7. Unit Hydrograph (UHG)

The relationship between the excess rainfall and streamflow is developed using the UHG.
UHG is a hydrograph for a unit volume of excess rainfall for a given storm duration. For example in SI units, UHG is a hydrograph for 1 cm of rainfall on a given watershed for a given storm duration.

Three principles for UHG method

1. For the same duration of rainfall, the runoff duration is same irrespective of the difference in the intensity (Fig. 7.10).
2. The runoff ordinates are in the same proportions with the intensities of the rainfall. That is, n times as much rain in a given time will produce a hydrograph with ordinates n times as large (Fig. 7.10).
3. The principle of superposition is valid (Fig. 7.11)

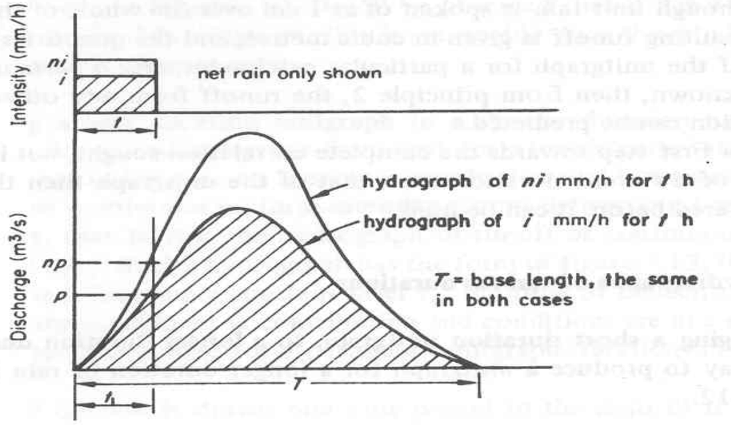


Figure 7.10 *Proportional principle of the unitgraph*

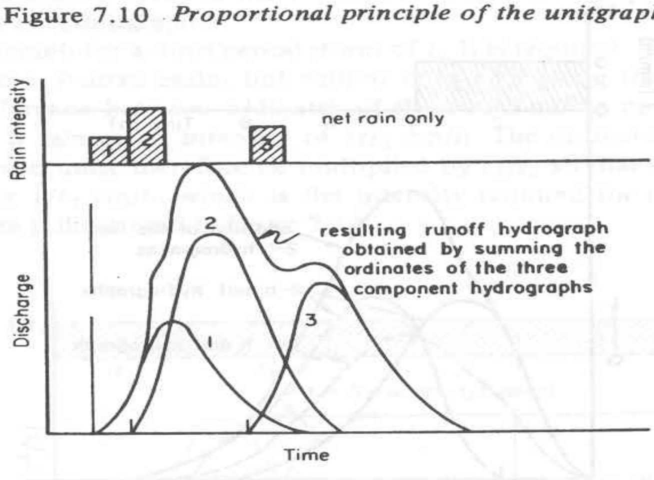


Figure 7.11 *Principle of superposition applied to unitgraphs*

Example: The gross rainfall was 14mm for a storm of 3 hour duration on a watershed having 13.6 km^2 area. The ϕ index for the catchment is estimated to be 1.3 mm/h. Derive the UHG for this catchment from the direct runoff data given below:

Time(h)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DRO(m^3/s)	0	1.8	6.1	8.4	6.5	5.2	4.0	3.0	2.2	1.6	1.1	0.7	0.4	0.2	0.1	0

Excess rainfall =

DRO volume =

UHG ordinates =

The UHG from multi-period storms

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

For example

$$Q_1 = P_1 U_1$$

$$Q_2 = P_1 U_2 + P_2 U_1$$

$$Q_3 = P_1 U_3 + P_2 U_2 + P_3 U_1$$

$$Q_4 = P_1 U_4 + P_2 U_3 + P_3 U_2$$

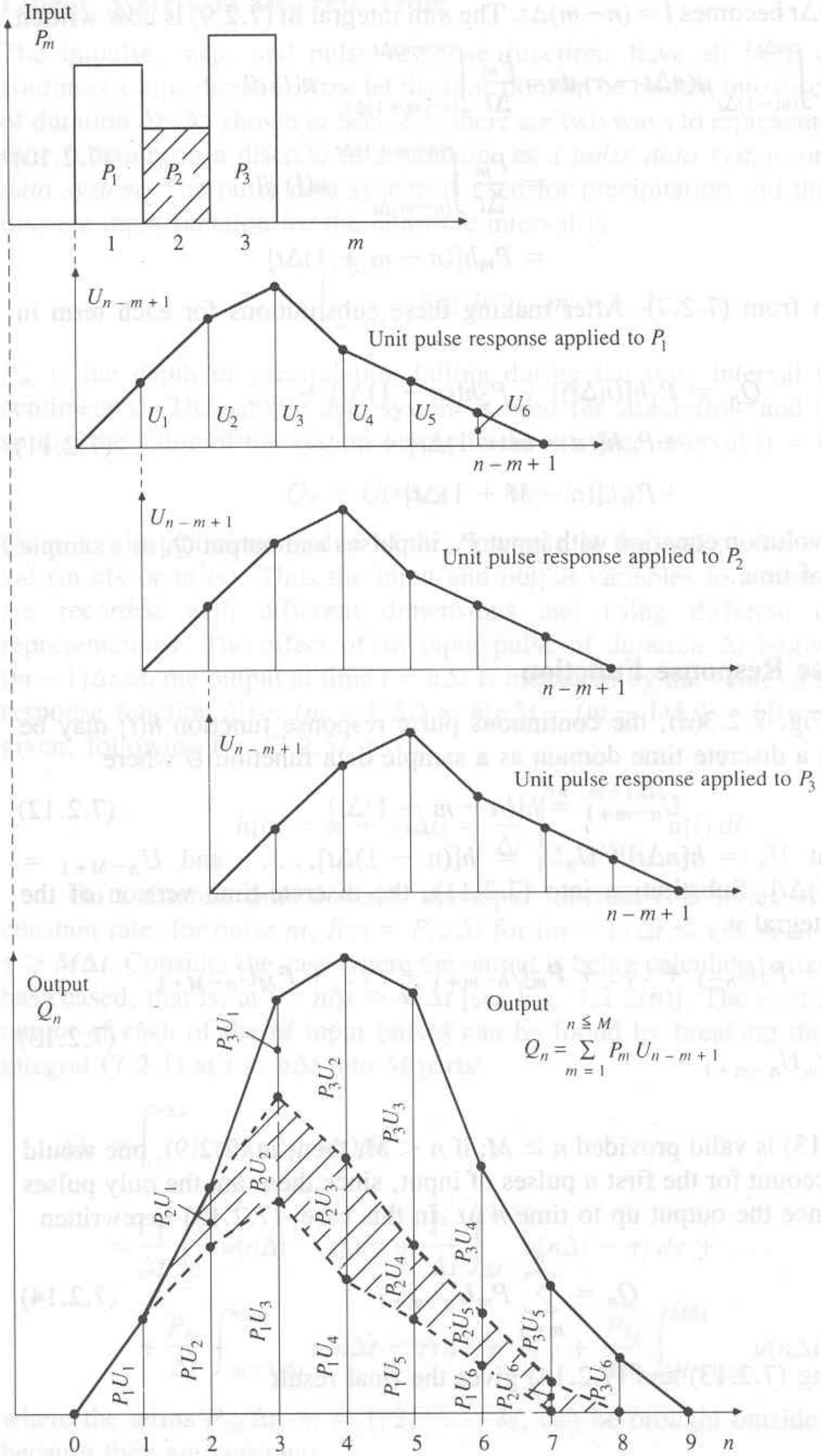


FIGURE 7.2.5
Application of the discrete convolution equation to the output from a linear system.

Example 7.4.1 Find the half-hour unit hydrograph using the excess rainfall hyetograph and direct runoff hydrograph given in Table 7.4.2 (these were derived in Example 5.3.1)

Time (1/2 h)	**Excess rainfall (in)**	**Direct runoff (cfs)**
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

$$U_2 = \frac{Q_2 - P_2U_1}{P_1} = \frac{1923 - 1.93 \times 404}{1.06} = 1079 \text{ cfs/in}$$

$$U_3 = \frac{Q_3 - P_3U_1 - P_2U_2}{P_1}$$

$$= \frac{5297 - 1.81 \times 404 - 1.93 \times 1079}{1.06} = 2343$$

The UHG is

n	1	2	3	4	5	6	7	8	9
U_n (cfs/in)	404	1079	2343	2506	1460	453	381	274	173

Check the volume of UHG to make sure it is 1 in.

UHG Application

Example 7.5.1

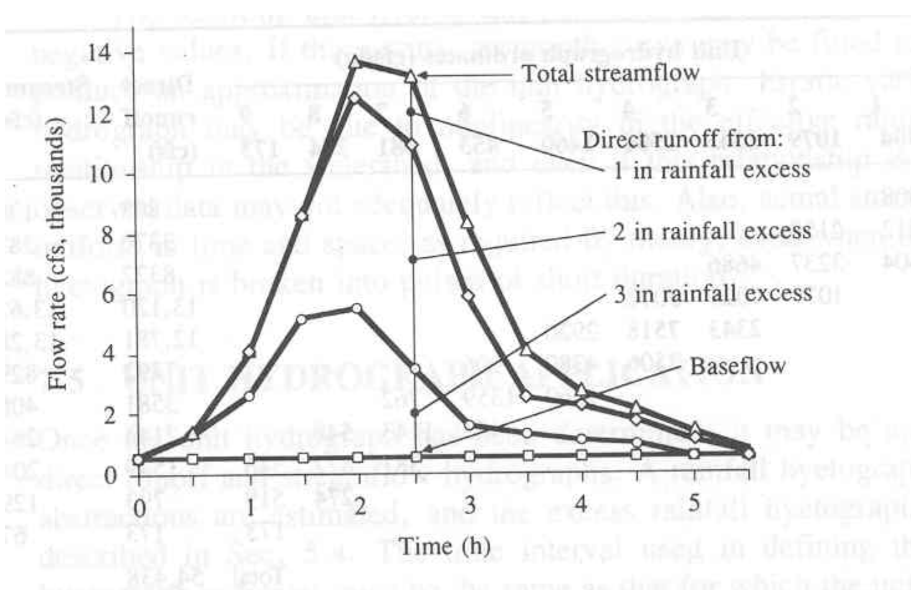
$$Q_1 = P_1U_1 = 2.00 \times 404 = 808\,cfs$$
$$Q_2 = P_1U_2 + P_2U_1 = 2.00 \times 1079 + 3.00 \times 404 = 3370\,cfs$$

and so on.

TABLE 7.5.1
Calculation of the direct runoff hydrograph and streamflow hydrograph for Example 7.5.1

Time ($\frac{1}{2}$-h)	Excess Precipitation (in)	Unit hydrograph ordinates (cfs/in) 1 404	2 1079	3 2343	4 2506	5 1460	6 453	7 381	8 274	9 173	Direct runoff (cfs)	Streamflow* (cfs)
$n =$ 1	2.00	808									808	1308
2	3.00	1212	2158								3370	3870
3	1.00	404	3237	4686							8327	8827
4			1079	7029	5012						13,120	13,620
5				2343	7518	2920					12,781	13,281
6					2506	4380	906				7792	8292
7						1460	1359	762			3581	4081
8							453	1143	548		2144	2644
9								381	822	346	1549	2049
10									274	519	793	1293
11										173	173	673
										Total	54,438	

*Baseflow = 500 cfs.



$$V_d = \sum_{n=1}^{N} Q_n \Delta t = 54438 \times 0.5 \quad cfs.h = 54438 \times 0.5 \times 3600$$

$$= 9.8 \times 10^7 \ ft^3$$

$$r_d = \frac{V_d}{A} = \frac{9.8 \times 10^7}{7.03 \times 5280^2} = 0.5 \ ft = 6 \ in$$

As can be observed the value of the r_d is equal to the amount of excess rainfall (6 in).

Synthetic UHG

Three types of synthetic UHG

1. Relating hydrograph characteristics to watershed characteristics
2. Based on dimensionless UHG
3. Based on models of watershed storage

Snyder's Synthetic UHG

Based on the study in U.S. for areas of 30 to 30,000 km^2

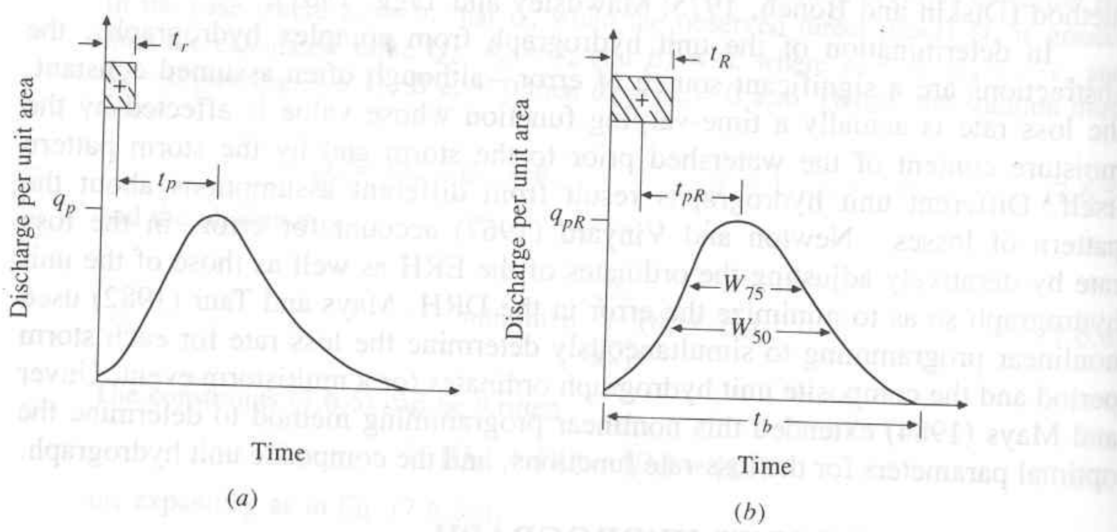


FIGURE 7.7.1
Snyder's synthetic unit hydrograph. (*a*) Standard unit hydrograph ($t_p = 5.5t_r$). (*b*) Required unit hydrograph ($t_{pR} \neq 5.5t_R$).

$t_p =$ Basin lag

$t_r =$ Rainfall duration

$q_p =$ Peak discharge per unit drainage area ($m^3/s.km^2$)

$W_p =$ Width of the UHG at a discharge equal to *p* percent of the peak discharge.

$$t_p = 5.5 t_r$$

1. The basin lag

$$t_p = C_1 C_t (L L_c)^{0.3} \text{ in hours} \qquad (7.7.2)$$

- $L =$ Length of the main stream in km or miles from the outlet to upstream divide
- $L_c =$ Distance in km or miles from the outlet to a point on the stream nearest the centroid of the watershed area.
- $C_1 = 0.75\,(\text{SI}), \quad 1\,(\text{English})$
- $C_t =$ A coefficient derived from gaged watersheds in the same region

2. The peak discharge

$$q_p = \frac{C_2 C_p}{t_p} \text{ in } m^3/s.km^2 \qquad (7.7.3)$$

- $C_2 = 2.75\,(\text{SI}), \quad 640\,(\text{English})$
- $C_p =$ A coefficient derived from gaged watersheds in the same region

To compute C_t and C_p for a gaged watershed, the values of L and L_c are measured from the basin map. From a derived UHG of the watershed, obtain the effective duration t_R in hours, basin lag t_{pR} in hours and q_{pR} in $m^3/s.km^2$.

If $t_{pR} = 5.5t_R$ then $t_R = t_r$ and $q_{pR} = q_p$

And C_t and C_p can be computed from Eq. 7.7.2 and 7.7.3

If $t_{pR} \neq 5.5t_R$ then $t_p = t_{pR} + \dfrac{t_r - t_R}{4}$ (7.7.4)

And 7.7.1 and 7.7.4 are solved simultaneously for t_r and t_p and the coefficients are found from 7.7.2 and 7.7.3 with $q_{pR} = q_p$ and $t_{pR} = t_p$.

3. The peak discharge in $m^3/s.km^2$ for the required UHG

$$q_{pR} = \frac{q_p t_p}{t_{pR}}$$

4. The base time in hours, assuming a triangular shape for the UHG,

$$t_b = \frac{C_3}{q_{pR}}$$ where, $C_3 = 5.56\ (\text{SI}), \quad 1290\ (\text{English})$

5. The width in hours

$$W = C_w q_{pR}^{-1.08}$$

For 75% width $C_w = 1.22\ (\text{SI}), \quad 440\ (\text{English})$

For 50% width $C_w = 2.14\ (\text{SI}), \quad 770\ (\text{English})$

Usually on-third of this width is distributed before the UHG peak time and two-thirds after the peak.

Example 7.7.1

Example 7.7.2

SCS dimensionless hydrograph

$q_p =$ Peak discharge

$T_p =$ Time to the rise of the UHG

For given peak discharge and lag time for the duration of the excess rainfall, the UHG can be estimated from the synthetic dimensionless hydrograph for the given watershed. SCS provides a hydrograph of this type as Fig. 7.7.4 (a).

For the area of the UHG to be unity, we may show that

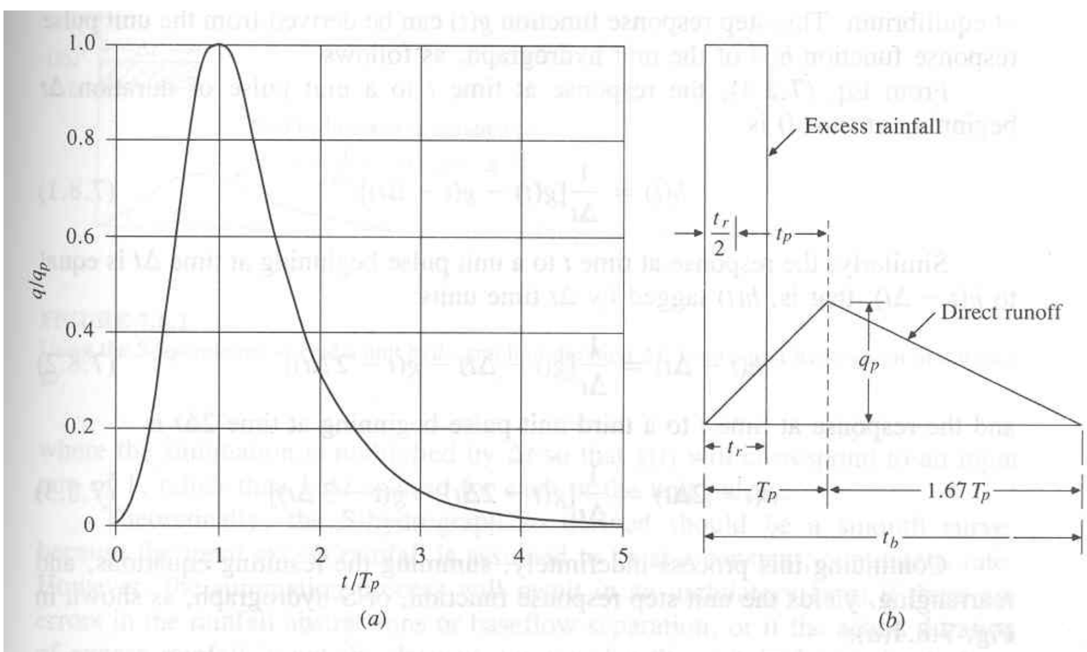
$$q_p = \frac{CA}{T_p} \qquad C = 2.08 \quad (483.4 \text{ English units})$$

where peak discharge is in m^3/s.cm (cfs/in in English units), time in hours and area A in square kilometers (square miles in English units).

Further it is given that, the basin lag

$$t_p = 0.6T_c \qquad T_c = \text{Time of concentration of the watershed}$$

$$T_p = \frac{t_r}{2} + t_p$$



Example 7.7.3

UHGs for different rainfall durations

S-Hydrograph is that resulting from a continuous excess rainfall at a constant rate of 1cm/h (or 1 in/h) for an indefinite period.

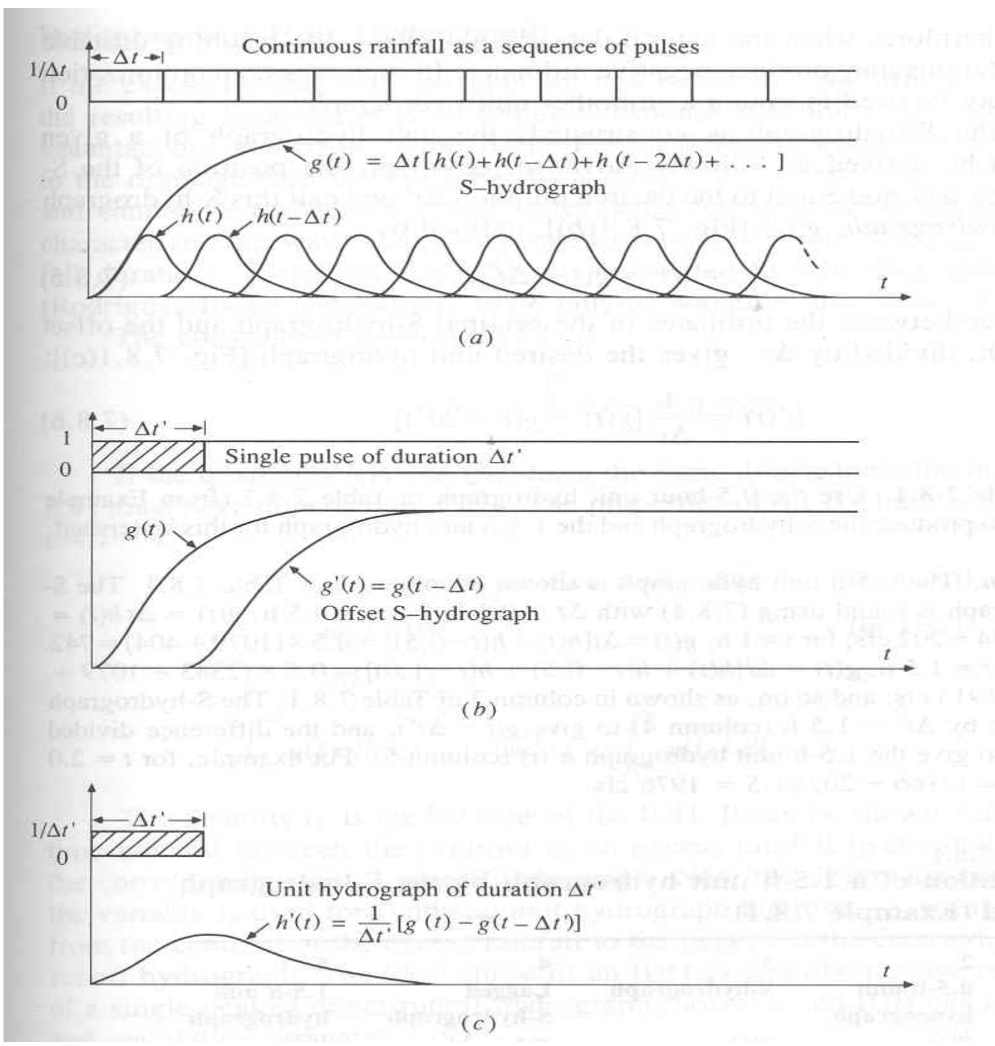
$h(t) =$ UHG ordinates for duration Δt

$h(t - \Delta t) =$ UHG ordinates for duration Δt lagged by Δt

$g(t) = \Delta t$ (Sum of UHG ordinates lagged by Δt) = S-hydrograph ordinates for Δt

$g(t - \Delta t') =$ S-hydrograph ordinates lagged by the new duration $\Delta t'$ = Offset S-Hydrograph

$$h'(t) = \frac{1}{\Delta t'}\left[g(t) - g(t - \Delta t')\right] =$$ UHG ordinates for duration $\Delta t'$



Example: 7.8.1
Use the 0.5-h unit hydrograph in Table 7.4.3 to produce the S-hydrograph and the 1.5-h unit hydrograph for the watershed.

Time	**0.5-h UHG**	**S-hydrograph**	**Lagged S-hydrograph**	**1.5-h UHG**
t	$h(t)$	$g(t)$	$g(t-\Delta t')$	$h'(t)$
(h)	(cfs/in)	(cfs)	(cfs)	(cfs/in)
0.5	404	202	0	135
1	1079	742	0	494
1.5	2343	1913	0	1275
2	2506	3166	202	1976
2.5	1460	3896	742	2103
3	453	4123	1913	1473
3.5	381	4313	3166	765
4	274	4450	3896	369
4.5	173	4537	4123	276
5	0	4537	4313	149
5.5	0	4537	4450	58
6	0	4537	4537	0