Chapter 6
Water Flow in Open Channels

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Expected student outcomes

- Ability to calculate discharge carrying capacity of open channels **[a, e]**
- Ability to calculate water surface profiles in open channels manually and with the help of computer **[a, e, k]**

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Difference between pipe and open channel flow

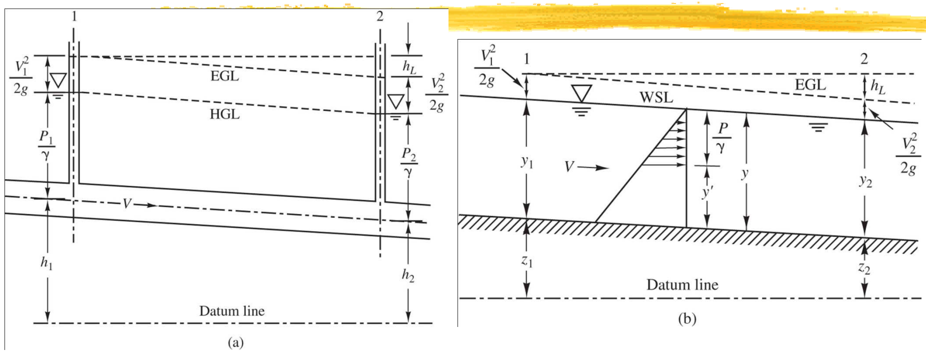
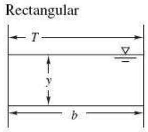
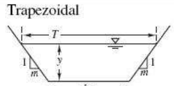
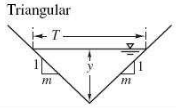
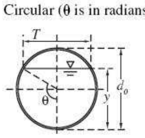


TABLE 6.1 Cross-Sectional Relationships for Open-Channel Flow

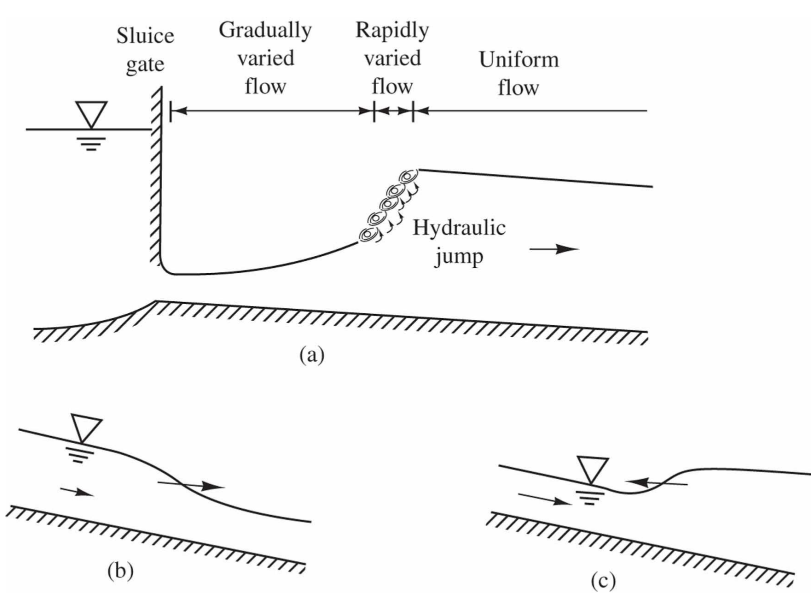
Section Type	Area (A)	Wetted perimeter (P)	Hydraulic Radius (R_h)	Top Width (T)	Hydraulic Depth (D)
Rectangular 	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y
Trapezoidal 	$(b + my)y$	$b + 2y\sqrt{1 + m^2}$	$\frac{(b + my)y}{b + 2y\sqrt{1 + m^2}}$	$b + 2my$	$\frac{(b + my)y}{b + 2my}$
Triangular 	my^2	$2y\sqrt{1 + m^2}$	$\frac{my}{2\sqrt{1 + m^2}}$	$2my$	$\frac{y}{2}$
Circular (θ is in radians) 	$\frac{1}{8}(2\theta - \sin 2\theta)d_0^2$	θd_0	$\frac{1}{4}(1 - \frac{\sin 2\theta}{2\theta})d_0$	$(\sin\theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{2\theta - \sin 2\theta}{\sin\theta}\right)d_0$

Source: V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

6.1 Open channel flow classification

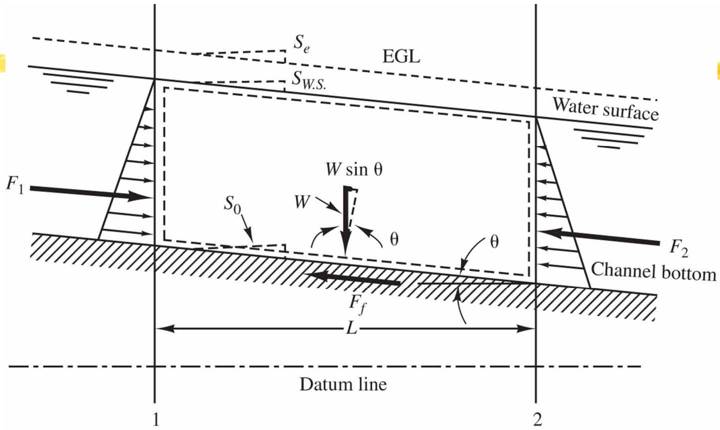
- **Steady and Unsteady flow**
- **Uniform and non-uniform flow**

Rapidly varied flow | **Gradually varied flow**



(a) Gradually varied flow (GVF), rapidly varied flow (RVF), and uniform flow (UF) (b) and (c) Unsteady varied flow

6.2 Uniform flow in open channels



Applying momentum equation, we get $\sum F = 0 = -\tau_0 PL + \gamma AL \sin\theta$

For very small angle of inclination (θ less than 10º) $\sin\theta \approx \tan\theta = S_0$

and for uniform flow $S_0 = S_{W.S.} = S_e$ $\tau_0 = \gamma R S_e$

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In 1769 a French engineer, Antoine Chezy, assumed that the resisting force per unit area of the channel bed is proportional to the square of the mean velocity, KV^2, where K is a constant of proportionality. The total resistance force may thus be written as

$$F_f = \tau_0 PL = KV^2 PL \tag{6.1c}$$

where τ_0 is the resisting force per unit area of the channel bed, also known as the *wall shear stress.*

Substituting Equation (6.1b) and Equation (6.1c) into Equation (6.1a), we have

$$\gamma ALS_0 = KV^2 PL$$

or

$$V = \sqrt{\left(\frac{\gamma}{K}\right)\left(\frac{A}{P}\right)S_0}$$

In this equation, $A/P = R_h$, and $\sqrt{\gamma/K}$ may be represented by a constant, C. For uniform flow, $S_0 = S_e$, the above equation may thus be simplified to

$$V = C\sqrt{R_h S_e} \tag{6.2}$$

in which R_h is the *hydraulic radius* of the channel cross section. The hydraulic radius is defined as the water area divided by the wetted perimeter for all shapes of open channel cross sections.

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Over the past two centuries many attempts have been made to determine the value of Chezy's C. The simplest relationship and the one most widely applied in the United States is derived from the work of an Irish engineer, Robert Manning (1891–1895).* Using the analysis performed on his own experimental data and on those of others, Manning derived the following empirical relation:

$$C = \frac{1}{n} R_h^{1/6} \tag{6.3}$$

in which n is known as the Manning's coefficient of the channel roughness. Some typical values of Manning's coefficient are given in Table 6.1.

Substituting Equation (6.3) into Equation (6.2), we have the *Manning's formula for uniform flow*

$$V = \frac{1}{n} R_h^{2/3} S_e^{1/2} \tag{6.4}$$

where V has units of m/sec, R_h is given in m, S_e in m/m, and n is dimensionless.

The discharge in a uniform flow channel may be determined by

$$Q = AV = \frac{1}{n} A R_h^{2/3} S_e^{1/2} \tag{6.5a}$$

where Q is given in m^3/sec.

On the right-hand side of this equation, the water area, A, and the hydraulic radius, R_h, are both functions of water depth, d, which is known as the *uniform depth* or *normal depth* when the flow is uniform.

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In the BG system, Manning's equation is

$$Q = \frac{1.49}{n} A R_h^{2/3} S_e^{1/2}$$

where Q is in ft^3/sec, A in ft^2, R_h in ft, S_e in ft/ft, and n is dimensionless.

TABLE 6.2 Typical Values of Manning's n

Channel Surface	n
Glass, PVC, HDPE	0.010
Smooth steel, metals	0.012
Concrete	0.013
Asphalt	0.015
Corrugated metal	0.024
Earth excavation, clean	0.022–0.026
Earth excavation, gravel and cobbles	0.025–0.035
Earth excavation, some weeds	0.025–0.035
Natural channels, clean and straight	0.025–0.035
Natural channels, stones or weeds	0.030–0.040
Riprap lined channel	0.035–0.045
Natural channels, clean and winding	0.035–0.045
Natural channels, winding, pools, shoals	0.045–0.055
Natural channels, weeds, debris, deep pools	0.050–0.080
Mountain streams, gravel and cobbles	0.030–0.050
Mountain streams, cobbles and boulders	0.050–0.070

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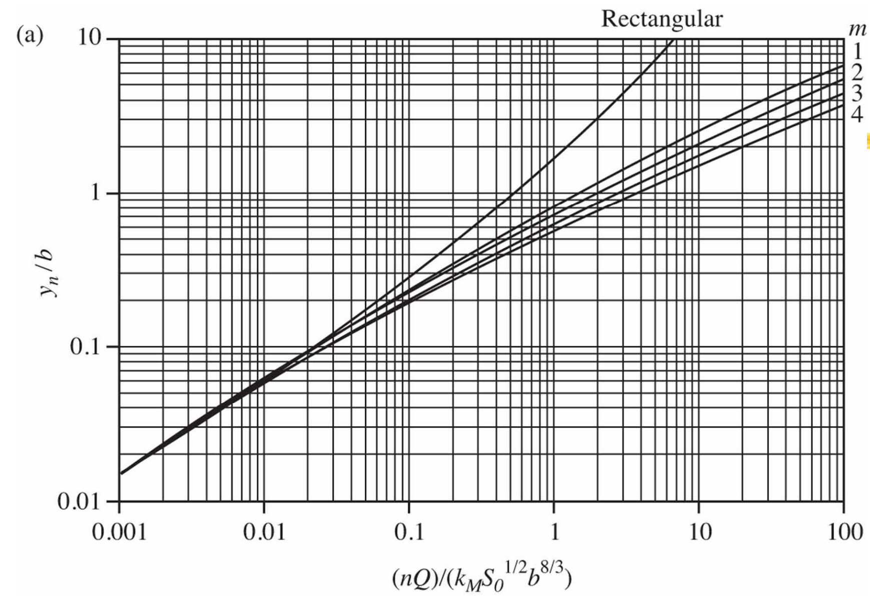

Normal depth of flow for trapezoidal channels

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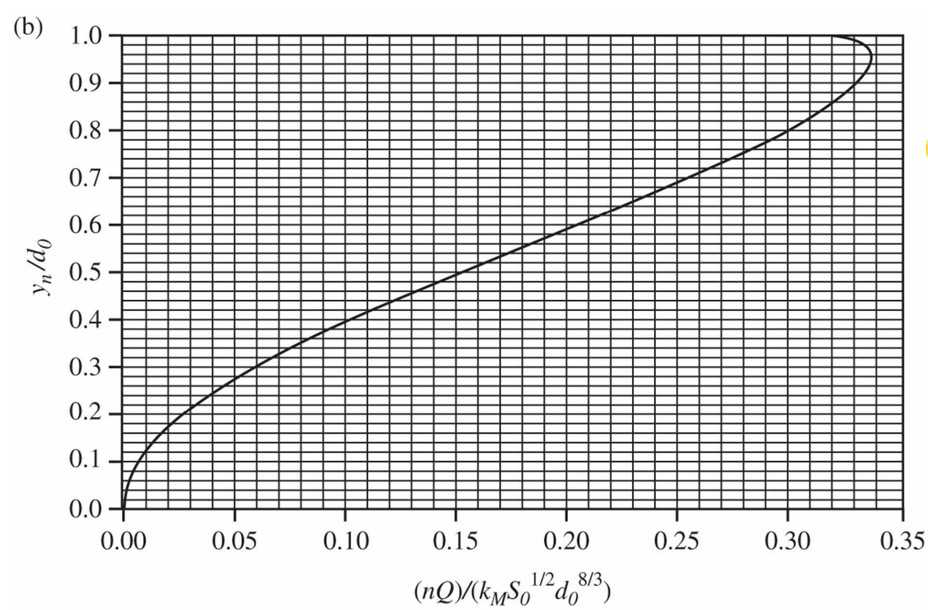

Normal depth of flow for circular channels

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Example 6.1

A 3-m-wide rectangular irrigation channel carries a discharge of 25.3 m^3/sec at a uniform depth of 1.2 m. Determine the slope of the channel if Manning's coefficient is n = 0.022.

$$A = by = (3)(1.2) = 3.6 \text{ m}^2$$

$$P = b + 2y = 5.4 \text{ m}$$

$$R_h = \frac{A}{P} = \frac{3.6}{5.4} = \frac{2}{3} = 0.667 \text{ m}$$

$$S_0 = S_e = \left(\frac{Qn}{AR_h^{2/3}}\right)^2 = 0.041$$

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Example 6.3

If the discharge in the channel in Example 6.1 is increased to 40 m^3/sec, what is the normal depth of the flow?

Area: $A = by = 3y$

Wetted Perimeter: $P = b + 2y = 3 + 2y$

Hydraulic Radius: $R_h = \frac{A}{P} = \frac{3y}{3 + 2y}$

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

$$40 = \frac{1}{0.022}(3y)\left(\frac{3y}{3 + 2y}\right)^{2/3}(0.041)^{1/2}$$

$$AR_h^{2/3} = (3y)\left(\frac{3y}{3 + 2y}\right)^{2/3} = \frac{(0.022)(40)}{(0.041)^{1/2}} = 4.346$$

$$\boxed{y = y_n = 1.69 \text{ m}}$$

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Example 6.3

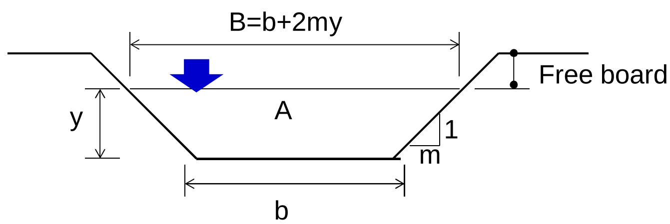
$$\frac{nQ}{(1.0)S_0^{1/2}b^{8/3}} = \frac{(0.022)(40)}{(1.0)(0.041)^{1/2}(3)^{8/3}} = 0.23$$

$$y_n/b = 0.56 \text{ and } y_n = (0.56)(3.0) = 1.68 \text{ m.}$$

Example 6.4

Prove that the best hydraulic trapezoidal section is a half-hexagon.

To find the best hydraulic section we need to minimize the cost of material used in the channel. In other words, we need to minimize the wetted perimeter, for given roughness, discharge and bed slope.



$$A = by + my^2 \qquad (1)$$

$$P = b + 2y\sqrt{m^2+1} \qquad (2)$$

Eliminating b from the above equations, we get $P = \frac{A}{y} - my + 2y\sqrt{m^2+1}$

Now consider both A and m constant and let the first derivative of P with respect to y equal zero to obtain the minimum value of P

$$\frac{dP}{dy} = \frac{A}{y^2} - m + 2\sqrt{m^2+1} = 0$$

Substituting A from Equation (1), we get $\frac{by+my^2}{y^2} = \left(2\sqrt{m^2+1}\right) - m$

$$b = 2y\left(\sqrt{m^2+1} - m\right) \qquad (3)$$

By definition, hydraulic radius is: $R_h = \frac{A}{P} = \frac{by+my^2}{b+2y\sqrt{m^2+1}}$

Substituting the value of b from Equation (3) into the above equation and simplifying, we have $R_h = \frac{y}{2}$

It shows that **the best hydraulic trapezoidal section has a hydraulic radius equal to one-half of the water depth**. Substituting Equation (3) into Equation (2) and solving for P, we have

$$P = 2y\left(2\sqrt{m^2+1} - m\right) \qquad (4)$$

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To determine the value of m that makes P the least, the first derivative of P is taken with respect to m. Equating it to zero and simplifying, we have

$$m = \frac{\sqrt{3}}{3} = \cot\left(60^\circ\right)$$

$$b = 2y\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} - \frac{1}{\sqrt{3}}\right) = \frac{2y}{\sqrt{3}} \Leftrightarrow y = \frac{\sqrt{3}}{2}b = b\sin\left(60^\circ\right)$$

Which shows that it is a hexagon. We can also observe that $P = 3B$

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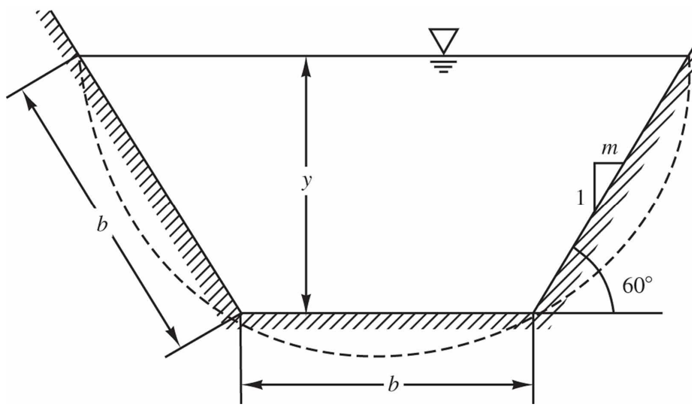

$m = \frac{1}{\tan 60°} = \frac{\sqrt{3}}{3}$

$y = b \sin 60° = \frac{\sqrt{3}}{3} b$

A=Area
P=Wetted perimeter
B= Top width

Shape	Best Geometry	A	P	B
Trapezoidal	Half hexagon	$1.73y^2$	$3.46y$	$2.31y$
Rectangle	Half square	$2y^2$	$4y$	$2y$
Triangle	Half square	y^2	$2.83y$	$2y$
Semicircle	-	$0.5\pi y^2$	πy	$2y$
Parabola	-	$1.89y^2$	$3.77y$	$2.83y$

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6.4 Energy principles in open channel flow

The elevation (potential) energy head in open-channel flow is measured with respect to a selected horizontal datum line. The vertical distance measured from the datum to the channel bottom (z) is commonly taken as the elevation energy head at the section.

Therefore, the total energy head at any section in an open channel is generally expressed as

$$H = \frac{V^2}{2g} + y + z \tag{6.7}$$

Specific energy in a channel section is defined as the energy head measured with respect to the channel bottom at the section. According to Equation 6.7, the specific energy at any section is

$$E = \frac{V^2}{2g} + y \tag{6.8}$$

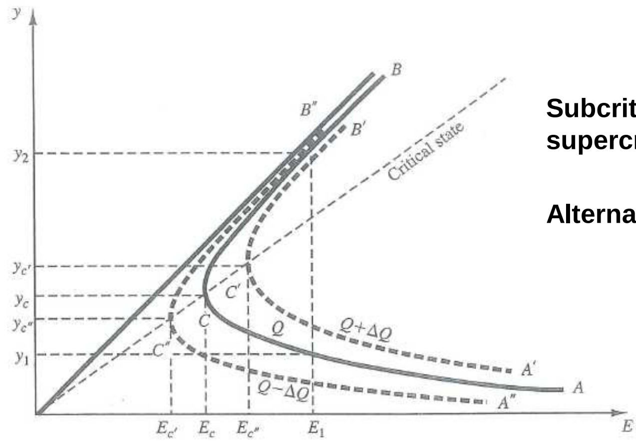
or the specific energy at any section in an open channel is equal to the sum of the velocity head and the water depth at the section.

Given the water area (A) and the discharge (Q) at a particular section, Equation 6.8 may be rewritten as

$$E = \frac{Q^2}{2gA^2} + y \tag{6.9}$$

Thus, for a given discharge Q, the specific energy at any section is a function of the depth of the flow only.

When the depth of the flow, y, is plotted against the specific energy for a given discharge at a given section, a *specific energy curve* is obtained (Figure 6.8). The specific energy curve has two limbs: AC and CB. The lower limb always approaches the horizontal axis toward the right and the upper limb approaches (asymptotically) a 45°-line that passes through the origin. At any point on the specific energy curve, the ordinate represents the depth of the flow at the section, and the abscissa represents the corresponding specific energy. Usually, the same scales are used for both the ordinate and the abscissa.



Subcritical and supercritical flow

Alternate depths

Figure 6.8 Specific energy curves of different discharges at a given channel section

At the critical state the specific energy of the flow takes a minimum value. This value can be computed by equating the first derivative of the specific energy with respect to the water depth to zero:

$$\frac{dE}{dy} = \frac{d}{dy}\left(\frac{Q^2}{2gA^2} + y\right) = -\frac{Q^2}{gA^3}\frac{dA}{dy} + 1 = 0$$

The differential water area (dA/dy) near the free surface is $dA/dy = T$, where T is the *top width* of the channel section. Hence,

$$-\frac{Q^2 T}{gA^3} + 1 = 0 \qquad (6.10a)$$

An important parameter for open-channel flow is defined by $A/T = D$, which is known as the *hydraulic depth* of the section. For rectangular cross sections, the hydraulic depth is equal to the depth of the flow. The above equation may thus be simplified to

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gDA^2} = 1 - \frac{V^2}{gD} = 0 \qquad (6.10b)$$

Froude Number, N_F = $\frac{V}{\sqrt{gD}} = 1$

From Equation 6.10, we may also write (for critical flow),

$$\frac{Q^2}{g} = \frac{A^3}{T} = DA^2 \tag{6.13}$$

In a rectangular channel, $D = y$ and $A = by$. Therefore,

$$\frac{Q^2}{g} = y^3 b^2$$

Because this relation is derived from the critical flow conditions stated above, $y = y_c$, which is the critical depth, and

$$y_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{q^2}{g}} \tag{6.14}$$

where $q = Q/b$, is the discharge per unit width of the channel.

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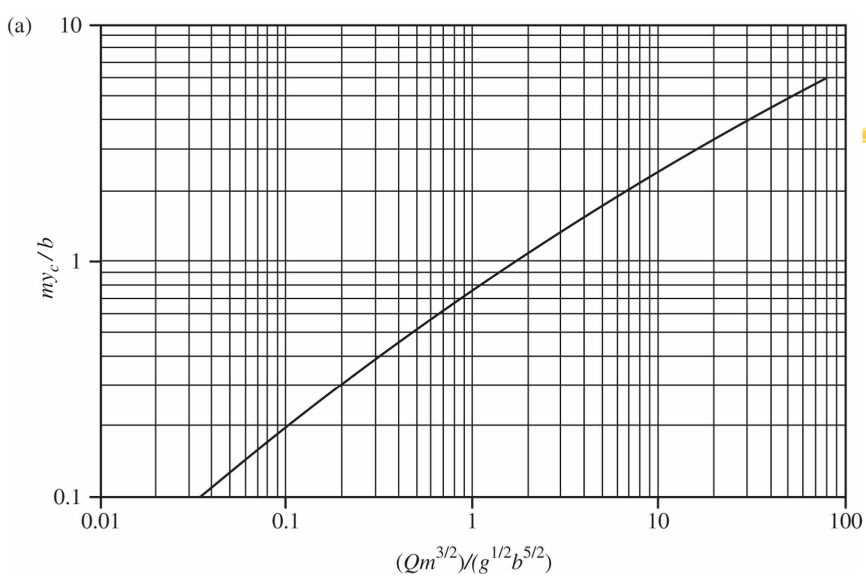

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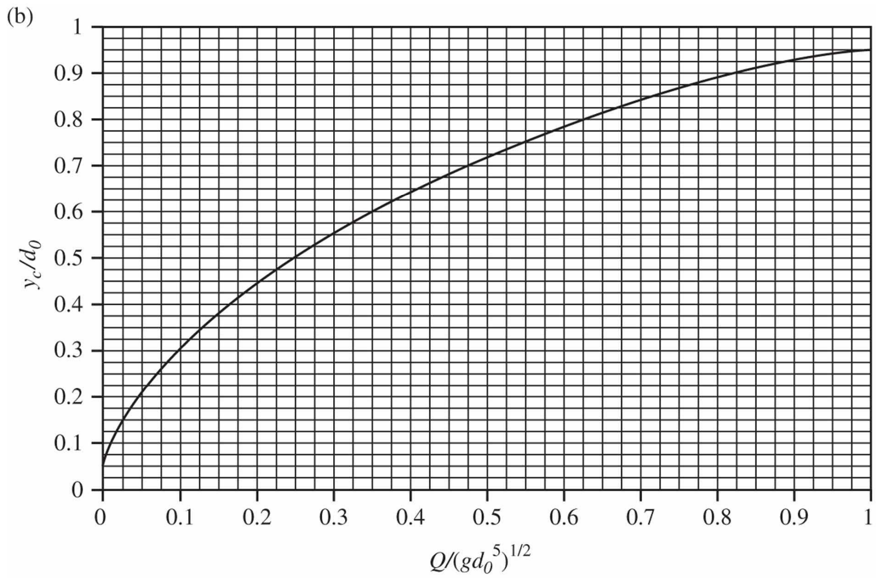

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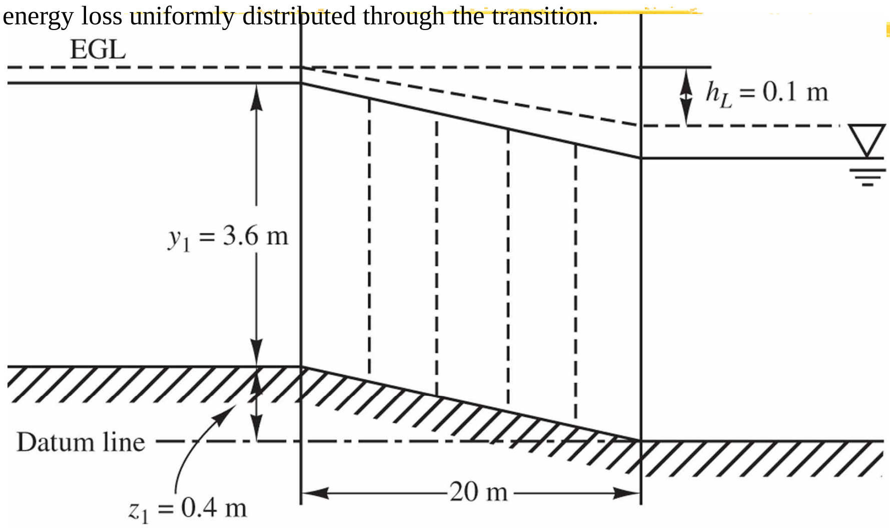
Example 6.5

A hydraulic transition is designed to connect two rectangular channels of the same width by a sloped floor. If the channel is 3 m wide and is carrying a discharge of 15 m^3/sec at 3.6 m depth, determine the water surface profile in the transition. Assume 0.1 m energy loss uniformly distributed through the transition.



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(a)

$$E = \frac{Q^2}{2gA^2} + y = \frac{(15)^2}{2(9.81)(3y)^2} + y = \frac{1.27}{y^2} + y$$

$$V_i = \frac{Q}{A_i} = \frac{15}{(3.6)(3)} = 1.39 \text{ m/sec}$$

$$\frac{V_i^2}{2g} = \frac{(1.39)^2}{2(9.81)} = 0.10 \text{ m}$$

$$H_i = \frac{V_i^2}{2g} + y_i + z_i = 0.10 + 3.60 + 0.40 = 4.10 \text{ m}$$

$$H_e = \frac{V_e^2}{2g} + y_e + z_e = H_i - 0.1 = 4.00 \text{ m}$$

$$E_e = H_e = 4.00 \text{ m}$$

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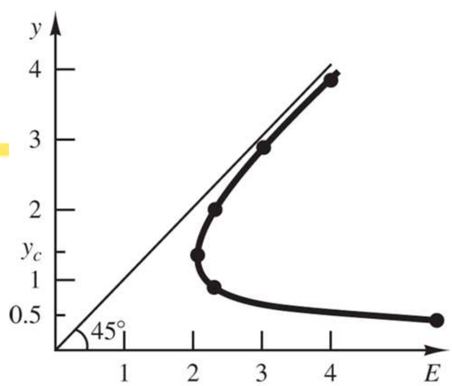

$$y_c = \sqrt[3]{\frac{5^2}{g}} = 1.37 \text{ m}$$

E_c(m) = 2.05 m

y (m)	E (m)
0.5	5.60
1.0	2.27
2.0	2.32
3.0	2.32
3.0	3.14
4.0	4.07

Section	Specific energy, E	E (m)	y (m)
Inlet		3.7	3.6
4.00 m	=3.7-0.1/5+0.4/5=	3.76	3.67
8.00 m	=3.76-0.1/5+0.4/5=	3.82	3.73
12.00 m	=3.82-0.1/5+0.4/5=	3.88	3.79
16.00 m	=3.94-0.1/5+0.4/5=	3.94	3.86
Exit	=3.94-0.1/5+0.4/5=	4	3.92

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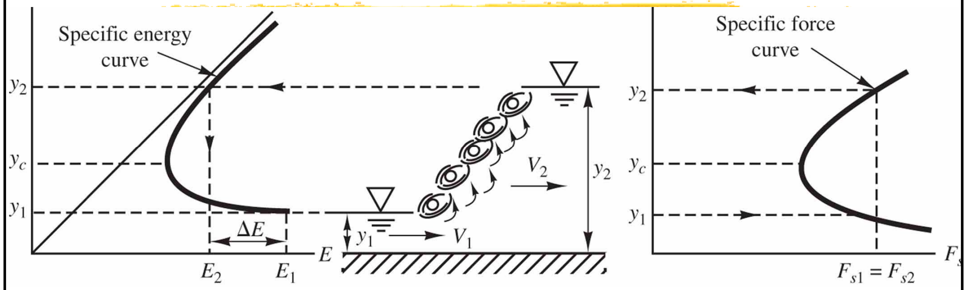
6.5 Hydraulic jumps

Initial depth and sequent depth of a hydraulic jump (Conjugate depths)



$$F_1 - F_2 = \rho q(V_2 - V_1)$$

$$F_1 = \frac{\gamma}{2}y_1^2, \qquad F_2 = \frac{\gamma}{2}y_2^2, \qquad V_1 = \frac{q}{y_1}, \qquad V_2 = \frac{q}{y_2}$$

$$\frac{q^2}{g} = y_1 y_2\left(\frac{y_1 + y_2}{2}\right) \qquad \boxed{\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8N_{F_1}^2} - 1\right)} \qquad N_{F_1} = \frac{V_1}{\sqrt{g y_1}}$$

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Example 6.7

A 10-ft-wide rectangular channel carries 500 cfs of water at a 2-ft depth before entering a jump. Compute the downstream water depth and the critical depth.

$$q = \frac{500}{10} = 50\ \text{ft}^3/\text{sec}\cdot\text{ft} \qquad y_c = \sqrt[3]{\frac{50^2}{32.2}} = 4.27\ \text{ft}$$

$$V_1 = \frac{q}{y_1} = \frac{50}{2} = 25\ \text{ft/sec}$$

$$N_{F_1} = \frac{V_1}{\sqrt{g y_1}} = 3.12$$

$$\frac{y_2}{2.0} = \frac{1}{2}\left(\sqrt{1 + 8(3.12)^2} - 1\right)$$

$$y_2 = 7.88\ \text{ft}$$

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Equation 6.15 may also be arranged as

$$F_1 + \rho q V_1 = F_2 + \rho q V_2$$

where

$$F_s = F + \rho q V \tag{6.19}$$

The quantity F_s is known as the *specific force* per unit width of the channel. For a given discharge, the specific force is a function of the water depth at a given section. When F_s is plotted against the water depth, the resulting curve is similar to a specific energy curve with a vertex that appears at the critical depth. A typical specific force curve is shown in Figure 6.11.

The energy head loss through the hydraulic jump (ΔE) may then be estimated by applying the definition

$$\Delta E = \left(\frac{V_1^2}{2g} + y_1\right) - \left(\frac{V_2^2}{2g} + y_2\right)$$

$$= \frac{1}{2g}(V_1^2 - V_2^2) + (y_1 - y_2) = \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) + (y_1 - y_2)$$

Substituting Equation 6.16 into the above equation and simplifying, we get

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

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Example 6.8

A long, rectangular open channel 3 m wide carries a discharge of 15 m^3/sec. The channel slope is 0.004, and the Manning's coefficient is 0.01. At a certain point in the channel, flow reaches normal depth.

(a) Determine the flow classification. Is it supercritical or subcritical?

(b) If a hydraulic jump takes place at this depth, what is the sequent depth?

(c) Estimate the energy head loss through the jump.

Solution

(a) The critical depth is calculated using Equation 6.14 and $y_c = 1.37$ m. The normal depth of this channel can be determined by the Manning equation (Equation 6.5):

$$Q = \frac{1}{n}A_1 R_{h_1}^{2/3} S^{1/2}$$

$$A = y_1 b, \quad R_h = \frac{A_1}{P_1} = \frac{y_1 b}{2y_1 + b}, \qquad b = 3 \text{ m}$$

$$15 = \frac{1}{0.01}(3y_1)\left(\frac{3y_1}{2y_1 + 3}\right)^{2/3}(0.004)^{1/2}$$

$$y_1 = 1.08 \text{ m}, \qquad V_1 = \frac{15}{3y_1} = 4.63 \text{ m/sec}$$

$$N_{F_1} = \frac{V_1}{\sqrt{gy_1}} = 1.42$$ Because $N_{F_1} > 1$, the flow is supercritical.

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(b) Applying Equation 6.17, we get

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8N_{F_1}^2} - 1\right) = 1.57y_1 = 1.70 \text{ m}$$

(c) The head loss can be estimated by using Equation 6.20:

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(0.62)^3}{4(1.70)(1.08)} = 0.032 \text{ m}$$

6.6 Gradually Varied Flow

The total energy head at any section in an open channel, as defined in Equation 6.7, is restated here as

$$H = \frac{V^2}{2g} + y + z = \frac{Q^2}{2gA^2} + y + z$$

To compute the water surface profile, we must first obtain the variation of the total energy head along the channel. Differentiating H with respect to the channel distance x, we obtain the energy gradient in the direction of the flow:

$$\frac{dH}{dx} = \frac{-Q^2}{gA^3}\frac{dA}{dx} + \frac{dy}{dx} + \frac{dz}{dx} = -\frac{Q^2T}{gA^3}\frac{dy}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

where $dA = T(dy)$. Rearranging the equation gives

$$\frac{dy}{dx} = \frac{\dfrac{dH}{dx} - \dfrac{dz}{dx}}{1 - \dfrac{Q^2T}{gA^3}} \tag{6.21}$$

Using the Manning formula (Equation 6.5), we get

$$S_e = -\frac{dH}{dx} = \frac{n^2Q^2}{R_h^{4/3}A^2} = \frac{n^2Q^2}{b^2y^{10/3}} \tag{6.22}$$

The slope of the channel bed may also be expressed in similar terms if uniform flow were assumed to take place in the channel. Because the slope of the channel bed is equal to the energy slope in uniform flow, the hypothetical uniform flow conditions are designated with the subscript n. We have

$$S_0 = -\frac{dz}{dx} = \left(\frac{n^2Q^2}{b^2y^{10/3}}\right)_n \tag{6.23}$$

From Equation 6.14 for rectangular channels,

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{Q^2}{gb^2}}$$

or

$$Q^2 = gy_c^3b^2 = \frac{gA_c^3}{b} \tag{6.24}$$

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Substituting Equations 6.22, 6.23, and 6.24 into Equation 6.21, we have

$$\frac{dy}{dx} = \frac{S_0\left[1 - \left(\frac{y_n}{y}\right)^{10/3}\right]}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]} \tag{6.25a}$$

For nonrectangular channels, Equation 6.24a can be generalized as

$$\frac{dy}{dx} = \frac{S_0\left[1 - \left(\frac{y_n}{y}\right)^{N}\right]}{\left[1 - \left(\frac{y_c}{y}\right)^M\right]} \tag{6.25b}$$

where the exponents M and N depend on the cross-sectional shape and the flow conditions as given by Chow.*

This form of the *gradually varied flow equation* is very useful for a qualitative analysis, which helps to understand the gradually varied flow classifications covered in the next sections. Other forms are often used to compute water surface profiles. Physically, the term dy/dx represents the slope of the water surface with respect to the bottom of the channel. For $dy/dx = 0$, the water depth remains constant throughout the reach or the special case of uniform flow. For $dy/dx < 0$, the water depth decreases in the direction of the flow. For $dy/dx > 0$, the water depth increases in the direction of the flow. Solutions of this equation under different conditions will yield the various water surface profiles that occur in open channels.

6.7 Classifications of Gradually Varied Flow

Depending on the channel slope, the surface conditions, the sectional geometry, and the discharge, open channels may be classified into five categories:

1. steep channels,
2. critical channels,
3. mild channels,
4. horizontal channels, and
5. adverse channels.

Steep channels:	$y_n/y_c < 1.0$	or	$y_n < y_c$
Critical channels:	$y_n/y_c = 1.0$	or	$y_n = y_c$
Mild channels:	$y_n/y_c > 1.0$	or	$y_n > y_c$
Horizontal channels:	$S_0 = 0$		
Adverse channels:	$S_0 < 0$		

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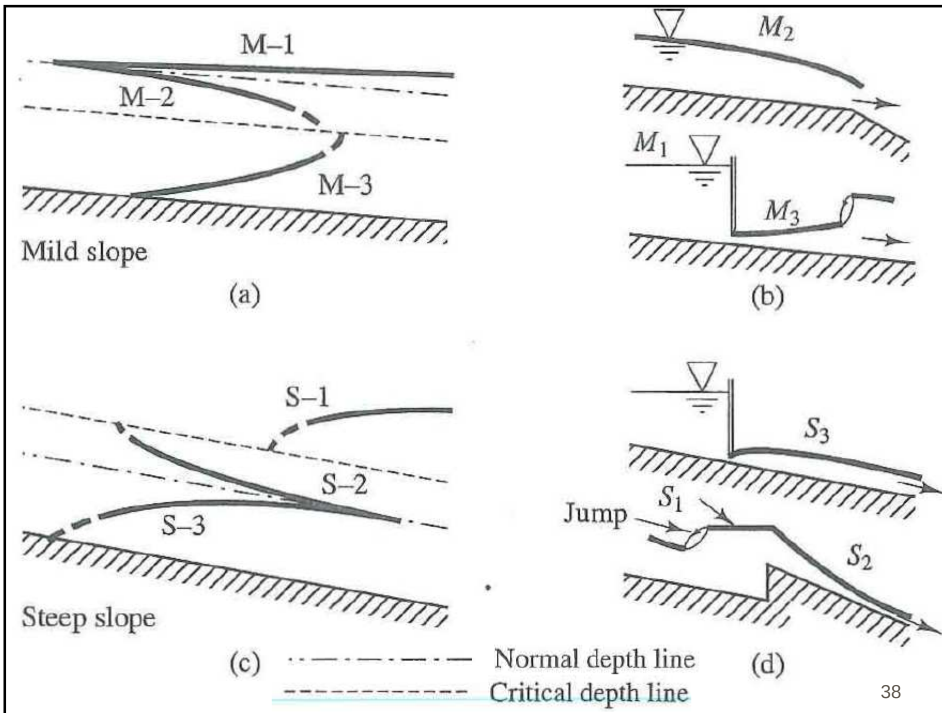

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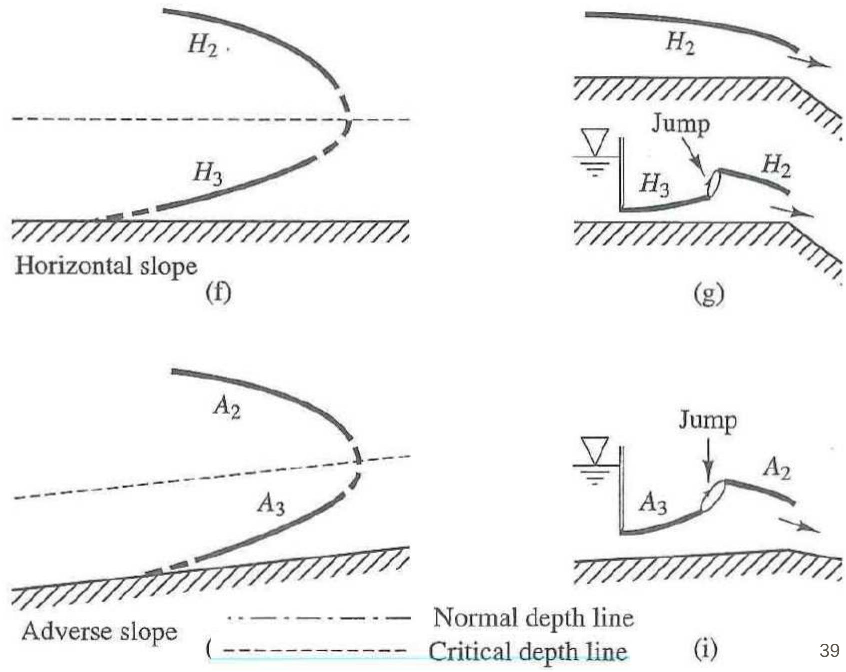

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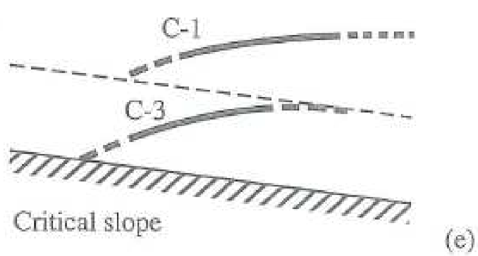

Rare occurrences

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TABLE 6.3 Characteristics of Water Surface Profile Curves

Channel	Symbol	Type	Slope	Depth	Curve
Mild	M	1	$S_0 > 0$	$y > y_n > y_c$	M-1
Mild	M	2	$S_0 > 0$	$y_n > y > y_c$	M-2
Mild	M	3	$S_0 > 0$	$y_n > y_c > y$	M-3
Critical	C	1	$S_0 > 0$	$y > y_n = y_c$	C-1
Critical	C	3	$S_0 > 0$	$y_n = y_c > y$	C-3
Steep	S	1	$S_0 > 0$	$y > y_c > y_n$	S-1
Steep	S	2	$S_0 > 0$	$y_c > y > y_n$	S-2
Steep	S	3	$S_0 > 0$	$y_c > y_n > y$	S-3
Horizontal	H	2	$S_0 = 0$	$y > y_c$	H-2
Horizontal	H	3	$S_0 = 0$	$y_c > y$	H-3
Adverse	A	2	$S_0 < 0$	$y > y_c$	A-2
Adverse	A	3	$S_0 < 0$	$y_c > y$	A-3

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6.8 Computation of Water Surface Profiles

Water surface profiles for gradually varied flow may be computed by using Equation 6.25. The computation normally begins at a section where the relationship between the water surface elevation (or flow depth) and the discharge is known. These sections are commonly known as *control sections* (or mathematically, boundary conditions). A few examples of common control sections in open channels are depicted in Figure 6.13. Locations where uniform flow occurs can also be viewed as a control section because the Manning equation describes a flow depth–discharge relationship. Uniform flow (i.e., flow at normal depth) tends to occur in the absence of or far away from other control sections and where the stream slope and cross section are relatively constant.

A successive computational procedure based on an energy balance is used to obtain the water surface elevation at the next section, either upstream or downstream from the control section. The distance between sections is critical because the water surface will be represented by a straight line. Thus, if the depth of flow is changing quickly over short distances, adjacent sections should be closely spaced to represent accurately the water surface profile. The step-by-step procedure is carried out in the downstream direction for rapid (supercritical) flows and in the upstream direction for tranquil (subcritical) flows.

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6.8 Computation of Water Surface Profiles

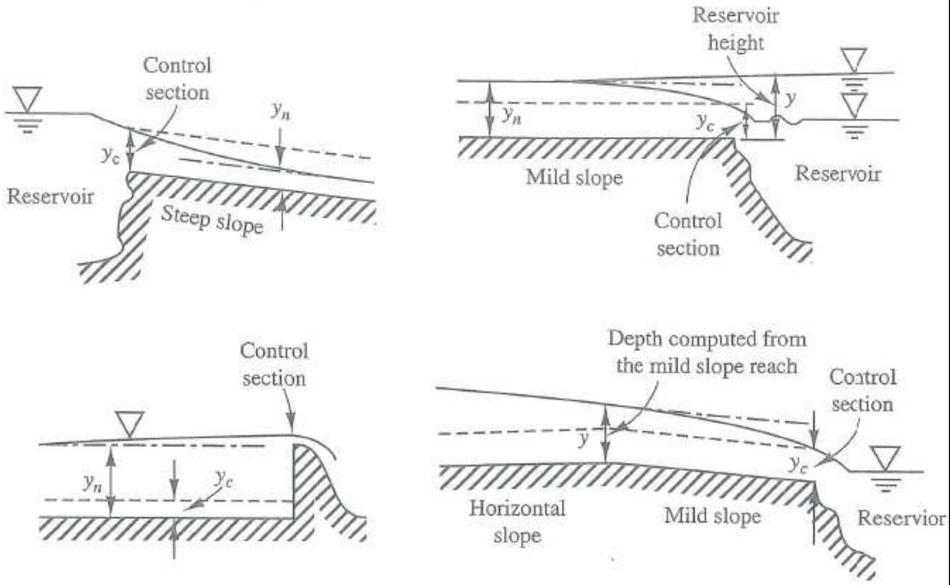

Figure 6.13 Control sections in open channels

6.8.1 Standard step method

The standard step method is presented in this section to calculate gradually varied flow water surface profiles. The method employs a finite difference solution scheme to solve the differential, gradually varied flow equation (Equation 6.25). It is the most common algorithm used in computer software packages that solve gradually varied flow profiles.

The standard step method is derived directly from an energy balance between two adjacent cross sections (Figure 6.14) that are separated by a sufficiently short distance so that the water surface can be approximated by a straight line. The energy relation between the two sections may be written as

$$\frac{V_2^2}{2g} + y_2 + \Delta z = \frac{V_1^2}{2g} + y_1 + h_L \qquad (6.26a)$$

$$\left(z_2 + y_2 + \frac{V_2^2}{2g}\right) = \left(z_1 + y_1 + \frac{V_1^2}{2g}\right) + \overline{S}_e \Delta L \qquad (6.26b)$$

$$E_2' = E_1' + \text{losses} \qquad (6.26c)$$

where z is the position head (channel bottom elevation with respect to some datum) and E' is the total energy head (position + depth + velocity). It is important to note that, in Equation 6.26, the sections 1 and 2 represent downstream and upstream sections, respectively. If the sections are numbered differently, the losses should always be added to the downstream side.

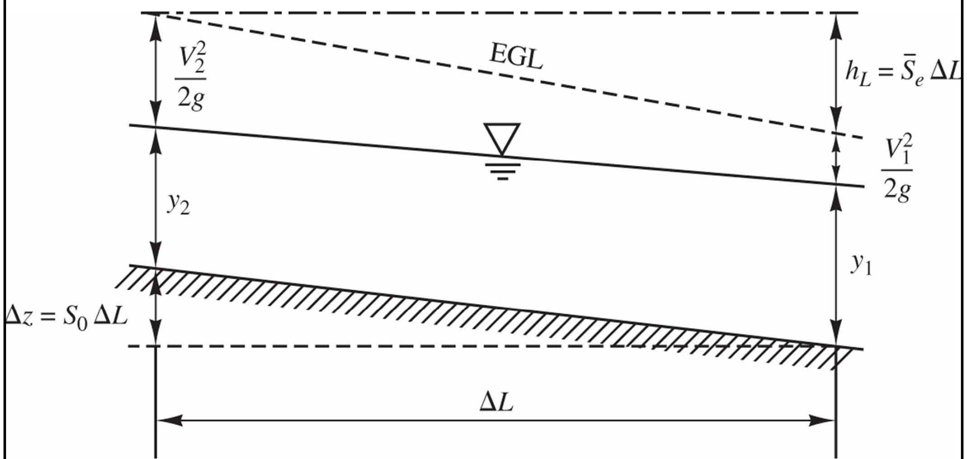

Figure 6.14 Energy relationships in a water surface profile

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Equation 6.26b cannot be solved directly for the unknown depth (e.g., y_2) because V_2 and $\bar{S}_e$ depend on y_2. Therefore, an iterative procedure is required using successive approximations of y_2 until the downstream and upstream energies balance (or come within an acceptable range). The energy slope (S_e) can be computed by applying the Manning equation, in either SI units

$$S_e = \frac{n^2 V^2}{R_h^{4/3}} \tag{6.27a}$$

where $\bar{S}_e$ is the average of the energy (EGL) slopes at the upstream and downstream sections. A tabulated computation procedure is recommended as illustrated in the example problems to follow.

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6.8.2 Direct step method

In the direct step method, the gradually varied flow equations are rearranged to determine the distance (ΔL) explicitly between two selected flow depths. This method is applicable to prismatic channels only because the same cross-sectional geometric relationships are used for all the sections along the channel.

Replacing sections 1 and 2 with U and D, respectively, and noting that $S_0 = (z_U - z_D)/\Delta L = \Delta z/\Delta L$ Equation 6.26b is rearranged as

$$\Delta L = \frac{\left(y_D + \frac{V_D^2}{2g}\right) - \left(y_U + \frac{V_U^2}{2g}\right)}{S_0 - \overline{S}_e} = \frac{E_D - E_U}{S_0 - \overline{S}_e} \tag{6.26d}$$

where $E = y + V^2/2g$ is *specific energy*. In Equation 6.26d, U and D represent upstream and downstream sections, respectively. For subcritical flow, the computations begin at the downstream end and progress upstream. In this case y_D and E_D would be known. An appropriate value for y_U is selected and the associated E_U is calculated. Then ΔL is determined by using Equation 6.26d. For supercritical flow, the computations begin at the upstream end and progress downstream. In this case, y_U and E_U would be known. An appropriate value for y_D is selected, and the associated E_D is calculated. Then ΔL is determined by using Equation 6.26d.

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Example 6.9

A grouted-riprap, trapezoidal channel ($n = 0.025$) with a bottom width of 4 meters and side slopes of $m = 1$ carries a discharge 12.5 m^3/sec on a 0.001 slope. Compute the backwater curve (upstream water surface profile) created by a low dam that backs water up to a depth of 2 m immediately behind the dam. Specifically, water depths are required at critical diversion points that are located at distances of 188 m, 423 m, 748 m, and 1,675 m upstream of the dam.

Solution

Normal depth for this channel can be calculated by using Equation 6.5 (iterative solution), Figure 6.4 (a), or appropriate computer software. Using Figure 6.4 (a),

$$\frac{nQ}{k_M S_0^{1/2} b^{8/3}} = \frac{(0.025)(12.5)}{(1.00)(0.001)^{1/2}\,(4)^{8/3}} = 0.245$$

From Figure 6.4 (a), with m = 1, we obtain

$$y_n/b = 0.415$$

therefore, $y_n = (4\text{ m})(0.415) = 1.66$ m.

Critical depth for this channel can be calculated by using Equation 6.13 (iterative solution), Figure 6.9 (a), or appropriate computer software. Using Figure 6.9 (a),

$$\frac{Qm^{3/2}}{g^{1/2}\,b^{5/2}} = \frac{(12.5)(1)^{3/2}}{(9.81)^{1/2}\,(4)^{5/2}} = 0.125$$

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From Figure 6.9 (a), we obtain

$$my_c/b = 0.230; \text{ therefore, } y_c = (4 \text{ m})(0.230)/1.0 = 0.92 \text{ m}$$

We will first use the standard step method. Water surface profile computations require the use of the Manning equation (Equation 6.27a), which contains the variables R_h and V. Recall from earlier discussions that $R_h = A/P$, where A is the flow area and P is the wetted perimeter, and $V = Q/A$.

The computation procedure displayed in Table 6.4 (a) is used to determine the water surface profile. The depth just upstream from the dam is the control section, designated as section 1. Energy balance computations begin here and progress upstream (backwater) because the flow is subcritical ($y_c < y_n$). The finite difference process is iterative; the depth of flow is assumed at section 2 until the energy at the first two sections match using Equation 6.26b. Once the water depth at section 2 is determined, the depth of flow at section 3 is assumed until the energies at sections 2 and 3 balance. This stepwise procedure continues upstream until the entire water surface profile is developed.

Because the starting depth of 2.00 m is greater than the normal depth and normal depth exceeds critical depth, the profile has an M-1 classification (Figure 6.12). The flow depth will approach normal depth asymptotically as the computations progress upstream, as depicted in Figure 6.13 (c). Once the depth becomes normal, or relatively close, the computation procedure is ended. The first few standard step computations are displayed in Table 6.4 (a); completion of the problem is left to the student in Problem 6.8.6.

TABLE 6.4 (a) Water Surface Profile (Backwater) Computations Using the Standard Step Method (Example 6.9)

(1) Section	(2) U/D	(3) y (m)	(4) z (m)	(5) A (m^2)	(6) V (m/sec)	(7) $V^2/2g$ (m)	(8) P (m)	(9) R_h (m)	(10) S_e	(11) $S_{e(avg)}$	(12) h_L (m)	(13) Total Energy (m)
1	D	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000538	0.1011	2.156
2	U	1.94	0.188	11.52	1.085	0.0600	9.487	1.215	0.000567	(ΔL = 188 m)		2.188
Note: The trial depth of 1.94 m is too high; the energy does not balance. Try a lower upstream depth.												
1	D	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000554	0.1042	2.159
2	U	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000600	(ΔL = 188 m)		2.160
Note: The trial depth of 1.91 m is correct. Now balance energy between sections 2 and 3.												
2	D	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000673	0.1582	2.319
3	U	1.80	0.423	10.44	1.197	0.0731	9.091	1.148	0.000745	(ΔL = 235 m)		2.296
Note: The trial depth of 1.80 m is too low; the energy does not balance. Try a higher upstream depth.												
2	D	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000659	0.1549	2.315
3	U	1.82	0.423	10.59	1.180	0.0710	9.148	1.158	0.000716	(ΔL = 235 m)		2.314
Note: The trial depth of 1.82 m is correct. Now balance energy between sections 3 and 4.												

Column (1) Section numbers are arbitrarily designated from downstream to upstream.

Column (2) Sections are designated as either downstream (D) or upstream (U) to assist in the energy balance.

Column (3) Depth of flow (meters) is known at section 1 and assumed at section 2. Once the energies balance, the depth is now known at section 2, and the depth at section 3 is assumed until the energies at sections 2 and 3 balance.

Column (4) The channel bottom elevation (meters) above some datum (e.g., mean sea level) is given. In this case, the datum is taken as the channel bottom at section 1. The bottom slope and distance interval are used to determine subsequent bottom elevations.

Column (5) Water cross-sectional area (square meters) corresponds to the depth in the trapezoidal cross section.

Column (6) Mean velocity (meters per second) is obtained by dividing the discharge by the area in column 5.

Column (7) Velocity head (meters).
Column (8) Wetted perimeter (meters) of the trapezoidal cross section based on the depth of flow.
Column (9) Hydraulic radius (meters) equal to the area in column 5 divided by the wetted perimeter in column 8.
Column (10) Energy slope obtained from Manning equation (Equation 6.27a).
Column (11) Average energy grade line slope of the two sections being balanced.
Column (12) Energy loss (meters) from friction between the two sections found using $h_L = S_{e(\text{avg})}(\Delta L)$ from Equation 6.26b.
Column (13) Total energy (meters) must balance in adjacent sections (Equation 6.26b). Energy losses are always added to the downstream section. Also, the energy balance must be very close before proceeding to the next pair of adjacent sections or errors will accumulate in succeeding computations. Thus, even though depths were only required to the nearest 0.01 m, energy heads were calculated to the nearest 0.001 m.

TABLE 6.4 (b) Water Surface Profile (Backwater) Computations Using the Direct Step Method (Example 6.9)

Section	U/D	y (m)	A (m^2)	P (m)	R_h (m)	V (m/sec)	$V^2/2g$ (m)	E (m)	S_e	ΔL(m)	Distance to Dam (m)
1	*D*	2.00	12.00	9.657	1.243	1.042	0.0553	2.0553	0.000508		0
2	*U*	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601	186	186
	A distance of 186 m separates the two flow depths (2.00 m and 1.91 m).										
2	*D*	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601		186
3	*U*	1.82	10.59	9.148	1.158	1.180	0.0710	1.8910	0.000716	239	425
	A distance of 239 m separates the two flow depths (1.91 m and 1.82 m).										

6.9 Hydraulic design of open channels

TABLE 6.6 Stable Side Slopes for Channels

Material	Side Slope[a] (Horizontal:Vertical)
Rock	Nearly Vertical
Muck and peat soils	$^1/_4$:1
Stiff clay or earth with concrete lining	$^1/_2$:1 to 1:1
Earth with stone lining or earth for large channels	1:1
Firm clay or earth for small ditches	$1^1/_2$:1
Loose, sandy earth	2:1 to 4:1
Sandy loam or porous clay	3:1

[a]If channel slopes are to be mowed, a maximum side slope of 3:1 is recommended.
Source: Adapted from V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

Freeboard is the vertical distance between the top of the channel and the water surface that prevails under the design flow conditions. This distance should be sufficient to allow variations in the water surface because of wind-driven waves, tidal action, occurrence of flows exceeding the design discharge, and other causes. There are no universally accepted rules to determine an acceptable freeboard. In practice, freeboard selection is often a matter of judgment, or it is stipulated as part of the prevailing design standards. For example, the U.S. Bureau of Reclamation recommends that unlined channel freeboard be computed as

$$F = \sqrt{Cy} \tag{6.28}$$

where F = freeboard, y = flow depth, and C = freeboard coefficient. If F and y are in ft, C varies from 1.5 for a channel capacity of 20 cfs to 2.5 for a channel capacity of 3,000 cfs or more. If metric units are used with F and y in meters, C varies from 0.5 for a flow capacity for 0.6 m^3/s to 0.76 for a flow capacity of 85 m^3/s or more. For lined channels, the Bureau recommends that the curves displayed in Figure 6.15 are used to estimate the height of the bank above the water surface (W.S.) and the height of the lining above the water surface.

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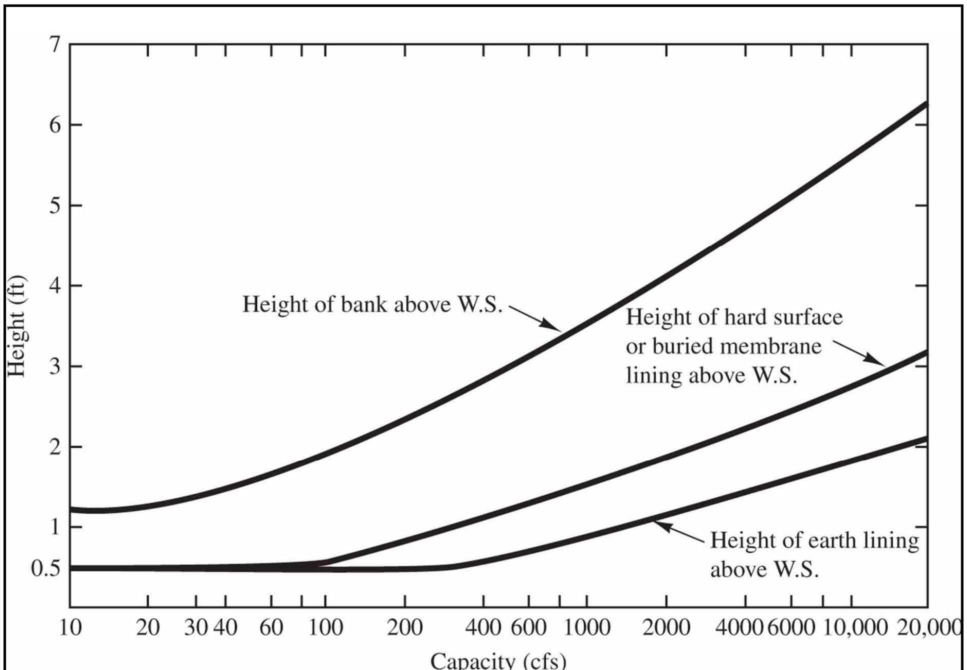

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TABLE 6.7 Suggested Maximum Permissible Channel Velocities

Channel Material	V_{max} (ft/sec)	V_{max} (m/sec)
Sand and Gravel		
Fine sand	2.0	0.6
Coarse sand	4.0	1.2
Fine gravel[a]	6.0	1.8
Earth		
Sandy silt	2.0	0.6
Silt clay	3.5	1.0
Clay	6.0	1.8

[a]Applies to particles with median diameter (D_{50}) less than 0.75 in (20 mm).
Source: U.S. Army Corps of Engineers. "Hydraulic Design of Flood Control Channels," Engineer Manual, EM 1110-2-1601. Washington, DC: Department of the Army, 1991.

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6.9.1 Unlined channels

1. For the specified channel material, determine the Manning roughness coefficient from Table 6.2, a stable side slope from Table 6.6, and the maximum permissible velocity from Table 6.7.
2. Compute the hydraulic radius (R_h) from the Manning equation rearranged as

$$R_h = \left(\frac{n\, V_{max}}{k_M \sqrt{S_0}} \right)^{3/2} \tag{6.29}$$

   where k_M = 1.49 ft$^{1/3}$/sec for the conventional U.S. unit system and 1.0 m$^{1/3}$/sec for the metric system.
3. Compute the required flow area from $A = Q/V_{max}$.
4. Compute the wetted perimeter from $P = A/R_h$.
5. Using the expressions for A and P given in Table 6.1, solve for the flow depth (y) and the bottom width (b) simultaneously.
6. Check the Froude number and ensure that it is not close to unity.
7. Add a freeboard (Equation 6.28) and modify the section for practical purposes.

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Example 6.11

An unlined channel to be excavated in stiff clay will convey a design discharge of $Q = 9.0$ m^3/sec on a slope of $S_0 = 0.0028$. Design the channel dimensions using the maximum permissible velocity method.

Solution

From Table 6.6, $m = 1.0$ for stiff clay; from Table 6.2, use $n = 0.022$ (clean and smooth surface). Also, from Table 6.7, $V_{max} = 1.8$ m/sec. Using Equation 6.29 with $k_M = 1.00$

$$R_h = \left[\frac{0.022(1.8)}{1.00\sqrt{0.0028}}\right]^{3/2} = 0.647 \text{ m}$$

Also, $A = Q/V_{max} = 9.0/1.8 = 5.0$ m^2. Hence, $P = A/R_h = 5.0/0.647 = 7.73$ m. Now, from expressions given in Table 6.1 and using $m = 1.0$,

$$A = (b + my)y = (b + y)y = 5 \text{ m}^2$$

and

$$P = b + 2y\sqrt{1 + m^2} = b + 2.83\,y = 7.73 \text{ m}$$

We now have two equations with two unknowns, y and b. From the second equation, $b = 7.73 - 2.83y$. Substituting this into the first equation and simplifying yields

$$1.83y^2 - 7.73y + 5.00 = 0$$

This equation has two roots, $y = 0.798$ m and 3.43 m. The first root results in a channel width of $b = 7.73 - 2.83(0.798) = 5.47$ m. The second root results in a channel width of $b = 7.73 - 2.83(3.43) = -1.98$ m. Obviously, a negative channel width has no physical meaning. Therefore $y = 0.798$ m will be used.

Next we will check to see if the Froude number is close to the critical value of 1.0. From the expression given for the top width in Table 6.1,

$$T = b + 2my = 5.47 + 2(1)0.798 = 7.07 \text{ m}$$

Then the hydraulic depth becomes $D = A/T = 5.0/7.07 = 0.707$ m, and finally

$$N_F = \frac{V}{\sqrt{gD}} = \frac{1.8}{\sqrt{9.81(0.707)}} = 0.683$$

This value indicates that, under the design flow conditions, the flow will not be near the critical state.

Finally, we will determine a freeboard using Equation 6.28. It is known that C varies from 0.5 for a channel capacity of 0.6 m^3/s 0 to 0.76 for a capacity of 85 m^3/s. Assuming this variation is linear, we determine C as being 0.526 for $Q = 9.0$ m^3/s by interpolation. Then,

$$F = \sqrt{0.526(0.798)} = 0.648 \text{ m}$$

The total depth for the channel is $y + F = (0.798 + 0.648) = 1.45 \text{ m} \approx 1.5$ m (for practicality in field construction). The bottom width of 5.47 m is increased to 5.5 m for the same reason. The top width of the excavated channel then becomes $b + 2m(y) = 5.5 + 2(1)(1.5) = 8.5$ m.

6.9.2 Rigid boundary channels

1. Select m and determine n for the specified lining material.
2. Evaluate the ratio b/y from Equation 6.30.
3. Rearrange the Manning formula as

$$y = \frac{\left[(b/y) + 2\sqrt{1+m^2}\right]^{1/4}}{\left[(b/y) + m\right]^{5/8}}\left(\frac{Qn}{k_M\sqrt{S_0}}\right)^{3/8} \qquad (6.31)$$

   and solve for y knowing all the terms on the right-hand side. Then find b.
4. Check the Froude number.
5. Determine the height of lining and the freeboard using Figure 6.15 and modify the section for practical purposes.

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Example 6.12
A trapezoidal, concrete-lined channel is required to convey a design discharge of 15 m^3/s ec. The channel bottom slope is $S_0 = 0.00095$, and the maximum side slope based on local ordinances is $m = 2.0$. Design the channel dimensions using the best hydraulic section approach.

Solution
From Table 6.2, $n = 0.013$ for concrete. Substituting $m = 2$ into Equation 6.30, we find

$$\frac{b}{y} = 2\left(\sqrt{1+2^2} - 2\right) = 0.472$$

Next, using Equation 6.31 with $k_M = 1.0$ for the metric unit system,

$$y = \frac{\left[(0.472) + 2\sqrt{1+2^2}\right]^{1/4}}{\left[(0.472) + 2\right]^{5/8}}\left[\frac{(15.0)(0.013)}{1.0\sqrt{0.00095}}\right]^{3/8} = 1.69 \text{ m}$$

Then, $b = 0.472(1.69) = 0.798$ m. For this section,

$$\begin{aligned}
A &= (b+my)y = [0.798 + 2(1.69)]1.69 = 7.06 \text{ m}^2,\\
T &= b + 2my = 0.798 + 2(2)1.69 = 7.56 \text{ m},\\
D &= A/T = 7.06/7.56 = 0.934 \text{ m},\\
V &= Q/A = 15.0/7.06 = 2.12 \text{ m/sec, and}\\
N_F &= V/(gD)^{1/2} = 2.12/[9.81(0.933)]^{1/2} = 0.701.
\end{aligned}$$

The Froude number is sufficiently lower than the critical value of 1.0.

Finally, from Figure 6.15 (with $Q = 15$ m^3/s $= 530$ cfs), the lining height above the free surface is 1.2 ft (0.37 m). Also, the freeboard (height of bank) above the free surface is 2.9 ft (0.88 m). Thus, the design channel depth is $y + F = (1.69 + 0.88) = 2.57$ m $\approx$ 2.6 m (for practicality in field construction). The bottom width of 0.798 m is increased to 0.8 m for the same reason. The top width of the channel is $b + 2m(y) = 0.8 + 2(2)(2.6) = 11.2$ m.

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