Chapter 5
Water Pumps

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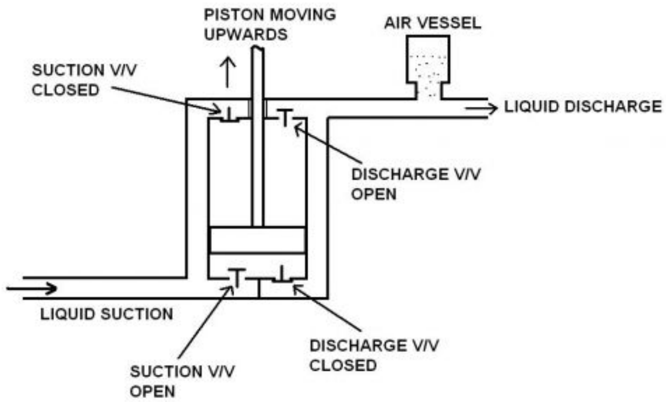
Expected student outcomes

- Differentiate among various types of pumps and apply pump curves and similarity principles for pumps. **[a,e]**
- Determine the pump properties experimentally when they are connected in series and parallel **[a,c,k]**

Types of pumps
Positive –Displacement Pumps

- These types of pumps displace fixed volumes of fluid during each cycle or revolution of the pump.
- No longer used for distribution system pumping in most water systems, but portable units may be used for dewatering excavations during construction.



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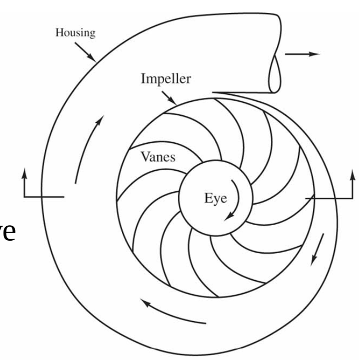
5.1 Centrifugal (radial flow) pumps

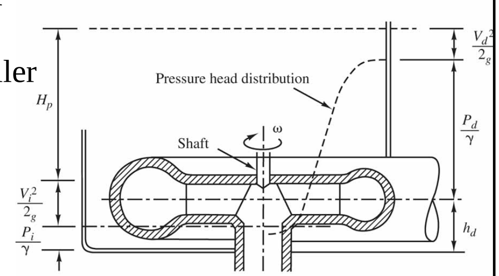
Frequently used in water distribution systems.

Water enters the pump through the eye of the spinning impeller and outward from the vanes to discharge pipe.

A centrifugal pump consists of: A rotating element (impeller) and housing which encloses the impeller and seals the pressurized liquid.



(a)



(b)

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Conservation of angular momentum

$$T=\int \vec{F}.\vec{r}=m\int \vec{a}.\vec{r}=\int \frac{d(m\vec{V})}{dt}.\vec{r}=\int(\vec{V}.\vec{r})d(\rho Q)=\rho\int|\vec{r}||\vec{V}|\cos\alpha dQ$$

ω : angular speed $\vec{F}$: Force $\vec{r}$: Moment arm $\vec{V}$: Velocity

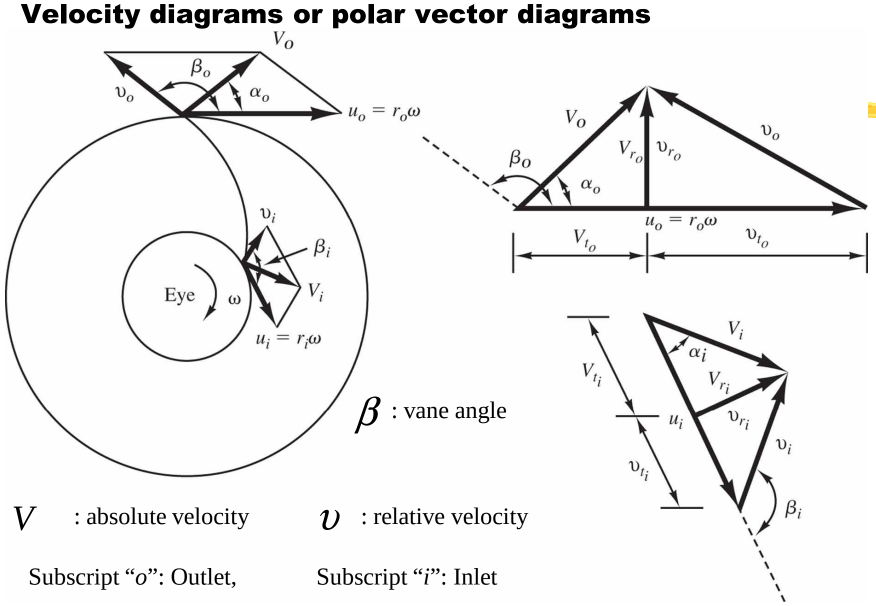
Torque : $$T=\rho\int_{out}|\vec{r}||\vec{V}|\cos\alpha dQ-\rho\int_{in}|\vec{r}||\vec{V}|\cos\alpha dQ$$

Using average velocity: $$T=\rho Q(r_oV_o\cos\alpha_o-r_iV_i\cos\alpha_i)$$

Power input to the pump: $$P_i=\omega T=\rho Q\omega(r_oV_o\cos\alpha_o-r_iV_i\cos\alpha_i)$$

Head supplied by pump: $$H_p=H_d-H_i=\left(h_d+\frac{p_d}{\gamma}+\frac{V_d^2}{2g}\right)-\left(\frac{p_i}{\gamma}+\frac{V_i^2}{2g}\right)$$

Velocity diagrams or polar vector diagrams



β : vane angle

V : absolute velocity υ : relative velocity

Subscript "*o*": Outlet, Subscript "*i*": Inlet

Subscript "*t*": tangential, Subscript "*r*": radial

Hydraulic efficiency of the pump

Power output from the pump: $P_o = \gamma Q H_p$

$\vec{T}$: Torque $\vec{F}$: Force $\vec{r}$: Moment arm $\vec{V}$: Velocity

$$T = \rho \int_{out} |\vec{r}||\vec{V}| \cos\alpha dQ - \rho \int_{in} |\vec{r}||\vec{V}| \cos\alpha dQ$$

Using average velocity: $T = \rho Q(r_0 V_0 \cos\alpha_0 - r_i V_i \cos\alpha_i)$

Power input to the pump: $P_i = \omega T = \rho\omega Q(r_0 V_0 \cos\alpha_0 - r_i V_i \cos\alpha_i)$

Pump efficiency: $e_p = \dfrac{P_o}{P_i}$ Motor efficiency: $e_m = \dfrac{P_i}{P_m}$

Overall efficiency of pump system: $e = e_p \times e_m = \dfrac{P_o}{P_i} \times \dfrac{P_i}{P_m} = \dfrac{P_o}{P_m}$

Example 5.1

A centrifugal pump has the following characteristics: $r_i = 12$ cm, $r_o = 40$ cm, $\beta_i = 118°$, $\beta_o = 140°$. The width of the impeller vanes is 10 cm and is uniform throughout. At the angular speed of 550 rpm the pump delivers 0.98 m^3/sec of water between two reservoirs with a 25-m elevation difference. If a 500-kW motor is used to drive the centrifugal pump, determine the efficiency of the pump and the overall efficiency of the system at this stage of operation.

Solution

The peripheral speeds of the vanes at the entrance and at the exit of the impeller are, respectively,

$$u_i = \omega r_i = 2\pi \cdot \frac{550}{60} \cdot 0.12 \text{ m} = 6.91 \text{ m/sec}$$

$$u_o = \omega r_o = 2\pi \cdot \frac{550}{60} \cdot 0.40 \text{ m} = 23.04 \text{ m/sec}$$

and the radial velocity of the water may be obtained by applying the continuity equation; $Q = A_i V_{r_i} = A_o V_{r_o}$, where $A_i = 2\pi r_i B$ and $A_o = 2\pi r_o B$. It can be easily shown that $V_{r_i} = \upsilon_{r_i}$ and $V_{r_o} = \upsilon_{r_o}$.

$$\upsilon_{r_i} = \frac{Q}{A_i} = \frac{Q}{2\pi r_i B} = \frac{0.98}{2\pi \cdot 0.12 \cdot 0.1} = 13.00 \text{ m/sec}$$

$$\upsilon_{r_o} = \frac{Q}{A_O} = \frac{Q}{2\pi r_o B} = \frac{0.98}{2\pi \cdot 0.4 \cdot 0.1} = 3.90 \text{ m/sec}$$

From the velocity vector diagram (see Figure 5.3)

$$v_{t_i} = \frac{v_{r_i}}{\tan \beta_i} = \frac{13.00}{\tan 118°} = -6.91 \text{ m/sec}$$

$$v_{t_o} = \frac{v_{r_o}}{\tan \beta_o} = \frac{3.90}{\tan 140°} = -4.65 \text{ m/sec}$$

and

$$V_i = \sqrt{v_{r_i}^2 + (u_i + v_{t_i})^2} = \sqrt{(13.00)^2 + (0.00)^2} = 13.00 \text{ m/sec}$$

$$\alpha_i = \tan^{-1}\frac{v_{r_i}}{(u_i + v_{t_i})} = \tan^{-1}\left(\frac{13.00}{0.00}\right) = 90°$$

$\cos \alpha_i = 0$ (Note that the absolute water velocity is completely in the radial direction. This minimizes the energy loss at the inlet.)

$$V_o = \sqrt{v_{r_o}^2 + (u_o + v_{t_o})^2} = \sqrt{(3.90)^2 + (18.39)^2} = 18.80 \text{ m/sec}$$

$$\alpha_o = \tan^{-1}\frac{v_{r_o}}{u_o + v_{t_o}} = \tan^{-1}\left(\frac{3.90}{18.39}\right) = 11.97°$$

$\cos \alpha_o = 0.978$

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Applying Equation (5.3), we get

$$P_i = \rho Q \omega (r_o V_o \cos \alpha_o - r_i V_i \cos \alpha_i)$$

$$P_i = 1000 \cdot 0.98 \cdot 2\pi \cdot \frac{550}{60} \cdot (0.40 \cdot 18.8 \cdot 0.978 - 0) = 415{,}120 \text{ w}$$

$= 415.12$ kW (Note: 1 N · m/sec = 1 watt)

Applying Equation (5.4) and assuming the only energy head added by the pump is the elevation [neglect losses in Equation (4.2)], we get $H_P = H_R - H_S$

$$P_o = \gamma Q H_P = 9.81(\text{kN/m}^3) \cdot 0.98(\text{m}^3/\text{sec}) \cdot 25(\text{m}) = 240.35\text{kW}$$

From Equation (5.5), the efficiency of the pump is

$$e_P = \frac{P_o}{P_i} = \frac{240.35}{415.12} = 0.579 \cong 58\%$$

From Equation (5.6), the overall efficiency of the system is

$$e = e_P e_m = \left(\frac{P_o}{P_i}\right)\left(\frac{P_i}{P_m}\right) = (0.58) \cdot \left(\frac{415.12}{500}\right)$$

$$= 0.48 = 48\%$$

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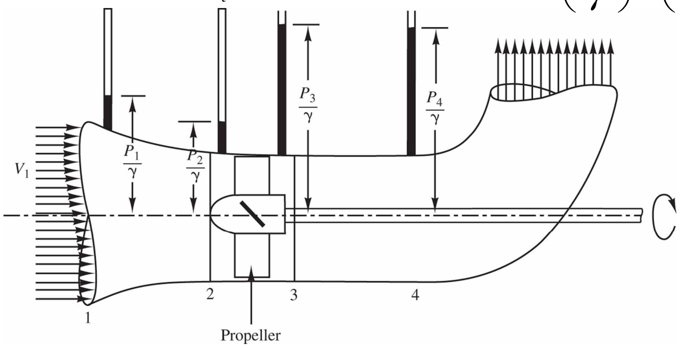
5.2 Propeller (Axial flow) pumps

Principle of impulse-momentum: The linear impulse of a force (or force system) acting on a body during a time interval be equal to the change in linear momentum in the body during that time.

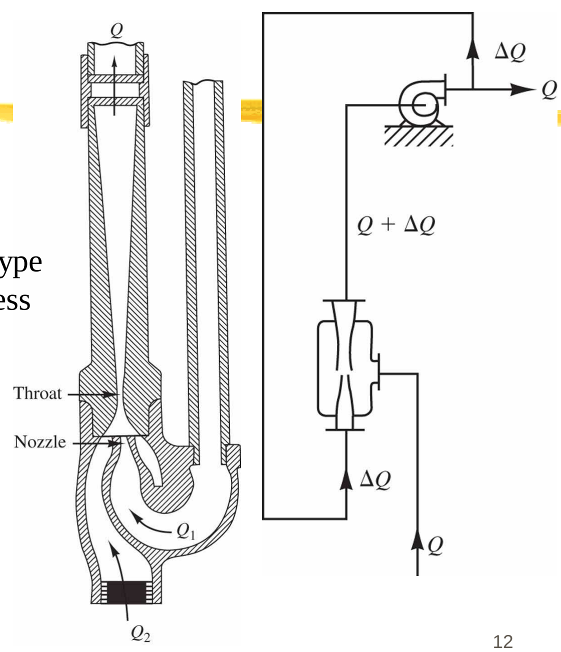
$$\text{Linear impulse}: I = \int_{t'}^{t''} F dt \qquad H_p = \left(\frac{p_3}{\gamma}\right) - \left(\frac{p_2}{\gamma}\right)$$



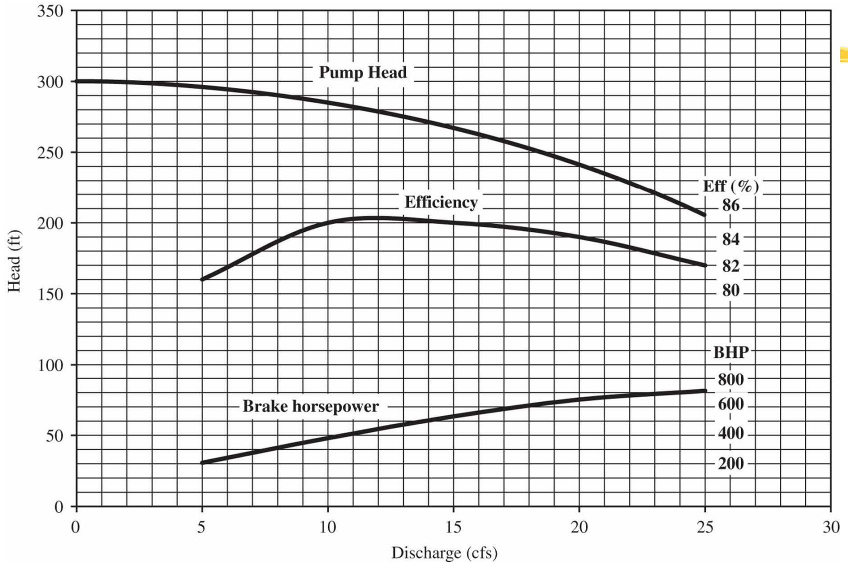
Generally used for low head (less than 12m) and high capacity (more than 20L/s) applications

5.3 Jet (Mixed flow) pumps

- Generally used in combination with a centrifugal pump.
- Used in dewatering construction sites.
- The efficiency of such type of pumps is very low (less than 25%)



5.4 Centrifugal pump characteristic curves



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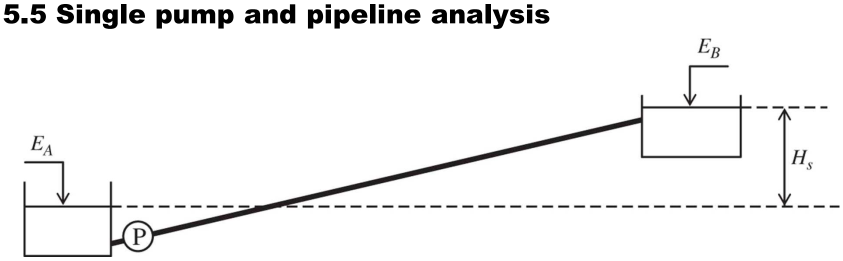
$$\frac{Q_2}{Q_1}=\frac{N_{r2}}{N_{r1}} \qquad \frac{H_{p2}}{H_{p1}}=\left(\frac{N_{r2}}{N_{r1}}\right)^2 \qquad \frac{BHP_2}{BHP_1}=\left(\frac{N_{r2}}{N_{r1}}\right)^3$$

5.5 Single pump and pipeline analysis



$$E_A+H_p=E_B+h_f \qquad H_p=(E_B-E_A)+h_f \qquad H_p=H_S+h_f$$

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Example 5.3

Consider the pump-pipeline system in Fig. 5.9. The reservoir water surface elevations are known: E_A = 100ft, E_B =220ft. The 2-ft-diameter pipe connecting the two reservoirs has a length of 12800ft and a Hazen-Williams coefficient of 100.

(a) Determine the discharge in the pipeline, the velocity of flow, and the energy grade line.

(b) Suppose the pump characteristics given in Part (a) are at a rotational speed of 2000 rpm. Determine the discharge in the pipeline and the pump head if the pump runs at 2200 rpm.

$$H_S = E_B - E_A = 220 - 100 = 120\,ft$$

$$h_f = KQ^{1.85} \qquad K = \frac{4.73L}{D^{4.87} C_{HW}^{1.85}} = \frac{4.73 \times 12800}{2^{4.87} \times 100^{1.85}}$$

$$= 0.413 s^{1.85}\,ft^{-4.55}$$

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Q(cfs)	H_p (ft)	H_f (ft)	H_S+h_f (ft)
0	300	0.0	120
5	295.5	8.1	128.1
10	282	29.2	149.2
15	259.5	61.9	181.9
20	225.5	105.4	225.4
25	187.5	159.3	279.3
30	138	223.2	343.2
35	79.5	296.8	416.8

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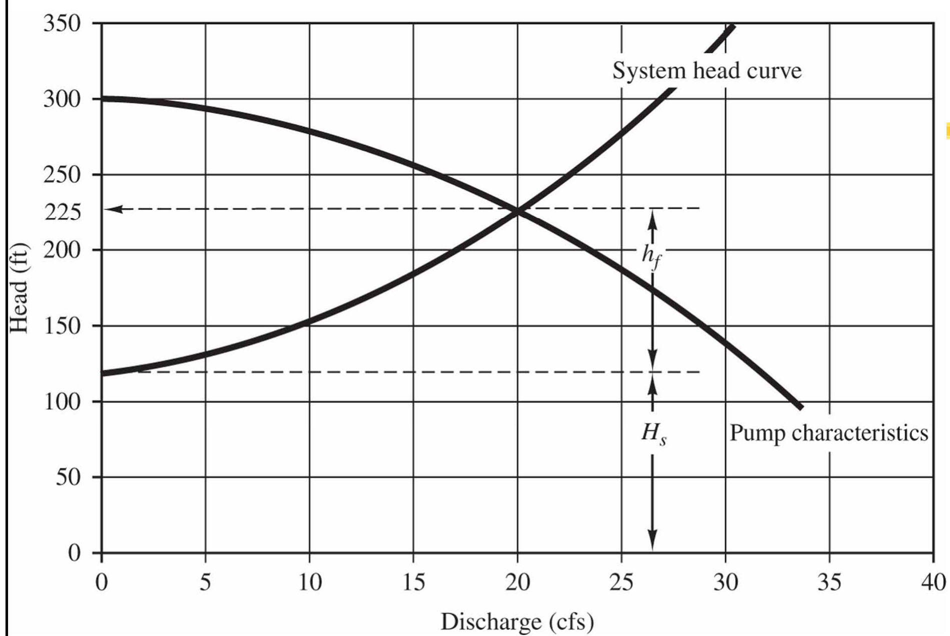

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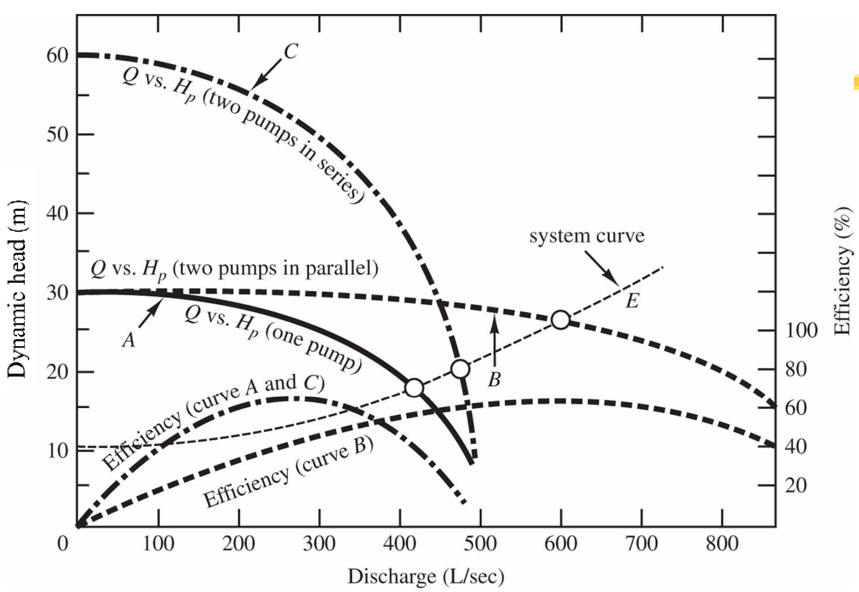
5.6 Pumps parallel or in series



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Example 5.4

Two reservoirs are connected by a 300-m-long asphalted cast-iron pipeline, 40 cm in diameter. The minor losses include the entrance, the exit, and a gate valve. The evaluation difference between the reservoirs is 10m and the water temperature is 10°C. Determine the discharge, head, and efficiency using (a) one pump, (b) two pumps in series, and (c) two pumps in parallel.
Use the pump described in Figure 5.13.

(a) $Q \approx 420L/s, \quad H_p \approx 18m, \quad e_p \approx 40\%$

(b) $Q \approx 470L/s, \quad H_p \approx 20m, \quad e_p \approx 15\%$

(c) $Q \approx 590L/s, \quad H_p \approx 26m, \quad e_p \approx 62\%$

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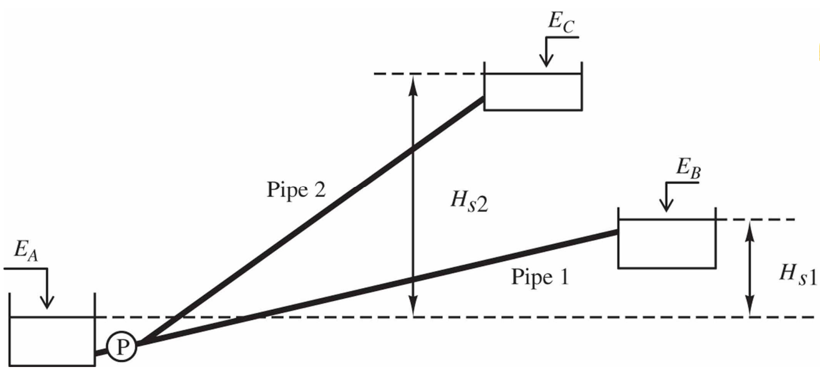
5.7 Pumps and branching pipes



$$H_{SH1} = H_{s1} + h_{f1} \qquad H_{SH2} = H_{s2} + h_{f2}$$

$$H_{SH1} = E_B - E_A \qquad H_{SH2} = E_C - E_A \qquad H_{SH1} = H_{SH2} = H_p$$

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Example 5.5

In Figure 5.15, E_A = 110 ft, E_B = 120 ft, and E_C = 140 ft. Both pipes have a Darcy-Weisbach friction factor of 0.02. Pipe 1 is 10,000 ft long and has a diameter of 2.5 ft. Pipe 2 is 15,000 ft long and has a diameter of 2.5 ft. The pump characteristics are given in the following table and plotted in Figure 5.16. Determine the discharge in each pipe.

Maximum discharge

Q(cfs)	0	10	20	30	40	50	60
H_P (ft)	80	78.5	74	66.5	56	42.5	26

Shut-off head

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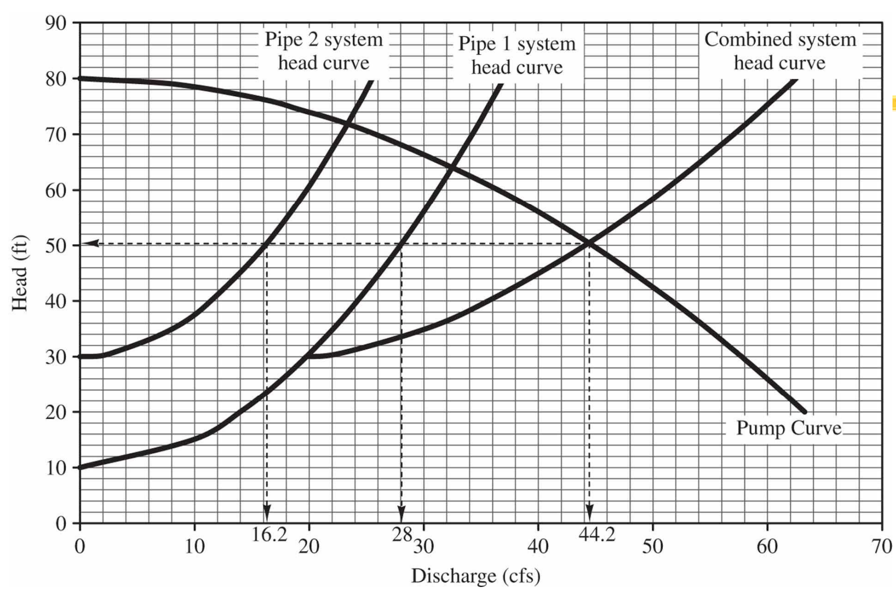

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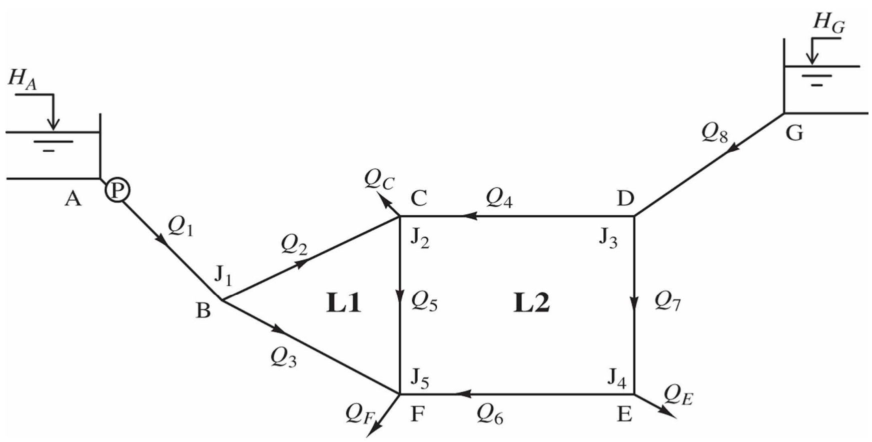
5.8 Pumps and pipe networks

$$H_P = a - bQ|Q| - cQ$$



Example 5.7

The pipe system shown in Figure 5.19 is identical to that of Examples 4.9 and 4.10. However, the demand has increased at junction F(Q_F = 0.30 m^3/sec, not 0.25 m^3 /sec), which necessitates adding a pump to the system just downstream of reservoir A. The pump characteristics can be expressed as

$$H_p = 30 - 50Q^2 - 5Q$$

where H_p is in m and Q is in m^3/sec. Determine the discharge in each pipe using the same initial values as in Example 4.10.

All the equations remain the same except for F8, we have to include head supplied by the pump as follows:

$$F_8 = H_A + \overbrace{a - bQ_1|Q_1| - cQ_1}^{H_p} - K_1Q_1|Q_1| - K_2Q_2|Q_2| + K_4Q_4|Q_4| + K_8Q_8|Q_8| - H_G$$

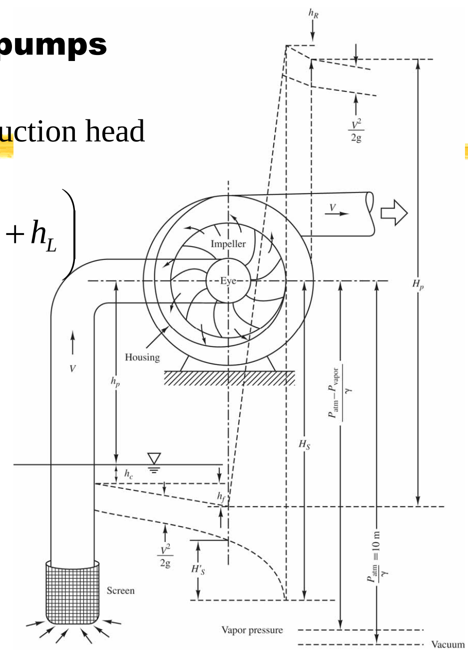
5.9 Cavitation in water pumps

$$\left(\frac{p_{atm}}{\gamma}-\frac{p_{vapor}}{\gamma}\right) > H_s$$: Total suction head

$$h_p \le \left(\frac{p_{atm}}{\gamma}-\frac{p_{vapor}}{\gamma}\right)-\left(H_s' + \frac{V^2}{2g} + h_L\right)$$

Cavitation parameter: $\sigma = \dfrac{H_s'}{H_p}$

$H_s' = \text{NPSH}$



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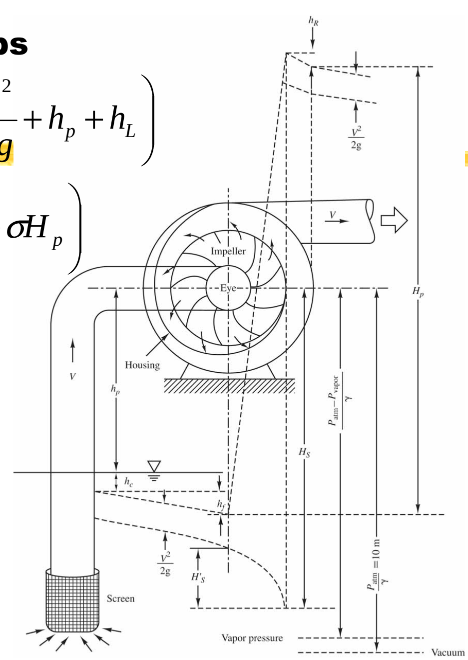
Cavitation in water pumps

$$H_s' = \sigma H_p = \left(\frac{p_{atm}}{\gamma}-\frac{p_{vapor}}{\gamma}\right)-\left(\frac{V_i^2}{2g} + h_p + h_L\right)$$

$$h_p = \left(\frac{p_{atm}}{\gamma}-\frac{p_{vapor}}{\gamma}\right)-\left(\frac{V_i^2}{2g} + h_L + \sigma H_p\right)$$



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Example 5.8

A pump is installed in a 15-cm, 300-m-long pipeline to pump 0.060 m^3 / sec of water at 20°C. The elevation difference between the supply reservoir and the receiving reservoir is 25 m. The pump has an 18-cm impeller intake diameter, a cavitation parameter of $\sigma = 0.12$, and experiences a total head loss of 1.3 m on the suction side. Determine the maximum allowable distance between the pump intake and the water surface elevation in the supply tank. Assume the pipeline has $C_{HW} = 120$.

$$h_f = KQ^m = 10.7L/(D^{4.87}C_{HW}^{1.85})Q^{1.85}$$

$$= 10.7 \times 300 \times 0.06^{1.85}/(0.15^{4.87}120^{1.85}) = 25.8m$$

$$V_i = \frac{0.06}{\pi(0.09)^2} = 2.36\text{m/s} \quad V_d = \frac{0.06}{\pi(0.075)^2} = 3.40\text{m/s}$$

$$h_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H_p = h_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$H_p = (h_2 - h_1) + h_L = 25 + \left(1.3 + 25.8 + \frac{3.4^2}{2 \times 9.81}\right) = 52.7\text{m}$$

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Example 5.8

$$P_{atm} = 1 \text{ bar} = 101400 \text{ Pa}$$

$$P_{vapor} = 0.023042 \text{ bar} = 2335 \text{ Pa}$$

$$\gamma = 9790 \text{ N/m}^3$$

Maximum allowable height of the pump above the supply reservoir is

$$h_p = \frac{P_{atm}}{\gamma} - \frac{P_{vapor}}{\gamma} - \frac{V_i^2}{2g} - \sum h_{Ls} - \sigma H_p$$

$$= \frac{101400}{9790} - \frac{2335}{9790} - \frac{2.36^2}{2 \times 9.81} - 1.3 - 0.12 \times 52.7$$

$$= 2.21 \text{ m}$$

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5.10 Specific speed and pump similarity

$$\text{Shape number}: S = \frac{\omega\sqrt{Q}}{(gH_p)^{3/4}}$$

$$\text{Specific speed}: N_s = \frac{\omega\sqrt{Q}}{(H_p)^{3/4}} \qquad N_s = \frac{\omega\sqrt{P_i}}{(H_p)^{5/4}}$$

TABLE 5.1 Conversion of Specific Speed

Units	Discharge Units	Head Units	Pump Speed	Equation	Symbol	Conversion	
United States	U.S. gal/min	ft	rev/min	(5.25)	N_{s1}	$N_{s1} = 45.6\ S$	$N_{s1} = 51.6\ N_{s3}$
English	Imp. gal/min	ft	rev/min	(5.25)	N_{s2}	$N_{s2} = 37.9\ S$	$N_{s2} = 43.0\ N_{s3}$
Metric	m^3/sec	m	rev/min	(5.25)	N_{s3}	$N_{s3} = 0.882\ S$	$N_{s3} = 0.019\ N_{s1}$
SI	m^3/sec	m	rad/sec	(5.24)	S	$S = 0.022\ N_{s1}$	$S = 1.134\ N_{s3}$

Note: $g = 9.81\ m/sec^2 = 32.2\ ft/sec^2$

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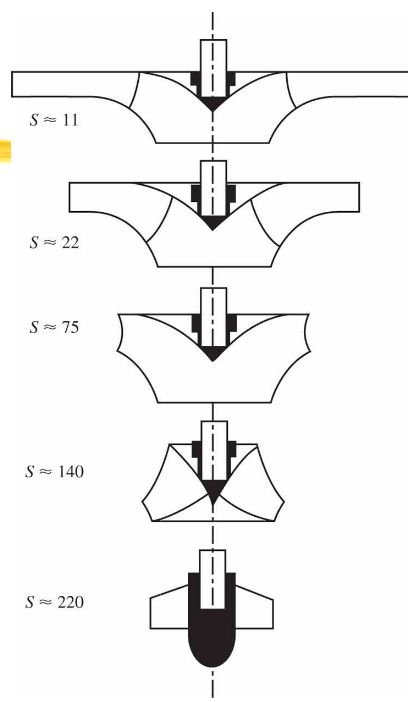
Impeller shapes and the values of shape numbers



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Example 5.9

A centrifugal water pump operating at its optimum efficiency delivers 2.5m^3/sec over a height to 20 m. The pump has a 36-cm diameter impeller and rotates at 300 rad/sec. Compute the specific speed of the pump in terms of (a) discharge and (b) power if the maximum efficiency of the pump is 80 percent.

$$N_s = \frac{300\sqrt{2.5}}{(20)^{3/4}} = 50$$

$$P_i = (\gamma Q H_p)/e_p = [(9{,}790)(2.5)(20)]/0.80 = 6.12 \times 10^5 \text{W} \,(612 \text{ kW})$$

$$N_s = \frac{300\sqrt{612}}{(20)^{5/4}} = 175$$

Example 5.10

The impeller of the pump in Example 5.9 has a diameter of 0.36 m. What diameter should the impeller of a geometrically similar pump be for it to deliver one half of the water discharge at the same head? What is the speed of the pump?

$$N_s = \frac{\omega\sqrt{\frac{1}{2} \times 2.5}}{(20)^{3/4}} = 50$$

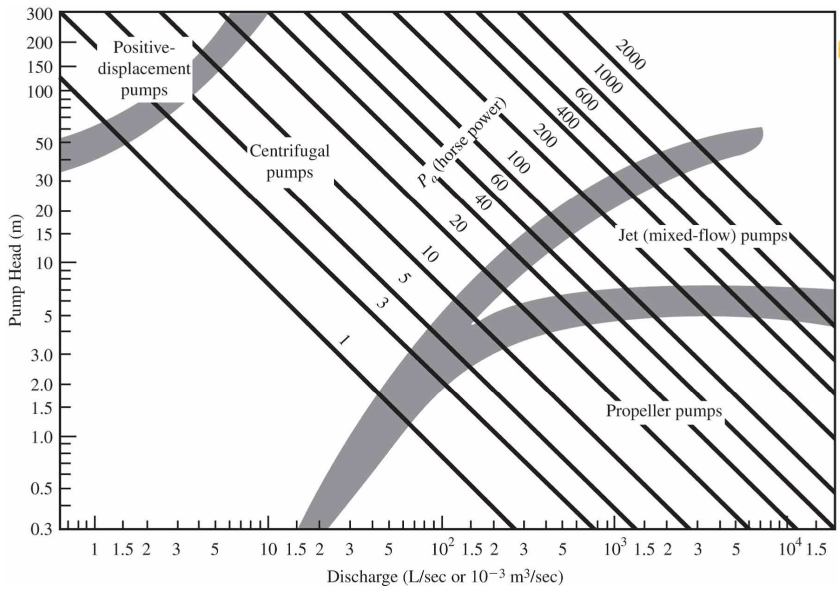
$$\text{The pump speed } \omega = \frac{50(20)^{3/4}}{(1.25)^{1/2}} = 423 \text{ rad/sec}$$

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \qquad \frac{2.5}{300 \cdot (0.36)^3} = \frac{1.25}{423 \cdot (D_2)^3}$$

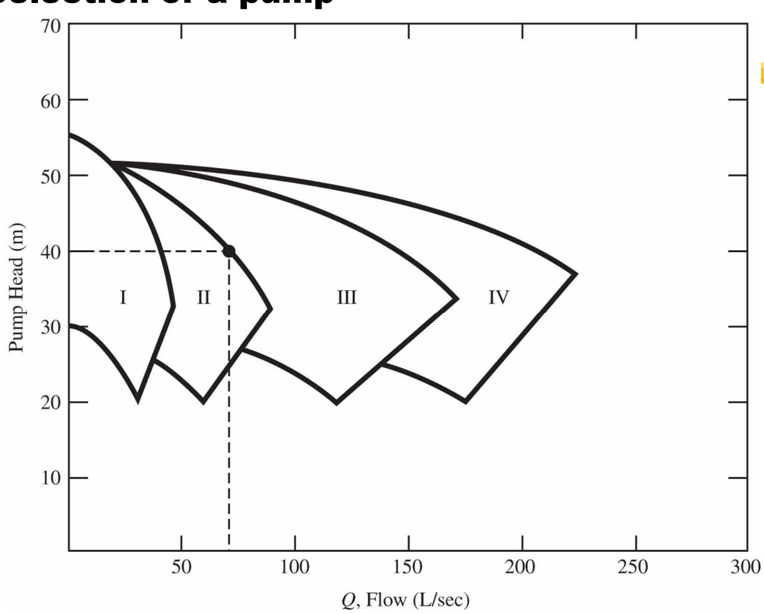
The diameter, $D_2 = 0.255$ m = 25.5 cm.

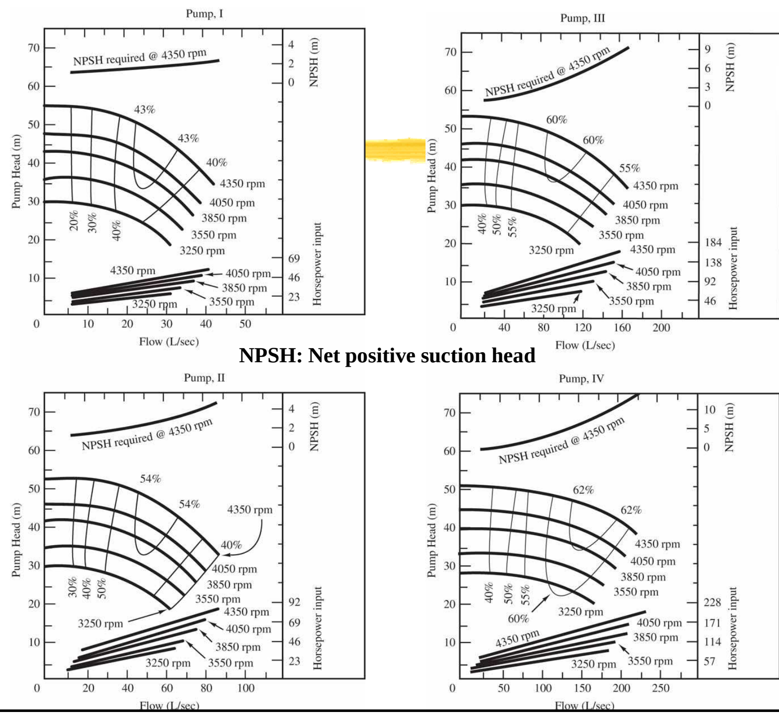
5.11 Selection of a pump

Head, discharge and power requirements of different types of pumps



Selection of a pump





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Example 5.11

A pump will be used to deliver a discharge of 70 L/sec of water between two reservoirs 1,000 m apart with an elevation difference of 20 m. Commercial steel pipes 20-cm in diameter are used for the project. Select the proper pump and determine the operating conditions for the pump based on the pump selection chart (Figure 5.23) and the pump characteristics curves (Figure 5.24), both provided by the manufacturer.

$$V = \frac{Q}{A} = \frac{0.070\ \text{m}^3/\text{sec}}{\frac{\pi}{4}(0.2\ \text{m})^2} = 2.23\ \text{m/sec}$$

$$N_R = \frac{VD}{\nu} = \frac{(2.23\ \text{m/sec})\,(0.2\ \text{m})}{1 \times 10^{-6}\ \text{m}^2/\text{sec}} = 4.5 \times 10^5$$

$$e/D = 0.045\ \text{mm}/200\ \text{mm} = 2.3 \times 10^{-4} = 0.00023$$

$$f = 0.016$$

$$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g} = 0.016\left(\frac{1000}{0.2}\right)\left(\frac{(2.23)^2}{2\,(9.81)}\right) = 20.3\ \text{m}$$

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$$H_{SH} = (\Delta \text{ elevation}) + (\text{friction loss})$$
$$= 20.0 \text{ m} + 20.3 \text{ m} = 40.3 \text{ m}$$

Q (L/s)	V (m/s)	N_R	f	h_f	H_{SH}
50	1.59	3.2×10^5	0.0165	10.7	30.7
60	1.91	3.8×10^5	0.0160	14.9	34.9
80	2.55	5.1×10^5	0.0155	25.6	45.6

First Selection: Pump II at 4350 rpm
Q = 70 L/s, H_p = 40.3 m, Hence, P_i = 71 hp and efficiency =52%

Second Selection: Pump III at 3850 rpm
Q = 68 L/s, H_p = 39 m, Hence, P_i = 61 hp and efficiency =58%

Third Selection: Pump III at 4050 rpm
Q = 73 L/s, H_p = 42 m, Hence, P_i = 70 hp and efficiency =59%