Chapter 4
Pipelines and Pipe Networks

Part 2

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Expected student outcomes

- Ability to calculate water hammer pressure in pipelines due to rapid and slow valve closures. **[a,c]**
- Ability to calculate and measure water level in surge tanks due to water hammer. **[a,c,e,k]**

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4.5 Water hammer phenomenon in pipelines

Occurrence of very high pressure due to valve closure, pump turn-off etc.

$$F = m\frac{dV}{dt} = \frac{m(V_0 - 0)}{0} = \infty$$

Pressure wave celerity and elasticity

$$C = \sqrt{\frac{E_c}{\rho}} \qquad \Delta P = E_c \frac{\Delta Vol}{Vol}$$

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Composite Modulus of Elasticity

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{Dk}{E_p e}$$

$k = (5/4 - \varepsilon)$: For pipes free to move longitudinally

$k = (1 - \varepsilon^2)$: For pipes anchored at both ends to prevent against longitudinal motion

$k = (1 - 0.5\varepsilon)$: For pipes with expansion joints

$\varepsilon = 0.25$: Poisson ratio for common pipe materials.

Neglecting the longitudinal stress in a pipe, (k=1), we can write

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{D}{E_p e}$$

Modulus of elasticity for common pipe materials are shown in Table 4.1.

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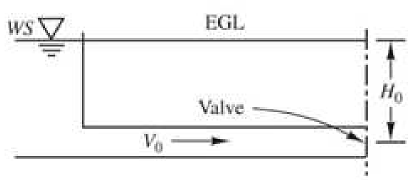
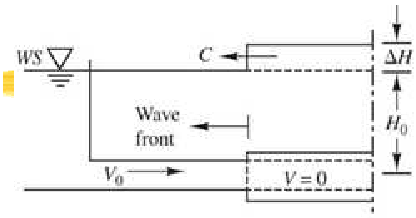
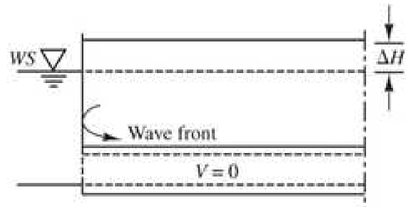
Propagation of water hammer pressure waves (friction in pipe neglected).



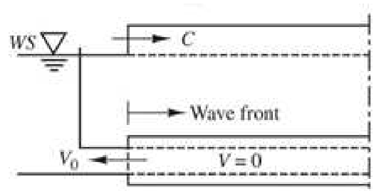
Steady-state condition prior to valve movement.



Transient conditions at $t < L/C$.



Transient conditions at $t = L/C$



Transient conditions at $L/C < t < 2L/C$

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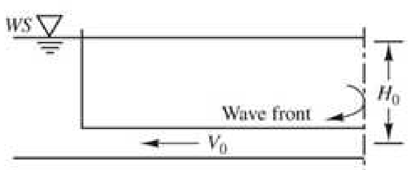
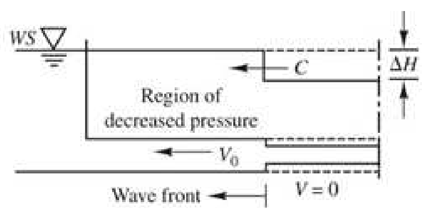
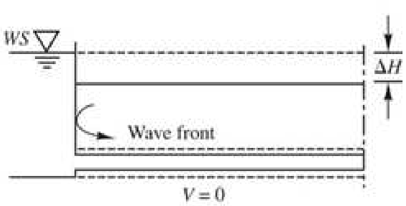


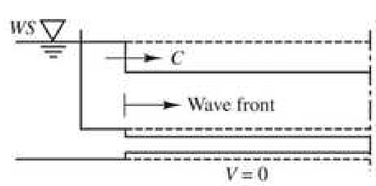
Transient conditions at $t = 2L/C$



Transient conditions at $2L/C < t < 3L/C$



Transient conditions at $t = 3L/C$



Transient conditions at $3L/C < t < 4L/C$

After $t = 4L/C$, the cycle repeats and continues indefinitely if the friction in the pipe is zero.

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Rapid valve closure

For a rapidly closing valve; $\Delta Vol = V_0 A \frac{L}{C}$

$$\Delta P = E_c \frac{\Delta Vol}{Vol} = E_c \frac{V_0 AL / C}{AL} = \frac{E_c V_0}{C} \qquad (4.23)$$

When the velocity of water is brought to zero in time, Δt, the total mass of water involved is: $m = \rho AC\Delta t$

Now , applying Newton's second law:

$$\Delta P.A = m\frac{\Delta V}{\Delta t} = \rho AC\Delta t \frac{V_0 - 0}{\Delta t} = \rho ACV_0 \qquad \Rightarrow C = \frac{\Delta P}{\rho V_0}$$

Putting the value of ΔP from Eq. (4.23) $C = \frac{E_c V_0}{C\rho V_0} \Rightarrow C = \sqrt{\frac{E_c}{\rho}}$

Now putting the value of C in Eq. (4.23)

$$\Delta P = \frac{E_c V_0}{\sqrt{E_c / \rho}} = V_0 \sqrt{\rho E_c} \qquad \Rightarrow \Delta H = \frac{\Delta P}{\rho g} = \frac{V_0}{g}\sqrt{\frac{E_c}{\rho}} = \frac{V_0}{g} C$$

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Slow valve closure

For slow valve closure, Allievi formula is used as follows:

$$\Delta P = P_0 \left(\frac{N}{2} + \sqrt{\frac{N^2}{4} + N} \right)$$

Where P_0 is static-state pressure in the pipe and $N = \frac{\rho L V_0}{P_0 t}$

In both situations (rapid and slow valve closure), the total pressure is

$$P = P_0 + \Delta P$$

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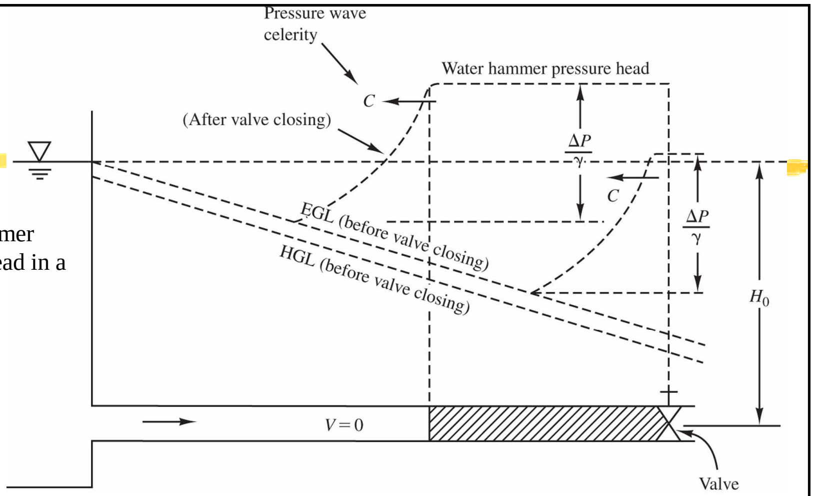
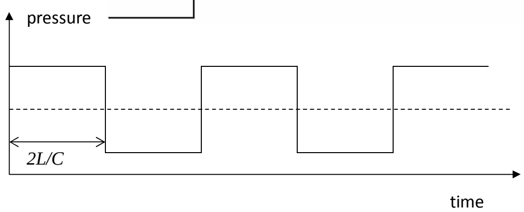


Water hammer pressure head in a pipeline



Pressure variation at the valve location with time.

Example 4.12

A ductile iron pipe 20 cm in diameter and with 15-mm thick walls is carrying water when the outlet is suddenly closed. If the design discharge is 40 L/sec, calculate the pressure head rise caused by water hammer if

(a) the pipe wall is rigid;

(b) The pipe is free to move longitudinally (negligible stresses), and

(c) the pipeline has expansion joints throughout its length.

$$A = \frac{\pi}{4} \cdot 0.2^2 = 0.0314\,\text{m}^2 \qquad V = \frac{Q}{A} = \frac{0.04}{0.0314} = 1.274 \text{ m/sec}$$

(a) For rigid pipe wall, $Dk/E_p e = 0$, Equation (4.20a) gives the following relation

$$\frac{1}{E_c} = \frac{1}{E_b}, \qquad \text{or} \qquad E_c = E_b = 2.2 \cdot 10^9 \text{ N/m}^2$$

$$\rho = 998 \text{ kg/m}^3$$

From Equation (4.19), we can calculate the speed of pressure wave,

$$C = \sqrt{\frac{E_c}{\rho}} = \sqrt{\frac{2.2 \cdot 10^9}{998}} = 1485 \text{ m/sec}$$

From Equation (4.24), we can calculate the rise of water hammer pressure

$$H = \frac{VC}{g} = \frac{1.274 \cdot 1485}{9.81} = 193 \text{ m } (H_2O)$$

$$\therefore P = \gamma H = 1.89 \cdot 10^6 \text{ N/m}^2$$

(b) For pipes with no longitudinal stress, $k = 1$, we may use Equation (4.20b),

$$E_c = \frac{1}{\left(\frac{1}{E_b} + \frac{D}{E_p e}\right)} = \frac{1}{\left(\frac{1}{2.2 \cdot 10^9} + \frac{0.2}{(1.6 \cdot 10^{11})(0.015)}\right)} = 1.86 \cdot 10^9$$

and

$$C = \sqrt{\frac{E_c}{\rho}} = 1365 \text{ m/sec}$$

Hence, the rise of water hammer pressure can be calculated

$$H = \frac{VC}{g} = \frac{1.274 \cdot 1365}{9.81} = 177 \text{ m } (H_2O) \qquad P = \gamma H = 1.74 \cdot 10^6 \text{ N/m}^2$$

(c) For pipes with expansion joints, $k = (1 - 0.5 \cdot 0.25) = 0.875$. From Equation (4.20a),

$$E_c = \frac{1}{\frac{1}{E_b} + \frac{0.875D}{E_p e}} = \frac{1}{\left(\frac{1}{2.2 \cdot 10^9} + \frac{(0.875)(0.2)}{(1.6 \cdot 10^{11})(0.015)}\right)} = 1.90 \cdot 10^9$$

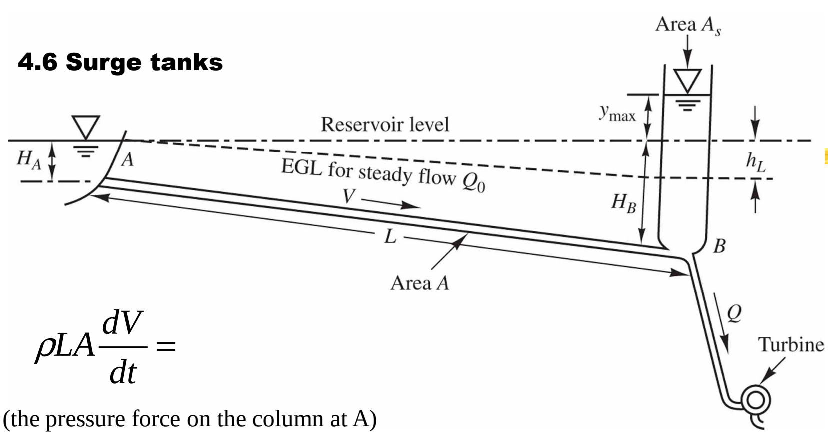
and

$$C = \sqrt{\frac{E_c}{\rho}} = 1378 \text{ m/sec}$$

Equation (4.24) is used to calculate the rise of water hammer pressure

$$H = \frac{VC}{g} = \frac{1.274 \cdot 1378}{9.81} = 179 \text{ m } (H_2O) \qquad P = \gamma H = 1.76 \cdot 10^6 \text{ N/m}^2$$

4.6 Surge tanks



$$\rho LA \frac{dV}{dt} =$$

(the pressure force on the column at A)
+ (the weight component of the column in the direction of the pipeline)
- (the force acting on the column at B)
+ (friction losses)

$$\begin{aligned} \rho LA \frac{dV}{dt} = \rho g A[&(H_A \pm \text{entrance loss}) \\ &+ (H_B - H_A) \\ &- (H_B + y \pm \text{throttle loss}) \\ &\pm \text{(pipeline losses)}] \end{aligned}$$

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$$h_L = K_f V \cdot |V| \qquad H_T = K_T U \cdot |U| \qquad K_f = fL/2gD$$

$$\boxed{\frac{L}{g}\frac{dV}{dt} + y + K_f V \cdot |V| + K_T U \cdot |U| = 0} \qquad \boxed{U = \frac{dy}{dt}}$$

Continuity at B: $\boxed{VA = UA_s + Q}$

The above three equations produce second order differential equation that can be solved for special cases. The solution for a simple constant area surge tank is given as follows, where β is the damping factor.

$$\frac{y_{max} + h_L}{\beta} = \ln\left(\frac{\beta}{\beta - y_{max}}\right) \qquad \beta = \frac{LA}{2gK_fA_s}$$

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Example 4.13

A simple surge tank 8 m in diameter is located at the downstream end of a 1500 m long pipe, 2.2 m in diameter. The head loss between the upstream reservoir and the surge tank is 15.1 m when the flow rate is 20 m^3/sec. Determine the maximum elevation of the water in the surge tank if a valve downstream suddenly closes.

For a smooth entrance the head loss at entrance may be neglected, we may write

$$h_L \cong h_f = K_f V^2 \qquad K_f = \frac{h_L}{V^2} = \frac{15.13}{(5.26)^2} = 0.5466$$

$$\beta = \frac{LA}{2gK_fA_s} = \frac{(1500)(3.80)}{2(9.81)(0.5466)(50.27)} = 10.58$$

$$\frac{y_{max} + 15.13}{10.58} = \ln\left(\frac{10.58}{10.58 - y_{max}}\right)$$

y_{max}	LHS	RHS
9.5	2.32	2.27
9.6	2.33	2.36
9.57	2.33	2.33

The maximum elevation for water is 9.57 m over the reservoir level.

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