CHAPTER 3
WATER FLOW IN PIPES

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Expected student outcomes

- Ability to calculate major and minor losses in pipelines **[a]**
- Ability to use laboratory equipment to measure major and minor losses in pipelines **[k]**

3.1 Description of pipe flow

Average velocity of flow: $V = \dfrac{Q}{A}$

3.2 The Reynolds numbers

Reynolds number:

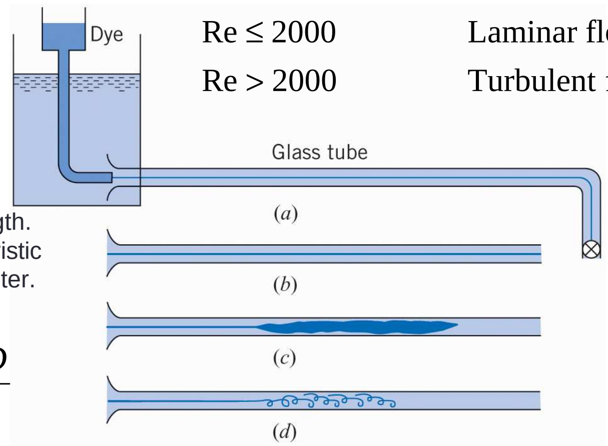
$$N_R = \frac{LV}{\nu} = \frac{\rho VL}{\mu}$$

L is a characteristic length. For pipes, the characteristic length is the pipe diameter.

$$N_R = \frac{DV}{\nu} = \frac{\rho VD}{\mu}$$

$\mathrm{Re} \leq 2000$ Laminar flow

$\mathrm{Re} > 2000$ Turbulent flow

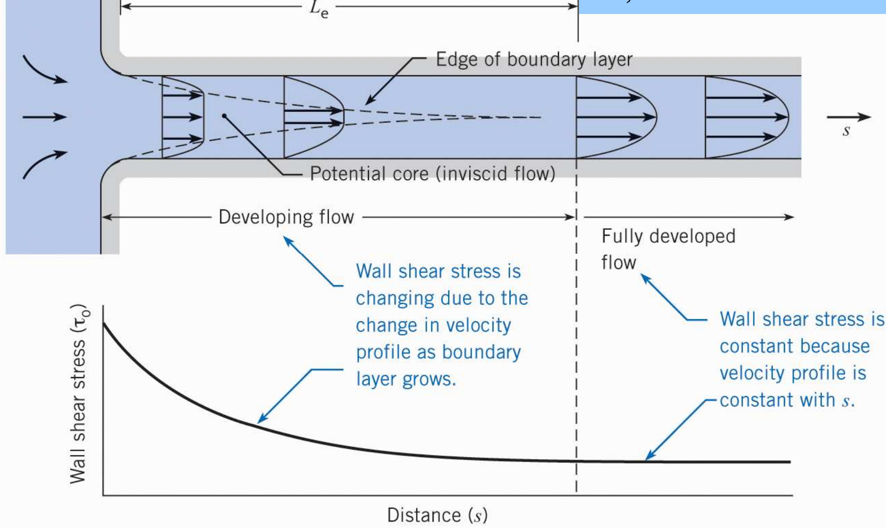


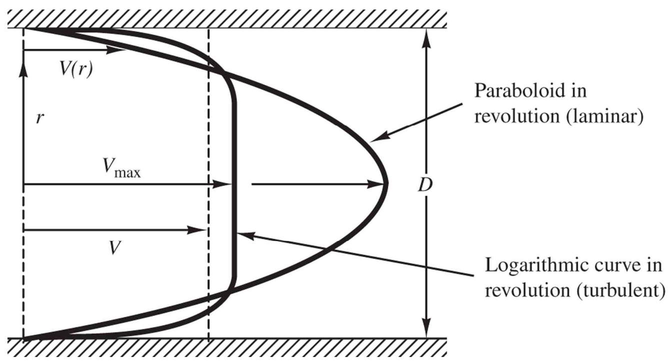
Developing and fully developed flow

Hydrodynamic entry region

$$L_{e,\text{laminar}} \cong 0.05\,\mathrm{Re}\,D$$

$$L_{e,\text{turbulent}} \cong 10D$$





For the fully developed laminar flows

Mean velocity = one-half of the maximum center line velocity

$V = 0.5\, V_{max}$ (Appendix 1 & 2)

Empirical formula to estimate kinematic viscosity of water:

$$10^6 \nu = 1.0049 - 0.02476(T - 20) + 0.00044(T - 20)^2$$

Continuity Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For an incompressible flow, mass density remains constant, so

$$V_1 A_1 = V_2 A_2 \qquad Q_1 = Q_2$$

3.3 Forces in pipe flow (for steady flow)

Momentum equation

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho\, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

$$\sum \vec{F} = \rho Q(\vec{V}_2 - \vec{V}_1)$$

Example 3.2

A horizontal nozzle, d_B=20mm, discharges 0.01m^3/s of water into the air. The supply pipe's diameter d_A=40mm. The nozzle is held in place by a hinge mechanism as shown in the figure. Determine the reaction force at the hinge if the gage-pressure at A is 500,000 N/m^2.

$$\vec{V}_A = \frac{Q}{A_A}\vec{i} = 7.96\vec{i}$$

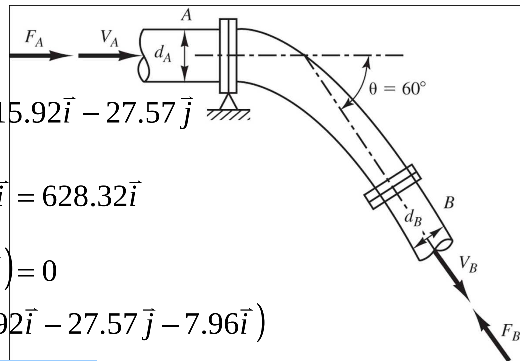
$$\vec{V}_B = \frac{Q}{A_B}\left(\cos 60^\circ \vec{i} - \sin 60^\circ \vec{j}\right) = 15.92\vec{i} - 27.57\vec{j}$$

$$\vec{F}_A = P_A A_A \vec{i} = 500000 \frac{\pi (0.04)^2}{4}\vec{i} = 628.32\vec{i}$$

$$\vec{F}_B = P_B A_B \left(-\cos 60^\circ \vec{i} + \sin 60^\circ \vec{j}\right) = 0$$

$$628.32\vec{i} + \vec{F} = (1000)(0.01)\left(15.92\vec{i} - 27.57\vec{j} - 7.96\vec{i}\right)$$

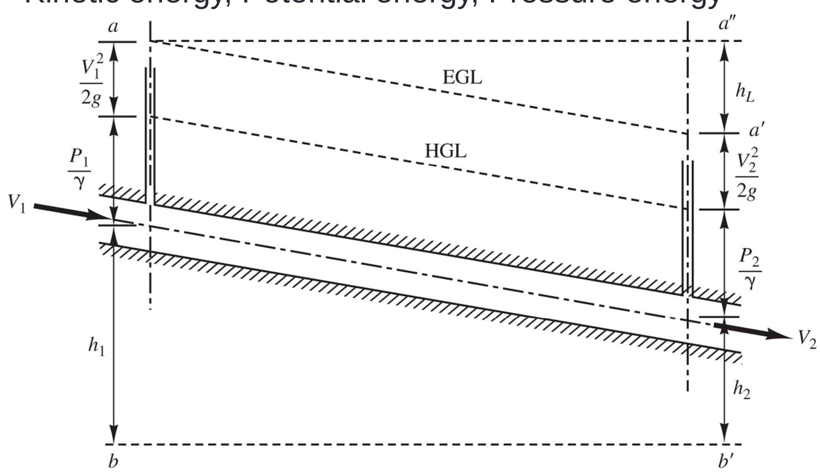
$$\vec{F} = -548.7\vec{i} - 275.7\vec{j}$$



3.4 Energy head in pipe flow

Basic forms of energy

Kinetic energy, Potential energy, Pressure energy



General form of energy equation for pipe flow

$$h_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H_p = h_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + H_t + h_L$$

$H_p =$ head provided by the pump

$H_t =$ head supplied by the turbine

$h_L =$ head loss

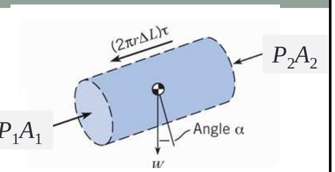
3.5 Loss of head from pipe friction

Darcy-Weisbach formula (Appendix 3)
(valid for both laminar and turbulent flows)

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

1. **Independent of the pressure under which the water flows,**
2. **Linearly proportional to the pipe length (*L*)**
3. **Inversely proportional to the pipe diameter (*D*)**
4. **Proportional to square of the mean velocity (*V*), and**
5. **Related to the surface roughness of the pipe, if the flow is turbulent.**

3.5 Loss of head from pipe friction

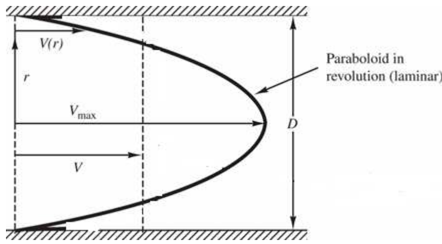


Laminar flow

$$(p_1 - p_2)(\pi r^2) - \tau_0(2\pi r L) - \gamma[(\pi r^2)L]\sin\alpha = 0$$

Horizontal pipe $$-2\pi r L\left(\mu \frac{dv}{dr}\right) = (P_1 - P_2)\pi r^2$$

Velocity profile $$u = \frac{P_1 - P_2}{4\mu L}\left(r_0^2 - r^2\right)$$



Total discharge through the pipe

$$Q = \int dQ = \int V dA = \int_{r=0}^{r=r_0} \frac{P_1 - P_2}{4\mu L}\left(r_0^2 - r^2\right)(2\pi r)dr$$

Hagen-Poiseuille law $$Q = \frac{\pi D^4 (P_1 - P_2)}{128\mu L}$$

Mean velocity = $$V = \frac{Q}{A} = \frac{\pi D^4 (P_1 - P_2)}{128\mu L}\left(\frac{1}{(\pi/4)D^2}\right)$$

$$V = \frac{(P_1 - P_2)D^2}{32\mu L}$$

Friction factor $$f = \frac{64}{N_R}$$ **(Appendix 3)**

If the flow is laminar, friction factor is <u>independent</u> of the surface roughness of the pipe.

Turbulent flow

Power-law Velocity profile
(n=6 to 10)

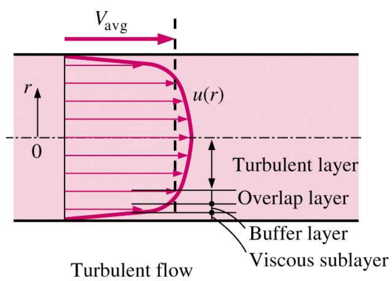
$$\frac{u}{u_{\max}} = \left(1 - \frac{r}{r_0}\right)^{1/n}$$

Logarithmic Velocity profile

$$\frac{u}{u_f} = \frac{1}{\kappa} \ln\left(\frac{y u_f}{\nu}\right) + 5.5$$

Shear velocity: $u_f = \sqrt{\tau_0 / \rho}$

Karman constant: $\kappa = 0.4$



When the Reynolds number approaches a higher value, the flow in the pipe becomes practically turbulent and the value of *f* then becomes less dependent on the Reynolds number but more dependent on the relative roughness (ε/D) of the pipe.

The quantity ε is a measure of the average roughness height of the pipe wall irregularities.

Friction factor
Colebrooke Equation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{N_R \sqrt{f}}\right)$$

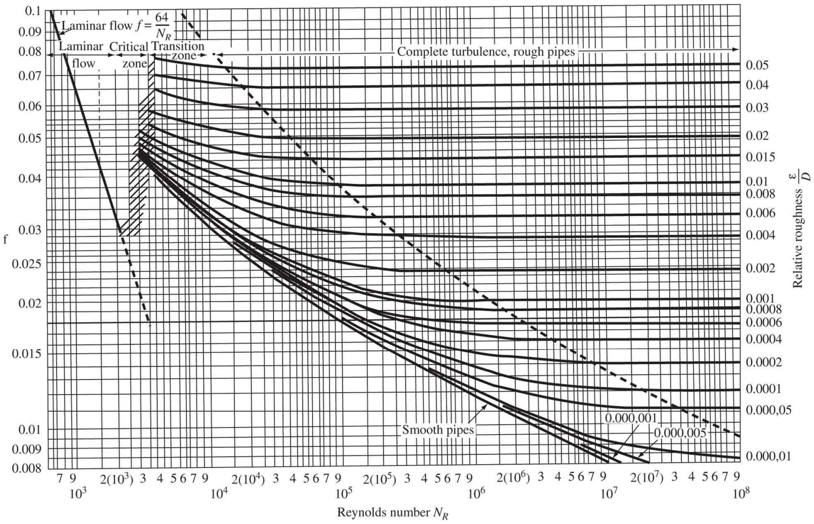
Approximate Equation by Swamee-Jain

$$f = \frac{0.25}{\left[\log\left(\left(\frac{\varepsilon / D}{3.7}\right) + \frac{5.74}{N_R^{0.9}}\right)\right]^2}$$

TABLE 3.1 Roughness Heights, ε, for Certain Common Pipe Materials

Pipe Material	ε (mm)	ε (ft)
Brass	0.0015	0.000005
Concrete		
Steel forms, smooth	0.18	0.0006
Good joints, average	0.36	0.0012
Rough, visible form marks	0.60	0.002
Copper	0.0015	0.000005

Moody's diagram



- Laminar flow zone: *f* is a simple linear function of the N_R
- Critical zone: *f* values are uncertain because the flow might be neither laminar nor truly turbulent
- Transitional zone: *f* is a function of both the N_R and the Relative roughness.
- Fully developed turbulence: *f* depends completely on the relative roughness.

Example 3.5

Compute the discharge capacity of a 3-m concrete (rough) pipe carrying water at 10ºC. It is allowed to have a head loss of 2m/km of pipe. From Table 3.1, ε=0.6mm, at 10ºC, ν=0.00000131m^2/s

3.6 Empirical equations for friction head loss

Hazen-Williams formula (SI units) $V = 0.85 C_{HW} R^{0.63} S^{0.54}$

S is the slope of the EGL (h_f / L)
R is the hydraulic radius (for a circular pipe R = D/4)

Manning's formula (SI units) $V = \frac{1}{n} R^{2/3} S^{1/2}$

For Hazen-Williams constant see Table 3.2
For Manning's n see Table 3.3

3.7 Friction head loss-discharge relationships

Friction head loss can be expressed as follows:

$$h_f = KQ^m$$

E.g. Darcy-Weisbach Equation $h_f = f \frac{L}{D} \frac{V^2}{2g}$ $Q = (\pi D^2 / 4) V$

$$h_f = \left[f \frac{L}{D^5} \left(\frac{16}{2g\pi^2} \right) \right] Q^2$$

Table 3.4

Equation	***m***	*K*
Darcy-Weisbach	2	$0.0826\, fL / D^5$
Hazen-Williams	1.85	$10.7 L / (D^{4.87} C_{HW}^{1.85})$
Manning	2	$10.3 n^2 L / D^{5.33}$

3.8 Loss of head in pipe contractions

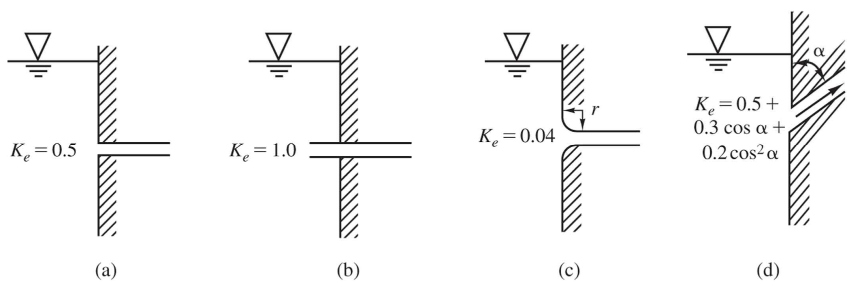
A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe because of both the increase in velocity and the loss of energy to turbulence.

$$h_c = K_c \frac{V_2^2}{2g} \qquad \text{Special Case: Entrance} \qquad h_e = K_e \frac{V_2^2}{2g}$$

K_c depends on the ratio of contraction (D_2/D_1), and the velocity in smaller pipe.

For Values of K_c see Table 3.5

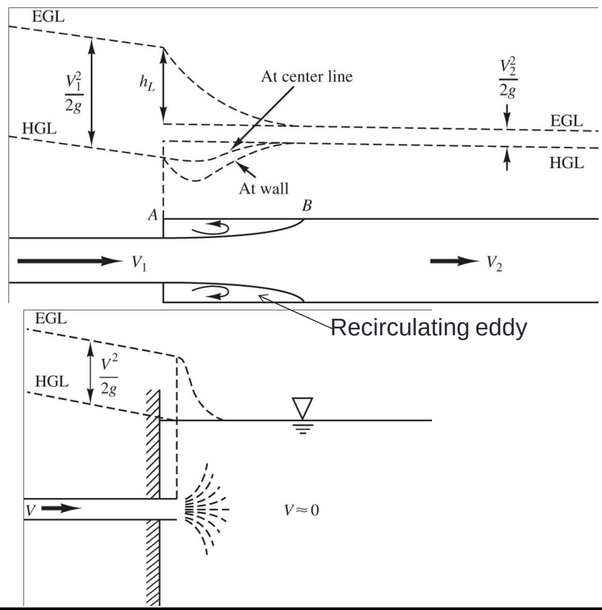
3.8 Loss of head in pipe contractions



Entrance type	K_e
Flushed	0.5
Projected	1.0
Bell-mouthed	0.04
With an angle α w.r.t. *y*-axis	$0.5 + 0.3\cos\alpha + 0.2\cos^2\alpha$

3.9 Loss of head in pipe expansions

$$h_E = \frac{(V_1 - V_2)^2}{2g}$$



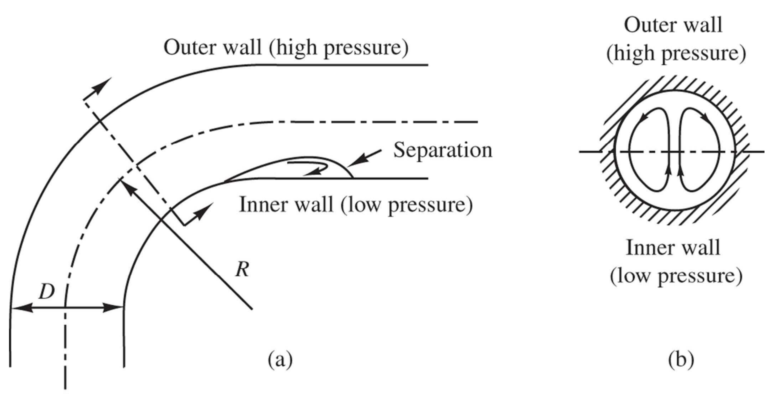
Special Case: Exit

$$h_d = \frac{V^2}{2g}$$

3.10 Loss of head in pipe bends

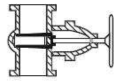
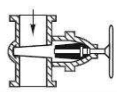
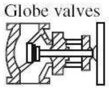
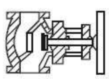
$$h_b = K_b \frac{V_2^2}{2g}$$

K_b depends on R/D ratio

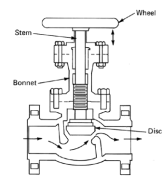


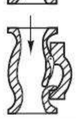
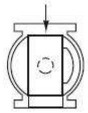
3.11 Loss of head in pipe valves

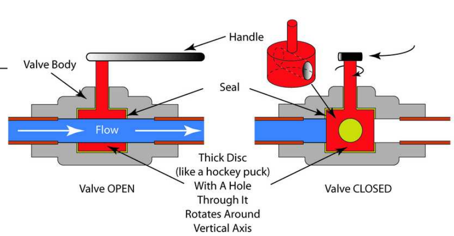
TABLE 3.6 Values of K_v for Common Hydraulic Valves

A. Gate valves		
	Closed	K_v = 0.15 (fully open)
	Open	
B. Globe valves		
	Closed	K_v = 10.0 (fully open)
	Open	

$$h_v = K_v \frac{V_2^2}{2g}$$

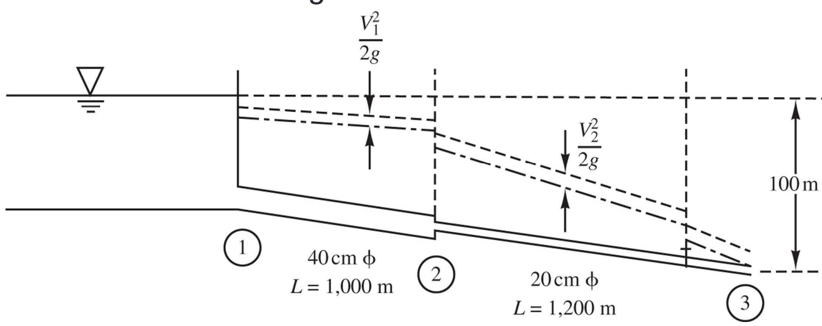


C. Check valves		
	Closed Hinge (swing type)	Swing type: K_v = 2.5 (fully open) Ball type: K_v = 70.0 (fully open) Lift type: K_v = 12.0 (fully open)
	Open	
D. Rotary valves		
	Closed	K_v = 10.0 (fully open)
	Open	



Example 3.9

Figure 3.17 shows two sections of cast iron pipe connected in series that bring water from a reservoir and discharge it into air through a rotary valve at a location 100m below the water surface elevation. If the water temperature is 10ºC and square connections are used, determine the discharge.

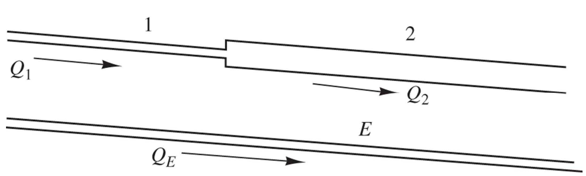


3.12 Method of equivalent pipes

- The method of equivalent pipes is used to facilitate the analysis of pipe systems containing several pipes in series or in parallel.
- The expressions presented for equivalent pipes account for the losses from friction only

3.12.1 Pipes in series

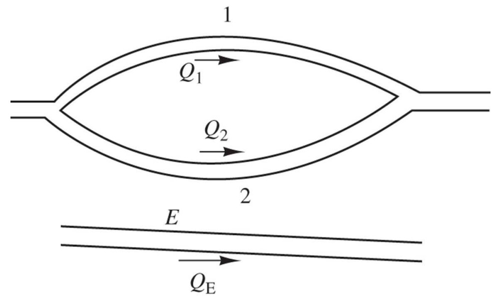
$$Q_1 = Q_2 = Q_E \qquad h_{fE} = h_{f1} + h_{f2}$$



$$h_f = f\frac{8LQ^2}{g\pi^2 D^5} \qquad f_E\frac{L_E}{D_E^5} = f_1\frac{L_1}{D_1^5} + f_2\frac{L_2}{D_2^5}$$

$$f_E\frac{L_E}{D_E^5} = \sum_{i=1}^{N} f_i\frac{L_i}{D_i^5}$$

3.12.1 Pipes in parallel



$$h_{fE} = h_{f1} = h_{f2}$$

$$Q_E = Q_1 + Q_2$$

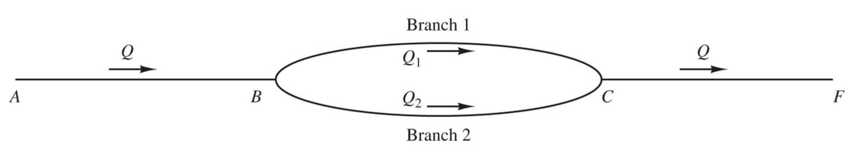
$$Q = \left[\frac{g\pi^2 D^5 h_f}{8fL}\right]^{1/2}$$

$$\left(\frac{D_E^5}{f_E L_E}\right)^{1/2} = \left(\frac{D_1^5}{f_1 L_1}\right)^{1/2} + \left(\frac{D_2^5}{f_2 L_2}\right)^{1/2}$$

$$\left(\frac{D_E^5}{f_E L_E}\right)^{1/2} = \sum_{i=1}^{N}\left(\frac{D_i^5}{f_i L_i}\right)^{1/2}$$

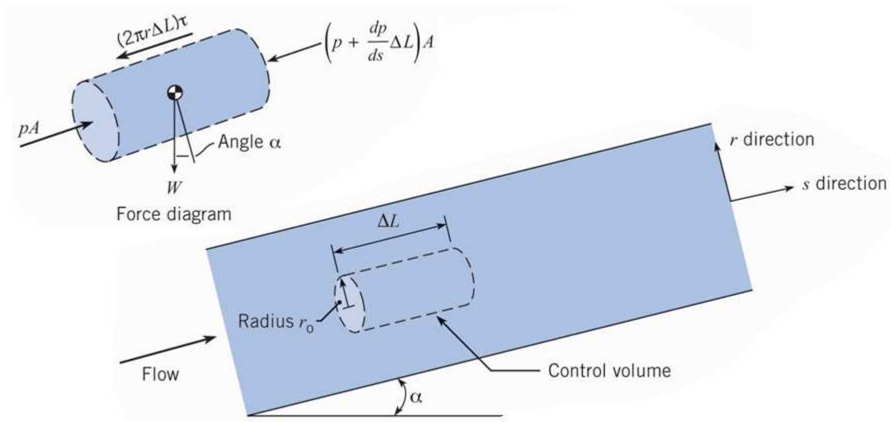
Example 3.10

Pipes AB and CF in Figure 3.20 have a diameter of 4 ft, possess a Darcy-Weisbach friction factor of 0.02, and carry a discharge of 120 cubic feet per second (cfs). The length of AB is 1,800 ft and that of CF is 1,500 ft. Branch 1 is 1,800 ft long and has a diameter of 3 ft and a friction factor of 0.018. Branch 2 has a length of 1,500 ft, a diameter of 2 ft, and a friction factor of 0.015. (a) Determine the total head loss resulting from friction between points A and F, and (b) determine the discharge in each of the two branches (1 and 2).



Appendix

Appendix 1: Shear stress distribution in pipe flow

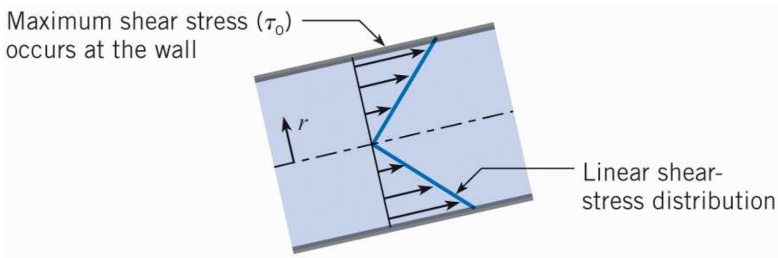


$$pA-\left(p+\frac{dp}{ds}\Delta L\right)A-W\sin\alpha-\tau(2\pi r)\Delta L=0$$

$$pA - \left(p + \frac{dp}{ds}\Delta L\right)A - W\sin\alpha - \tau(2\pi r)\Delta L = 0$$

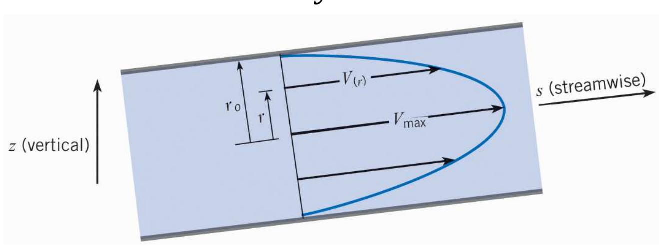
$$W = \gamma A \Delta L \quad \sin\alpha = \Delta z / \Delta L$$

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$



Appendix 2: Laminar flow in a round tube

Velocity profile $\tau = \mu\frac{dV}{dy} = -\mu\frac{dV}{dr} \quad (y = r_0 - r)$



$$-\mu\frac{dV}{dr} = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

$$\left(\frac{2\mu}{r}\right)\frac{dV}{dr} = \left[\frac{d}{ds}(p + \gamma z)\right] = \text{constant} = \frac{\gamma \Delta h}{\Delta L}$$

$$\frac{dV}{dr} = \left(\frac{r}{2\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = \left(\frac{r^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right] + C$$

$$V(r = r_0) = 0 \quad C = -\left(\frac{r_0^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = V_{\max} \quad at \quad r = 0$$

$$V_{\max} = -\left(\frac{r_0^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right] \Rightarrow V = V_{\max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

Discharge and mean velocity

$$Q = \int V dA$$

$$Q = -\int_0^{r_0} \left(\frac{r_0^2 - r^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right](2\pi r dr)$$

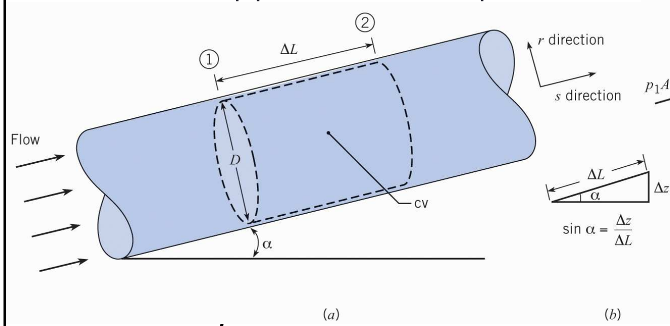
$$Q = -\left(\frac{\pi}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]\left|\frac{r^2 - r_0^2}{2}\right|_0^{r_0} = -\left(\frac{\pi r_0^4}{8\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

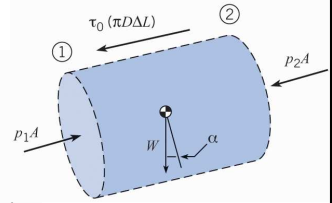
$$\bar{V} = Q / A = Q / \left(\pi r_0^2\right)$$

$$\bar{V} = -\left(\frac{r_0^2}{8\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right] = \frac{V_{\max}}{2}$$

Appendix 3: Pipe head loss (**Darcy-Weisbach formula**)

Total head loss= pipe head loss+ component head





$$\sum \vec{F} = \frac{d}{dt}\int_{cv} \vec{V}\rho dV\!\!\!\!- + \int_{cs} \vec{V}\rho \vec{V}.d\vec{A} = 0$$

$$\sum \vec{F} = (p_1 - p_2)\left(\frac{\pi D^2}{4}\right) - \tau_0(\pi D \Delta L) - \gamma\left[\left(\frac{\pi D^2}{4}\right)\Delta L\right]\sin\alpha = 0$$

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \frac{4\Delta L\,\tau_0}{D} \qquad \left(\sin\alpha = \frac{\Delta z}{\Delta L}\right)$$

From energy equation:

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + h_L$$

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma h_L$$

$$h_f = \frac{4L\tau_0}{D\gamma} = \left(\frac{L}{D}\right)\left(\frac{4\tau_0}{\rho \bar{V}^2/2}\right)\left(\frac{\rho \bar{V}^2/2}{\gamma}\right) = \left(\frac{4\tau_0}{\rho \bar{V}^2/2}\right)\left(\frac{L}{D}\right)\left(\frac{\bar{V}^2}{2g}\right)$$

$$f = \frac{4\tau_0}{(\rho \bar{V}^2/2)}$$

$$h_f = f\,\frac{L}{D}\frac{\bar{V}^2}{2g}$$

Head loss and friction factor

$$\left(\frac{p_1}{\gamma}+z_1\right)=\left(\frac{p_2}{\gamma}+z_2\right)+h_f \qquad \left(h_L=h_f\right)$$

$$\overline{V}=-\left(\frac{D^2}{32\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]=-\left(\frac{D^2}{32\mu}\right)\left(\frac{1}{\Delta L}\right)\left[\left(p_2+\gamma z_2\right)-\left(p_1+\gamma z_1\right)\right]$$

$$=-\left(\frac{D^2}{32\mu}\right)\left(\frac{\gamma}{\Delta L}\right)\left[\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)\right]$$

$$\left(\frac{p_1}{\gamma}+z_1\right)=\left(\frac{p_2}{\gamma}+z_2\right)+\frac{32\mu L\overline{V}}{\gamma D^2}$$

$$h_f=\frac{32\mu L\overline{V}}{\gamma D^2}$$

$$h_f=\frac{32\mu L\overline{V}}{\gamma D^2}=f\,\frac{L}{D}\frac{\overline{V}^2}{2g}$$

$$f=\frac{64\mu}{\rho D\overline{V}}=\frac{64}{\mathrm{Re}}$$