1/2}$)

K= soil hydraulic conductivity (length/time)

ϕ index



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Example 1-9

PHI INDEX METHOD FOR INFILTRATION

Use the rainfall data below to determine the ϕ index for a watershed that is 0.875 square miles, where the runoff volume is 228.7 ac-ft.

Time (hr)	Rainfall (in./hr)
0–2	1.4
2–5	2.3
5–7	1.1
7–10	0.7
10–12	0.3

The first step involves graphing the given data, as in Figure E1–9. To approach the problem, we must first change the area of the watershed into acres:

$$\text{area (ac)} = 0.875 \text{ sq mi } (640 \text{ acres/sq mi}),$$
$$\text{area} = 560 \text{ acres},$$

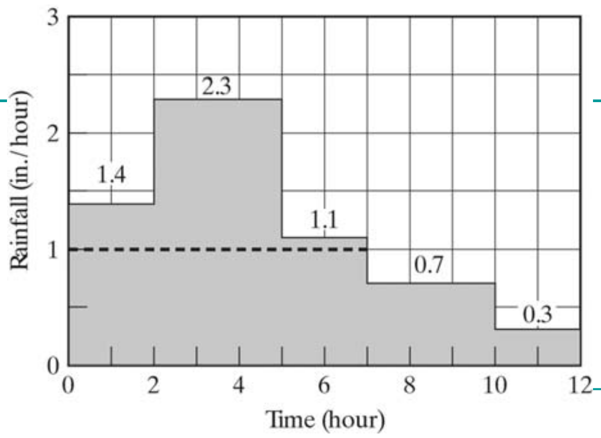
$$2(1.4 - \phi) + 3(2.3 - \phi) + 2(1.1 - \phi) + 3(0.7 - \phi) + 2(0.3 - \phi) = 4.9$$

Note that if ϕ is greater than the net rainfall for a specific time period, no negative rainfall is added into the runoff calculation.

The rate of infiltration can be found only by trial and error:

Assume $\phi = 1.5$ in./hr. The runoff is the volume of water above the line at which $y = 1.5$ on the graph in Fig. E1–9. This ϕ index would then account for $3(2.3 - 1.5) = 2.4$ in. of runoff (neglecting negative components), which is less than 4.9 in. Try again.

Assume $\phi = 0.5$ in./hr. This ϕ would account for $2(1.4 - 0.5) + 3(2.3 - 0.5) + 2(1.1 - 0.5) + 3(0.7 - 0.5) = 9.0$ in. of runoff, which is greater than 4.9 in. Try again.



Assume $\phi = 1.0$ in./hr, and the solution is found where the runoff is equal to $2(1.4 - 1.0) + 3(2.3 - 1.0) + 2(1.1 - 1.0) = 4.9$ in., the required amount.

From the calculations, one can see that below the dotted line at which $\phi = 1.0$ in./hr, the rainfall infiltrates into the ground and the rainfall above this line (a total of 4.9 in. in 12 hours) runs off, as required.

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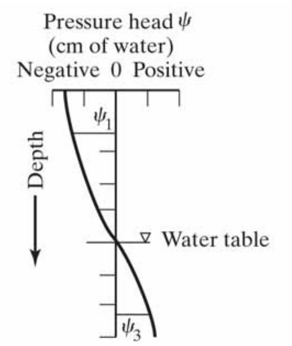
Green and Ampt method

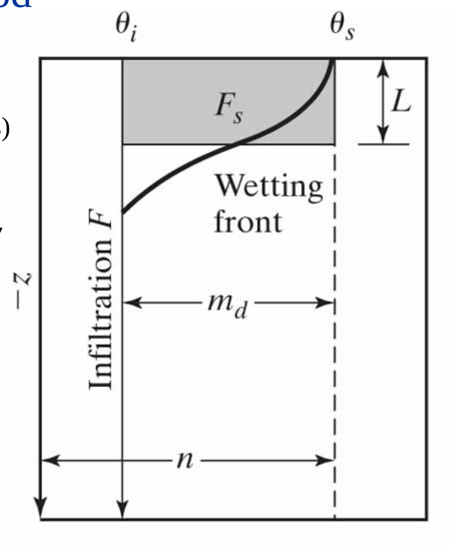
$$f = K_s(1 - M_d\psi / F)$$

M_d = moisture deficit (dimensionless)

K_s= saturated hydraulic conductivity (length/time), see Table 1-7

ψ = suction head (cm), see Table 1-7





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Three rainfall-infiltration scenarios are possible:

- **Case 1:** $i < f$, The rainfall intensity is less than the maximum downward hydraulic conductivity, meaning that runoff will never occur
- **Case 2:** $K_s < i < f$, The rainfall intensity is greater than the saturated conductivity but less than the infiltration rate. The time to ponding varies for different rainfall intensities.
- **Case 3:** $i > f$**,** The rainfall intensity is greater than the infiltration rate and runoff can occur.

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GREEN AND AMPT INFILTRATION EQUATION

For the following soil properties, develop a plot of infiltration rate f vs. infiltration volume F using the Green and Ampt equation:

$$K_s = 1.97 \text{ in./hr},$$
$$\theta_s = 0.518,$$
$$\theta_i = 0.318,$$
$$\psi = -9.37 \text{ in.},$$
$$i = 7.88 \text{ in./hr}.$$

Noting that $M_d = \theta_s - \theta_i$, we can solve Eq. (1–31) to obtain the volume of water that will infiltrate before surface saturation is reached:

$$F_s = \frac{\psi M_d}{(1 - i/K_s)}$$
$$= \frac{-(9.37 \text{ in.})(0.518 - 0.318)}{1 - [(7.88 \text{ in./hr})/(1.97 \text{ in./hr})]}$$
$$F_s = 0.625 \text{ in.}$$

Until 0.625 in. has infiltrated, the rate of infiltration is equal to the rainfall rate. After that point (surface saturation) the rate of infiltration is given by the equation [Eq. (1–30)]

$$f = K_s(1 - M_d\psi/F)$$

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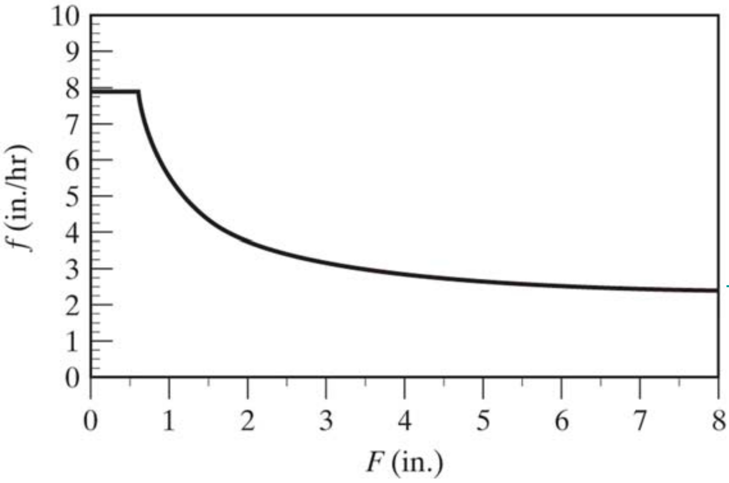

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Tutorial:

1.23 A class A pan is maintained near a small lake to determine daily evaporation (see the table on the following page). The level in the pan is observed every day, and water is added if the level falls near 7 in. For each day the difference in height level is calculated between the current and previous day, and the precipitation value is from the current day. Determine the daily lake evaporation if the pan coefficient is 0.70.

1.28 Determine the ϕ index of Figure PI–28 if the runoff depth was 5.6 in. of rainfall over the watershed area.

Day	Rainfall (In.)	Water Level (in.)
1	0	8.00
2	0.23	7.92
3	0.56	7.87
4	0.05	7.85
5	0.01	7.76
6	0	7.58
7	0.02	7.43
8	0.01	7.32
9	0	7.25
10	0	7.19
11	0	7.08*
12	0.01	7.91
13	0	7.86
14	0.02	7.80

*Refilled at this point.