Chapter 4
Pipelines and Pipe Networks

Part 1

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Expected student outcomes

- Ability to analyze and design pipe networks manually and with the help of computer. **[a,c,e,k]**

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Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For an incompressible flow, mass density remains constant, so

$$V_1 A_1 = V_2 A_2 \qquad Q_1 = Q_2$$

General form of energy equation for pipe flow

$$h_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H_p = h_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + H_t + h_L$$

$H_p =$ head provided by the pump

$H_t =$ head supplied by the turbine

$h_L =$ head loss

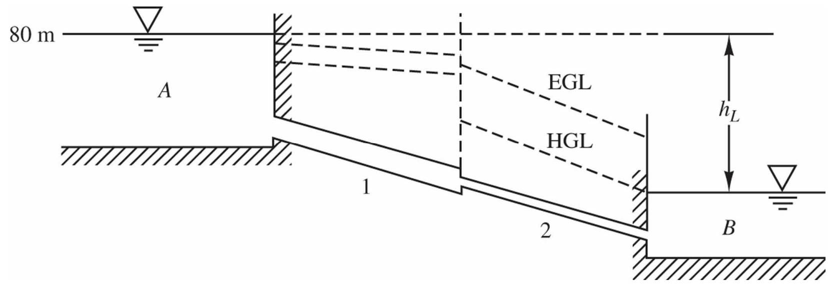
4.1 Pipelines connecting two reservoirs

Three types of problems

- **Given**: Q and pipe properties **Required**: Total h_L
- **Given**: Allowable h_L and pipe properties **Required**: Q
- **Given**: h_L and Q **Required**: Pipe diameter

Example 4.1

Two new cast iron pipes in series connect two reservoirs. Both pipes are 300 m long and have diameters of 0.6m and 0.4m, respectively. The elevation of the water surface in reservoir A is 80m. The discharge of 10ºC water from reservoir A to B is 0.5 m³/s. Find the elevation of the surface of reservoir B. Assume a sudden contraction at the junction and a square edge entrance.



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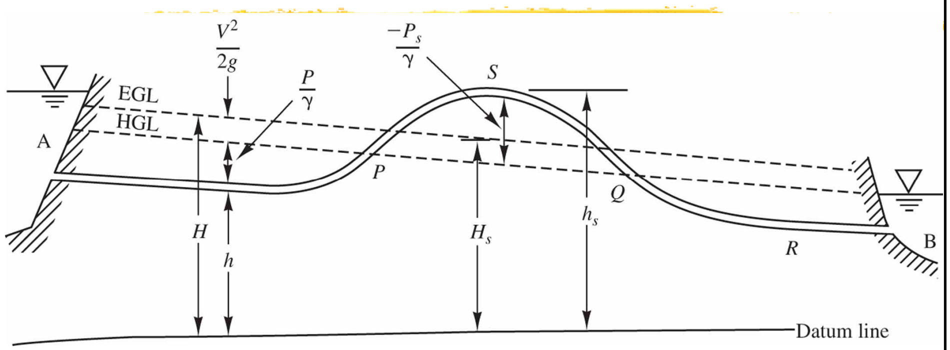
4.2 Negative pressure scenarios (pipelines and pumps)



$$H = h + \frac{P}{\gamma} + \frac{V^2}{2g} \qquad \Rightarrow \frac{P_S}{\gamma} = H_S - h_S - \frac{V_S^2}{2g}$$

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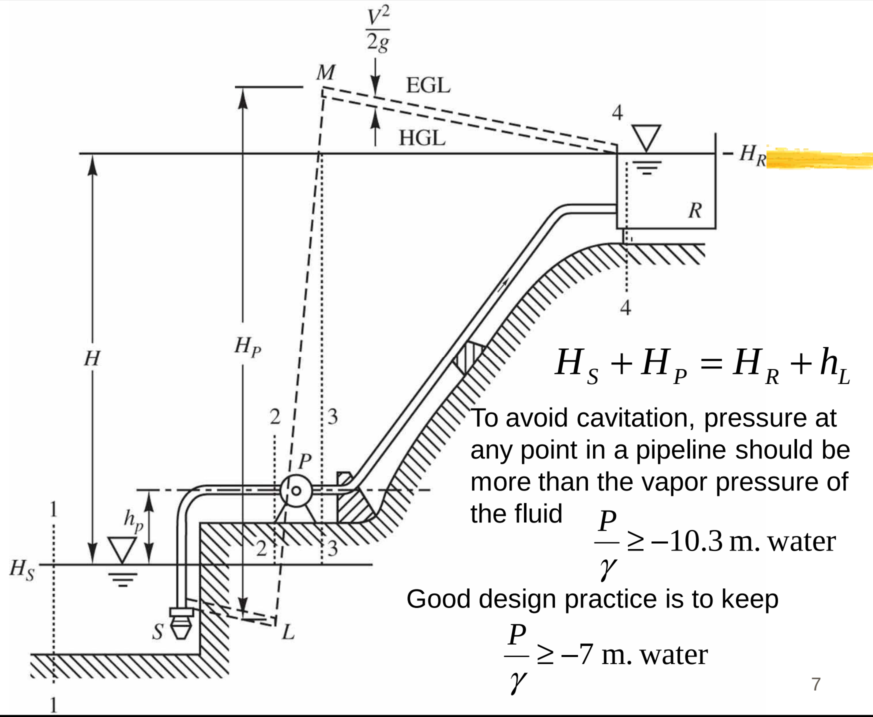

$$H_S + H_P = H_R + h_L$$

To avoid cavitation, pressure at any point in a pipeline should be more than the vapor pressure of the fluid $\frac{P}{\gamma} \geq -10.3 \text{ m. water}$

Good design practice is to keep

$$\frac{P}{\gamma} \geq -7 \text{ m. water}$$

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Example 4.5

A pump is necessary to lift water from the clear well at a water treatment plant to a storage tower 50 ft high and some distance away. A flow rate of 15cfs is required (68ºF). The 15-in pipeline (ε/D=0.00008) between the two reservoirs is 1500 ft long and contains minor losses that amount to 15 times the velocity head. Determine the pressure head required from the pump. Also determine the pressure head on the suction side of the pump if it is 10ft above the clear well and 100 ft down the pipeline.

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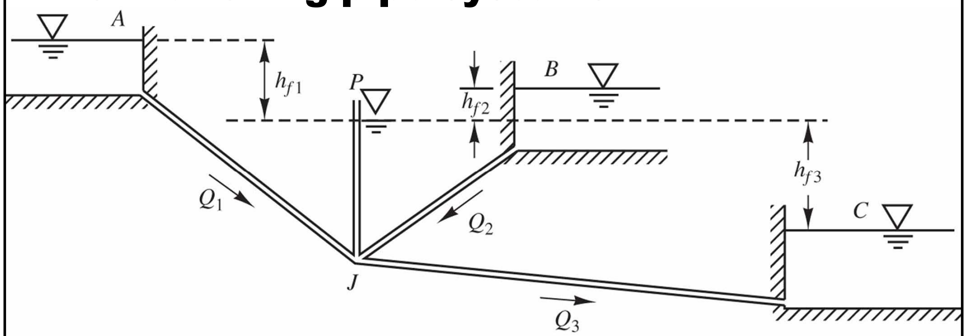
4.3 Branching pipe systems



$$Q_3 = Q_1 + Q_2 \Rightarrow \sum Q = 0$$

Solution procedure:

1. Assume piezometric level P at the junction
2. Calculate head losses in three pipes
3. Calculate the sum of all pipe discharges, ΣQ
4. Plot P versus ΣQ and read h_P for which ΣQ =0

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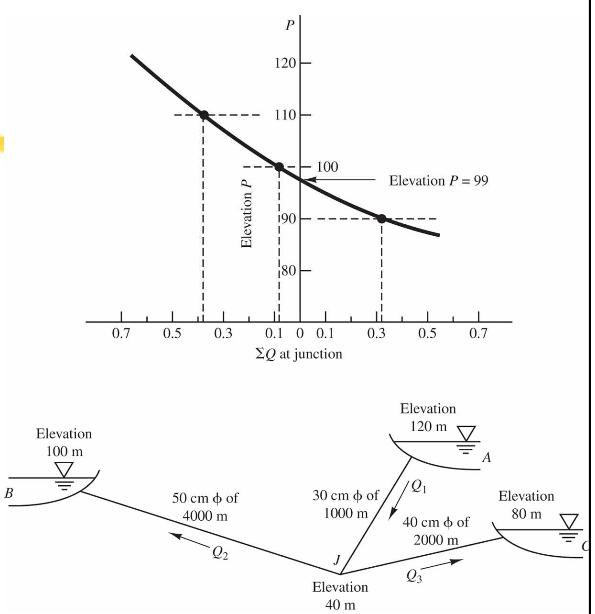
Example 4.6

In Figure 4.5 the three reservoirs A, B and C are connected by pipes to a common joint J. Pipe AJ is 1000m long and 30cm in diameter, pipe BJ is 4000m long and 50cm in diameter, and pipe CJ is 2000m long and 40cm in diameter. The pipes are made of concrete for which ε=0.6mm may be assumed. Determine the discharge in each pipe if the water temperature is 20ºC. (Assume that minor losses are negligible).



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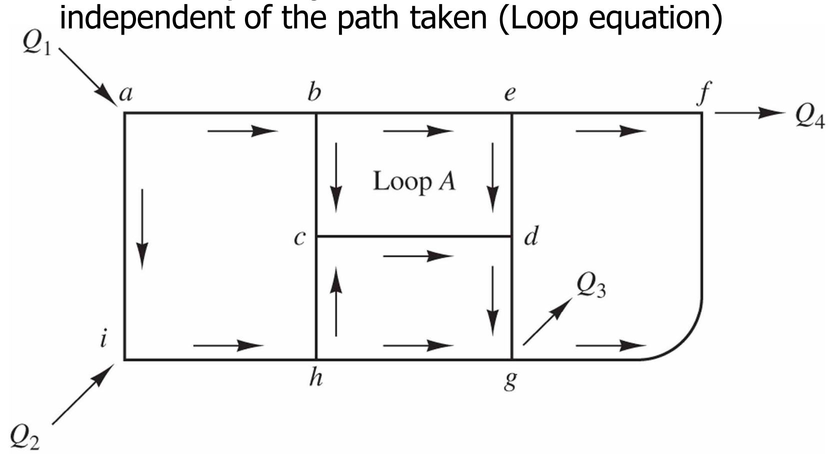
4.4 Pipe networks

1. At any junction, $\Sigma Q=0$ (Junction equation)
2. Between any two junction the total head loss is independent of the path taken (Loop equation)



4.4.1 Hardy-Cross Method

$$Q = Q_a + \Delta$$

Actual discharge: Q   Assumed discharge: Q_a

Correction: Δ

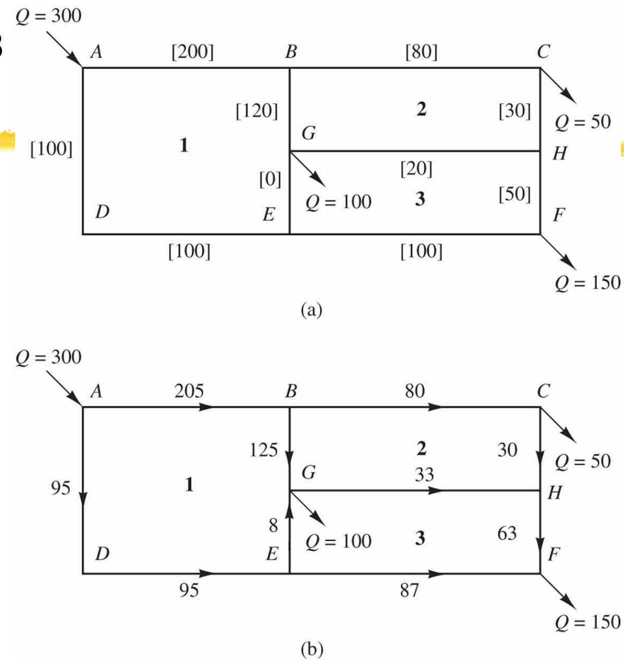
$$\sum K(Q_a + \Delta)^x = 0$$

$$\sum KQ_a^x + \sum xK\Delta Q_a^{x-1} + \sum \frac{x-1}{2} xK\Delta^2 Q_a^{x-2} + \cdots = 0$$

For small values of correction

$$\sum KQ_a^x + \Delta \sum KxQ_a^{x-1} = 0 \qquad \Delta = -\frac{\sum KQ_a^x}{\sum \left| xKQ_a^{x-1} \right|}$$

Example 4.8



4.4.2 The Newton method

At every junction and loop: $F_i(Q_1, Q_2, \ldots. Q_N) = 0$

$$
\begin{bmatrix}
\dfrac{\partial F_1}{\partial Q_1} & \dfrac{\partial F_1}{\partial Q_2} & .. & .. & \dfrac{\partial F_1}{\partial Q_N} \\
\dfrac{\partial F_2}{\partial Q_1} & \dfrac{\partial F_2}{\partial Q_2} & .. & .. & \dfrac{\partial F_2}{\partial Q_N} \\
.. & .. & .. & .. & .. \\
.. & .. & .. & .. & .. \\
\dfrac{\partial F_N}{\partial Q_1} & \dfrac{\partial F_N}{\partial Q_2} & .. & .. & \dfrac{\partial F_N}{\partial Q_N}
\end{bmatrix}
\begin{bmatrix}
\Delta Q_1 \\ \Delta Q_2 \\ .. \\ .. \\ \Delta Q_N
\end{bmatrix}
=
\begin{bmatrix}
-F_1 \\ -F_2 \\ .. \\ .. \\ -F_N
\end{bmatrix}
$$

Correction can be applied to get the discharge values for next iteration:

$$(Q_i)_{k+1} = (Q_i)_k + (\Delta Q_i)_k$$

In order to preserve the direction of flow, head loss is expressed as follows:

$$h_f = KQ|Q|^{m-1}$$

K and m values depend on the head loss formula used as shown in Table 3.4

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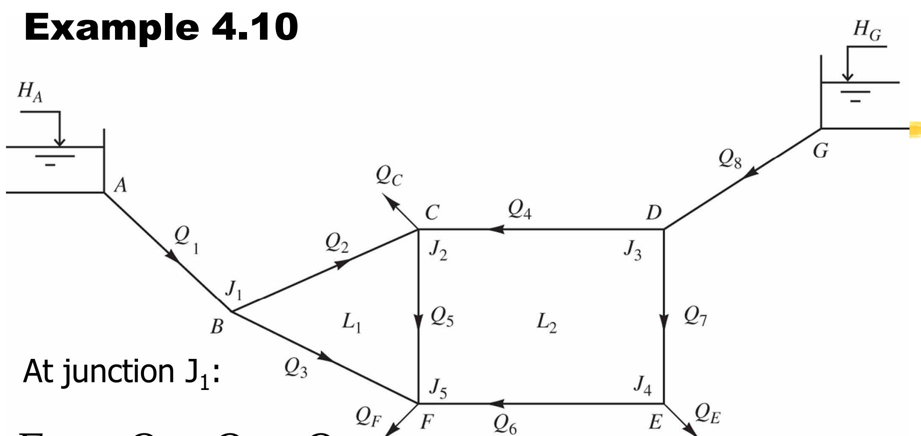
Example 4.10



At junction J_1:

$$F_1 = -Q_1 + Q_2 + Q_3$$

For loop L_1:

$$F_6 = -K_2 Q_2|Q_2| - K_3 Q_3|Q_3| + K_5 Q_5|Q_5|$$

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$$F_1 = -Q_1 + Q_2 + Q_3 \qquad F_2 = -Q_2 - Q_4 + Q_5 + Q_C$$

$$F_3 = Q_4 + Q_7 - Q_8 \qquad F_4 = Q_6 - Q_7 + Q_E$$

$$F_5 = -Q_3 - Q_5 - Q_6 + Q_F$$

$$F_6 = -K_2 Q_2 |Q_2| - K_3 Q_3 |Q_3| + K_5 Q_5 |Q_5|$$

$$F_7 = -K_4 Q_4 |Q_4| - K_5 Q_5 |Q_5| + K_6 Q_6 |Q_6| + K_7 Q_7 |Q_7|$$

$$F_8 = H_A - K_1 Q_1 |Q_1| - K_2 Q_2 |Q_2| + K_4 Q_4 |Q_4|$$
$$+ K_8 Q_8 |Q_8| - H_G$$

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The matrices can be solved using MATLAB or Excel. The final solution of this problem by Excel is shown below:

Iteration 7

No.	Q	F	1	2	3	4	5	6	7	8	Del. Q
1	0.2	0	-1	1	1	0	0	0	0	0	0
2	0.093	0	0	-1	0	-1	1	0	0	0	0
3	0.107	0	0	0	0	1	0	0	1	-1	0
4	0.096	0	0	0	0	0	0	1	-1	0	0
5	0.089	0	0	0	-1	0	-1	-1	0	0	0
6	0.053	0	0	352.538	-406.5	0	121	0	0	0	0
7	0.153	0	0	0	0	-312.7	-121	172.8	207.2	0	0
8	0.25	0	-77.57	-352.54	0	312.7	0	0	0	209.3	0

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