Chapter 10

Flow in Conduits

Ahmad Sana
Department of Civil and Architectural Engineering
Sultan Qaboos University
Sultanate of Oman
Email: sana@squ.edu.om
Webpage: http://ahmadsana.tripod.com

Significant learning outcomes

Conceptual Knowledge

- Describe laminar flow, turbulent flow, developing flow, and fully developed flow in a conduit.
- Describe how to characterize total head loss by using component and pipe head loss.
- List the steps used to derive the Darcy-Weisbach equation and Poiseuille flow solution.
- Describe the main features of the Moody diagram.

Procedural Knowledge

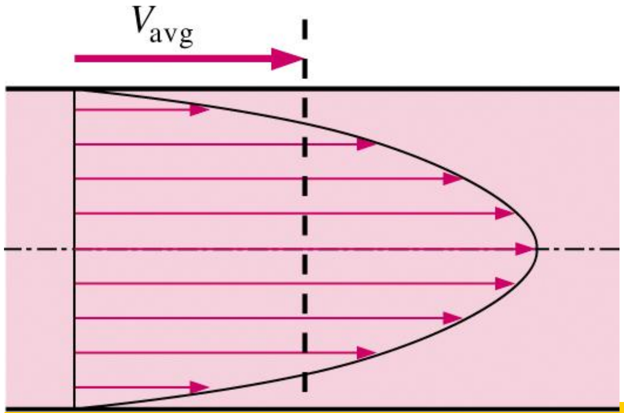
- Classify flow as laminar or turbulent and developing or fully developed.
- Using equations or the Moody diagram, find values of the friction factor f.
- Calculate pipe head loss, component head loss, and total head loss.

Typical Applications

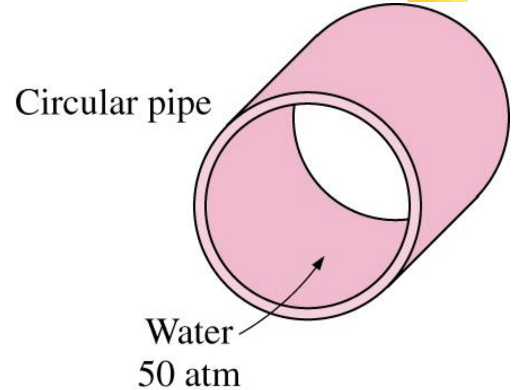
- For flow in a pipe, find the pressure drop or head loss.
- For a specified system, find the flow rate.
- For a specified flow rate and pressure drop, determine the size of pipe required.

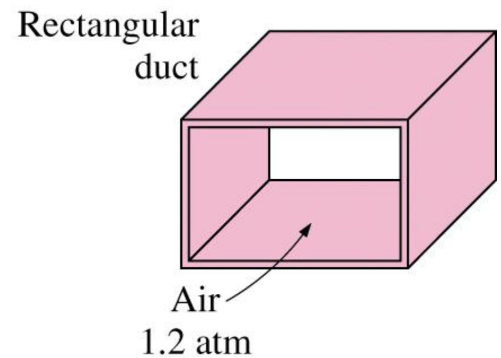
10.1 Classifying flow

- **Pipe, Duct or Conduit**: A closed fluid conveying section having pressure greater than atmospheric.
- **Average velocity**



$$V_{avg} = \frac{\int_{A_c} (\rho u(r))(2\pi r dr)}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r dr$$





10.1 Classifying flow

Laminar and Turbulent Flows

Reynolds number: The ratio of inertial to viscous forces

$$\mathrm{Re} = \frac{F_I}{F_v} = \frac{\rho V^2 L^2}{\mu V L} = \frac{\rho V L}{\mu}$$

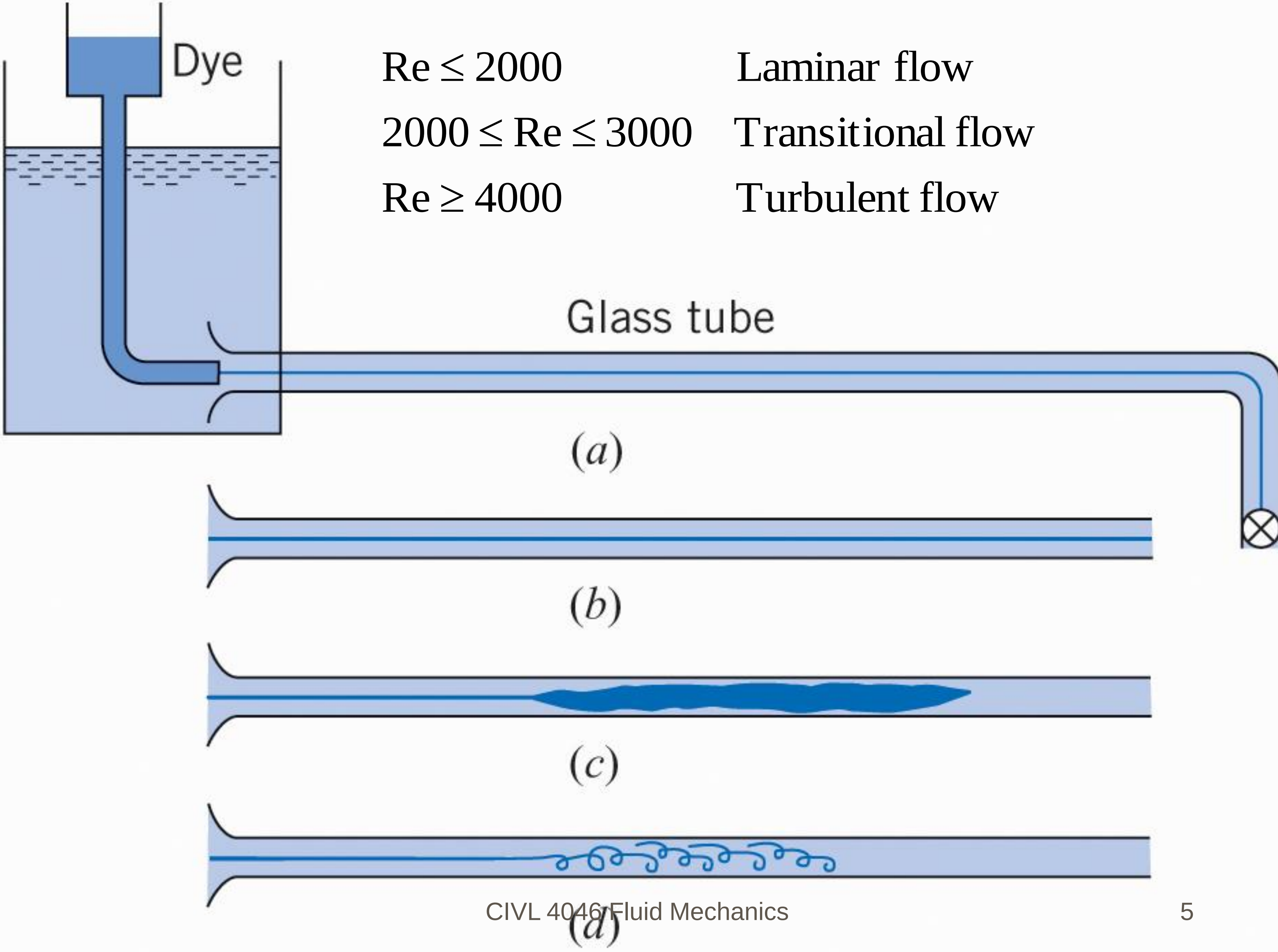
L is a characteristic length. For pipes, the characteristic length is the pipe diameter. So,

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{4Q}{\pi D \nu} = \frac{4\dot{m}}{\pi D \mu}$$

$Re \leq 2000$ Laminar flow

$2000 \leq Re \leq 3000$ Transitional flow

$Re \geq 4000$ Turbulent flow

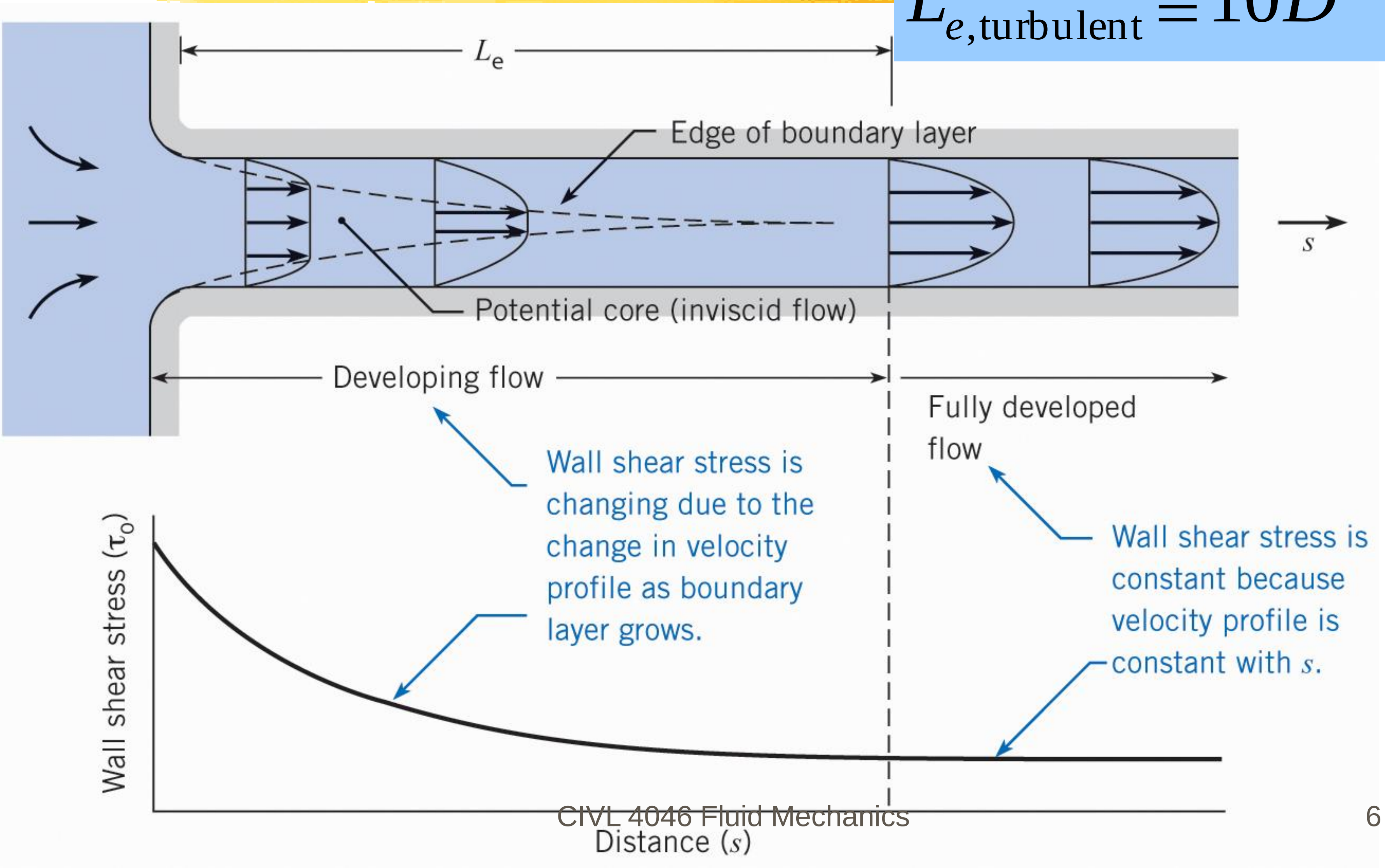


Developing and fully developed flow

$$L_{e,\text{laminar}} \cong 0.05\,\text{Re}\,D$$

$$L_{e,\text{turbulent}} \cong 10D$$

Hydrodynamic entry region

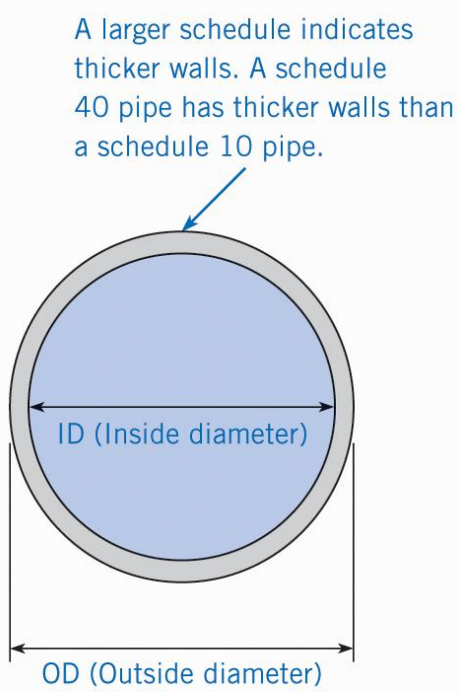


10.2 Specifying pipe sizes

Nominal pipe size (NPS)

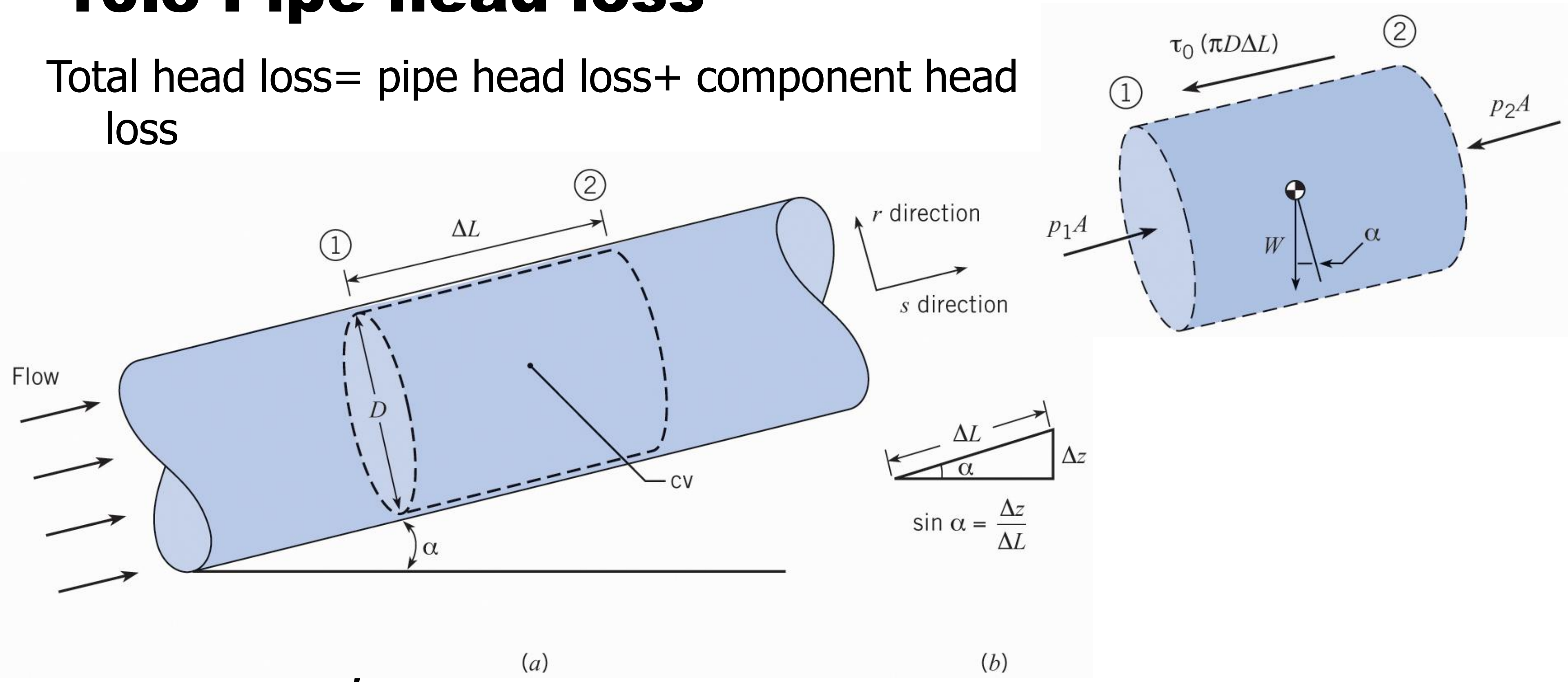
Table 10.1 NOMINAL PIPE SIZES

NPS (in)	OD (in)	Schedule	Wall Thickness (in)	ID (in)
1/2	0.840	40	0.109	0.622
		80	0.147	0.546
1	1.315	40	0.133	1.049
		80	0.179	0.957
2	2.375	40	0.154	2.067
		80	0.218	1.939
4	4.500	40	0.237	4.026
		80	0.337	3.826
8	8.625	40	0.322	7.981
		80	0.500	7.625
14	14.000	10	0.250	13.500
		40	0.437	13.126
		80	0.750	12.500
		120	1.093	11.814
24	24.000	10	0.250	23.500
		40	0.687	22.626
		80	1.218	21.564
		120	1.812	20.376



10.3 Pipe head loss

Total head loss= pipe head loss+ component head loss



$$\sum \vec{F} = \frac{d}{dt}\int_{cv} \vec{V}\rho d\forall + \int_{cs} \vec{V}\rho \vec{V}.d\vec{A} = 0$$

$$\sum \vec{F} = (p_1 - p_2)\left(\frac{\pi D^2}{4}\right) - \tau_0\left(\pi D \Delta L\right) - \gamma\left[\left(\frac{\pi D^2}{4}\right)\Delta L\right]\sin\alpha = 0$$

$$(p_1+\gamma z_1)-(p_2+\gamma z_2)=\frac{4\Delta L\,\tau_0}{D} \qquad \left(\sin\alpha=\frac{\Delta z}{\Delta L}\right)$$

From energy equation:

$$\left(\frac{p_1}{\gamma}+z_1\right)=\left(\frac{p_2}{\gamma}+z_2\right)+h_L$$

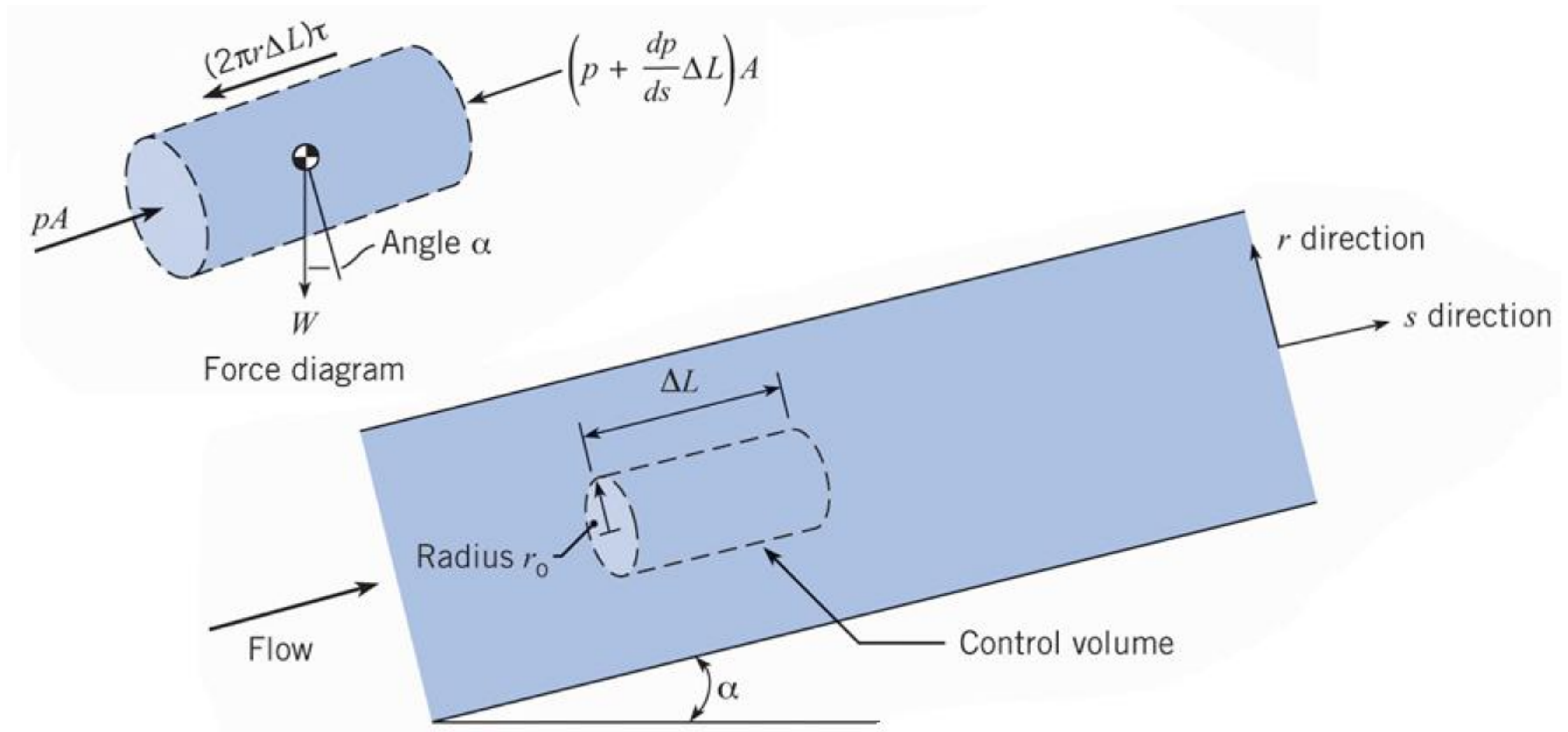
$$(p_1+\gamma z_1)-(p_2+\gamma z_2)=\gamma h_L$$

$$h_f=\frac{4L\,\tau_0}{D\gamma}=\left(\frac{L}{D}\right)\left(\frac{4\tau_0}{\rho\overline{V}^2/2}\right)\left(\frac{\rho\overline{V}^2/2}{\gamma}\right)=\left(\frac{4\tau_0}{\rho\overline{V}^2/2}\right)\left(\frac{L}{D}\right)\left(\frac{\overline{V}^2}{2g}\right)$$

$$f=\frac{4\tau_0}{\left(\rho\overline{V}^2/2\right)}$$

$$h_f=f\,\frac{L}{D}\frac{\overline{V}^2}{2g}$$

10.4 Shear stress distribution in pipe flow

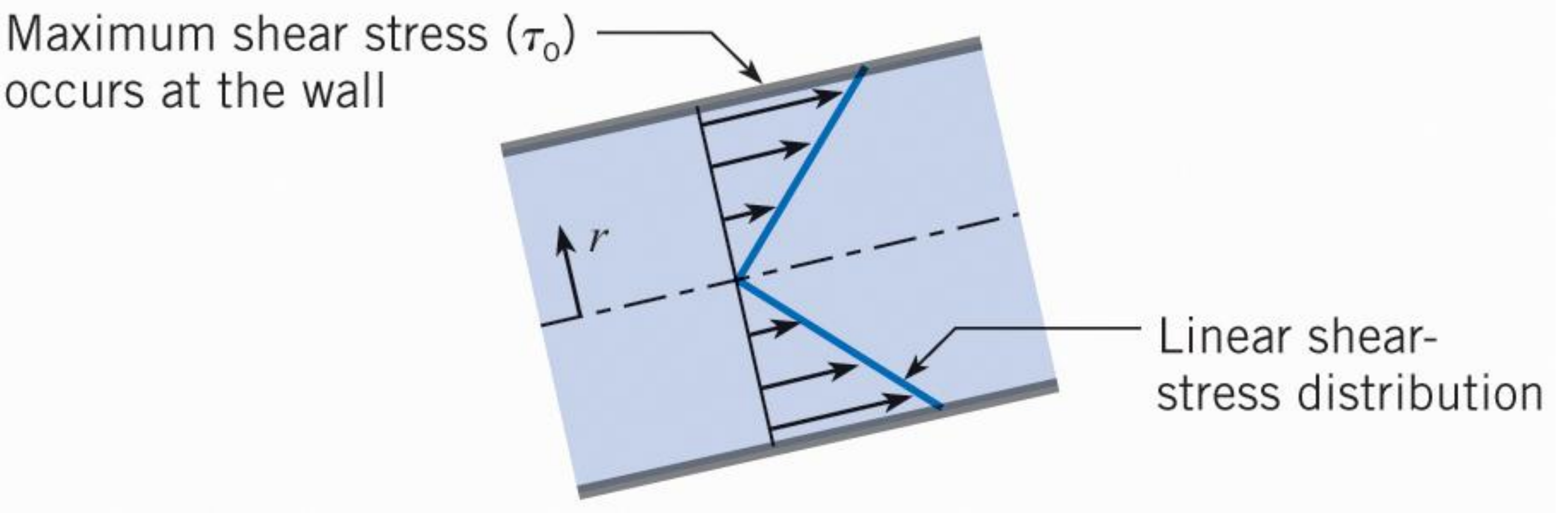


$$pA - \left(p + \frac{dp}{ds}\Delta L\right)A - W\sin\alpha - \tau(2\pi r)\Delta L = 0$$

$$pA - \left(p + \frac{dp}{ds} \Delta L \right) A - W \sin\alpha - \tau (2\pi r) \Delta L = 0$$

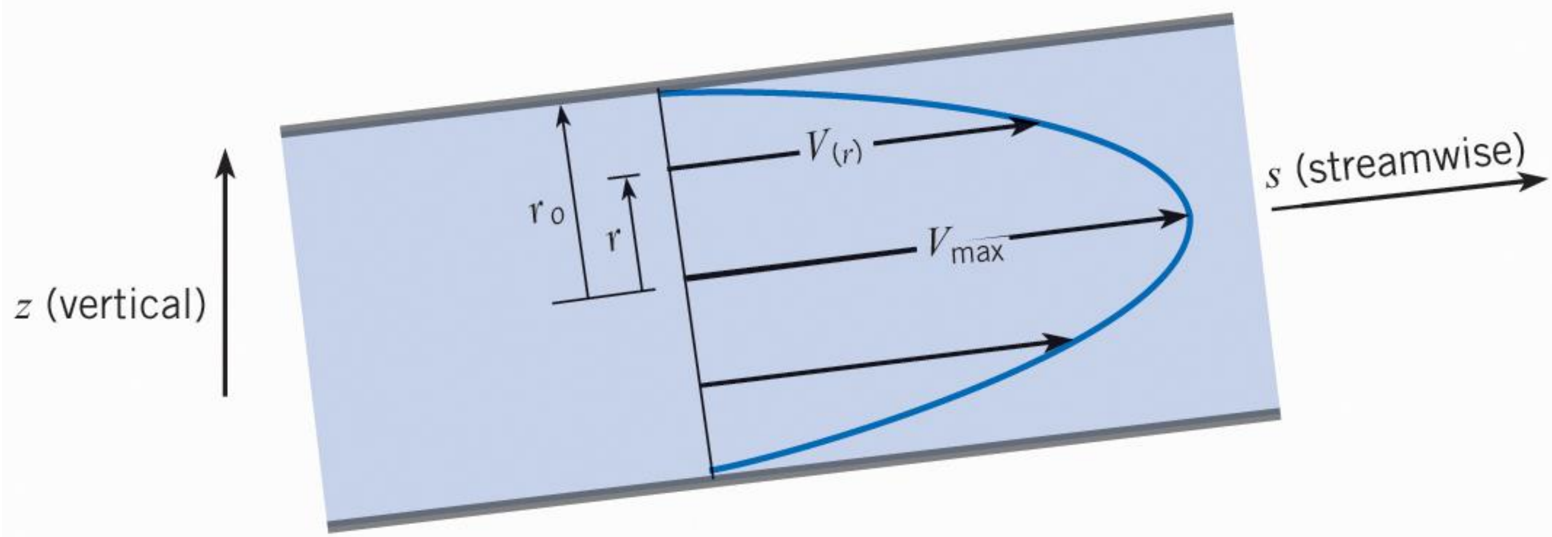
$$W = \gamma A \Delta L \qquad \sin\alpha = \Delta z / \Delta L$$

$$\tau = \frac{r}{2} \left[-\frac{d}{ds} (p + \gamma z) \right]$$



10.5 Laminar flow in a round tube

Velocity profile $\tau = \mu \frac{dV}{dy} = -\mu \frac{dV}{dr} \quad (y = r_0 - r)$



$$-\mu \frac{dV}{dr} = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

$$\left(\frac{2\mu}{r}\right)\frac{dV}{dr} = \left[\frac{d}{ds}(p + \gamma z)\right] = \text{constant} = \frac{\gamma \Delta h}{\Delta L}$$

$$\frac{dV}{dr} = \left(\frac{r}{2\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = \left(\frac{r^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right] + C$$

$$V(r = r_0) = 0 \quad C = -\left(\frac{r_0^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]$$

$$V = V_{\max} \quad at \quad r = 0$$

$$V_{\max} = -\left(\frac{r_0^2}{4\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right] \Rightarrow V = V_{\max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

Discharge and mean velocity

$$Q = \int V dA$$

$$Q = -\int_0^{r_0} \left(\frac{r_0^2 - r^2}{4\mu} \right) \left[\frac{\gamma \Delta h}{\Delta L} \right] (2\pi r dr)$$

$$Q = -\left(\frac{\pi}{4\mu} \right) \left[\frac{\gamma \Delta h}{\Delta L} \right] \left| \frac{r^2 - r_0^2}{2} \right|_0^{r_0} = -\left(\frac{\pi r_0^4}{8\mu} \right) \left[\frac{\gamma \Delta h}{\Delta L} \right]$$

$$\overline{V} = Q / A = Q / \left(\pi r_0^2 \right)$$

$$\overline{V} = -\left(\frac{r_0^2}{8\mu} \right) \left[\frac{\gamma \Delta h}{\Delta L} \right] = \frac{V_{max}}{2}$$

Head loss and friction factor

$$\left(\frac{p_1}{\gamma}+z_1\right)=\left(\frac{p_2}{\gamma}+z_2\right)+h_f \qquad \left(h_L=h_f\right)$$

$$\overline{V}=-\left(\frac{D^2}{32\mu}\right)\left[\frac{\gamma\Delta h}{\Delta L}\right]=-\left(\frac{D^2}{32\mu}\right)\left(\frac{1}{\Delta L}\right)\left[\left(p_2+\gamma z_2\right)-\left(p_1+\gamma z_1\right)\right]$$

$$=-\left(\frac{D^2}{32\mu}\right)\left(\frac{\gamma}{\Delta L}\right)\left[\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)\right]$$

$$\left(\frac{p_1}{\gamma}+z_1\right)=\left(\frac{p_2}{\gamma}+z_2\right)+\frac{32\mu L\overline{V}}{\gamma D^2}$$

$$h_f=\frac{32\mu L\overline{V}}{\gamma D^2}$$

$$h_f=\frac{32\mu L\overline{V}}{\gamma D^2}=f\,\frac{L}{D}\frac{\overline{V}^2}{2g}$$

$$f=\frac{64\mu}{\rho D\overline{V}}=\frac{64}{\mathrm{Re}}$$

EXAMPLE 10.2 HEAD LOSS FOR LAMINAR FLOW

Oil (S = 0.85) with a kinematic viscosity of $6 \times 10^{-4}\ \text{m}^2/\text{s}$ flows in a 15 cm pipe at a rate of $0.020\ \text{m}^3/\text{s}$. What is the head loss per 100 m length of pipe?

Problem Definition

Situation:

1. Oil is flowing in a pipe at a flow rate of $Q = 0.02\ \text{m}^3/\text{s}$.
2. Pipe diameter is $D = 0.15$ m.

Find: Head loss (in meters) for a pipe length of 100 m.

Assumptions: Fully developed, steady flow.

Properties: Oil: $S = 0.85$, $\nu = 6 \times 10^{-4}\ \text{m}^2/\text{s}$.

Solution

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.020\ \text{m}^3/\text{s}}{(\pi D^2)/4} = \frac{0.020\ \text{m}^3/\text{s}}{\pi((0.15\ \text{m})^2/4)} = 1.13\ \text{m/s}$$

2. Reynolds number

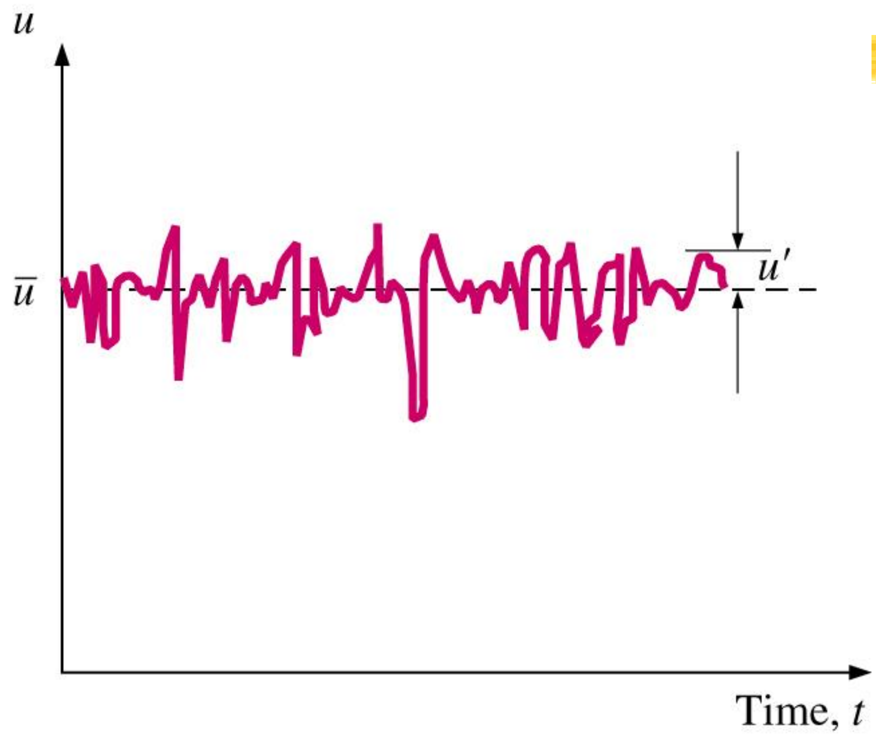
$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13\ \text{m/s})(0.15\ \text{m})}{6\times 10^{-4}\ \text{m}^2/\text{s}} = 283$$

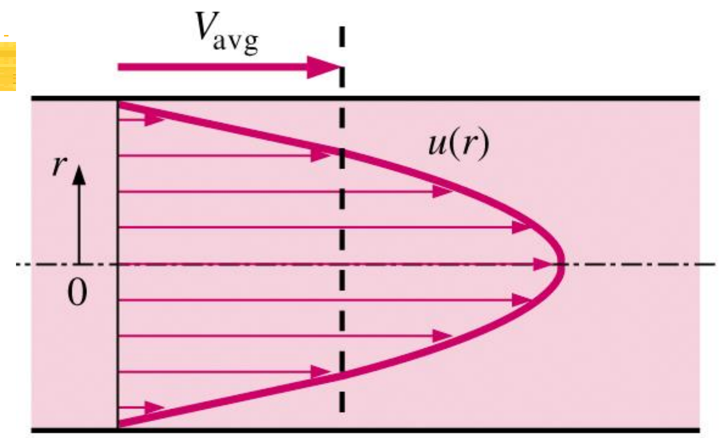
3. Since Re $<$ 2000, the flow is laminar.
4. Head loss (laminar flow).

$$h_f = \frac{32\mu LV}{\gamma D^2} = \frac{32\rho\nu LV}{\rho g D^2} = \frac{32\nu LV}{gD^2}$$

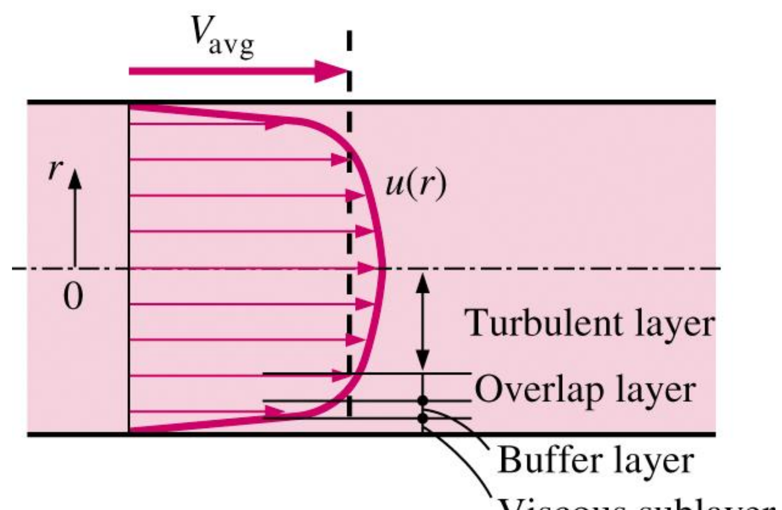
$$= \frac{32(6\times 10^{-4}\ \text{m}^2/\text{s})(100\ \text{m})(1.13\ \text{m/s})}{(9.81\ \text{m/s}^2)(0.15\ \text{m})^2} = \boxed{9.83\ \text{m}}$$

10.6 Turbulent flow and the Moody diagram





Laminar flow



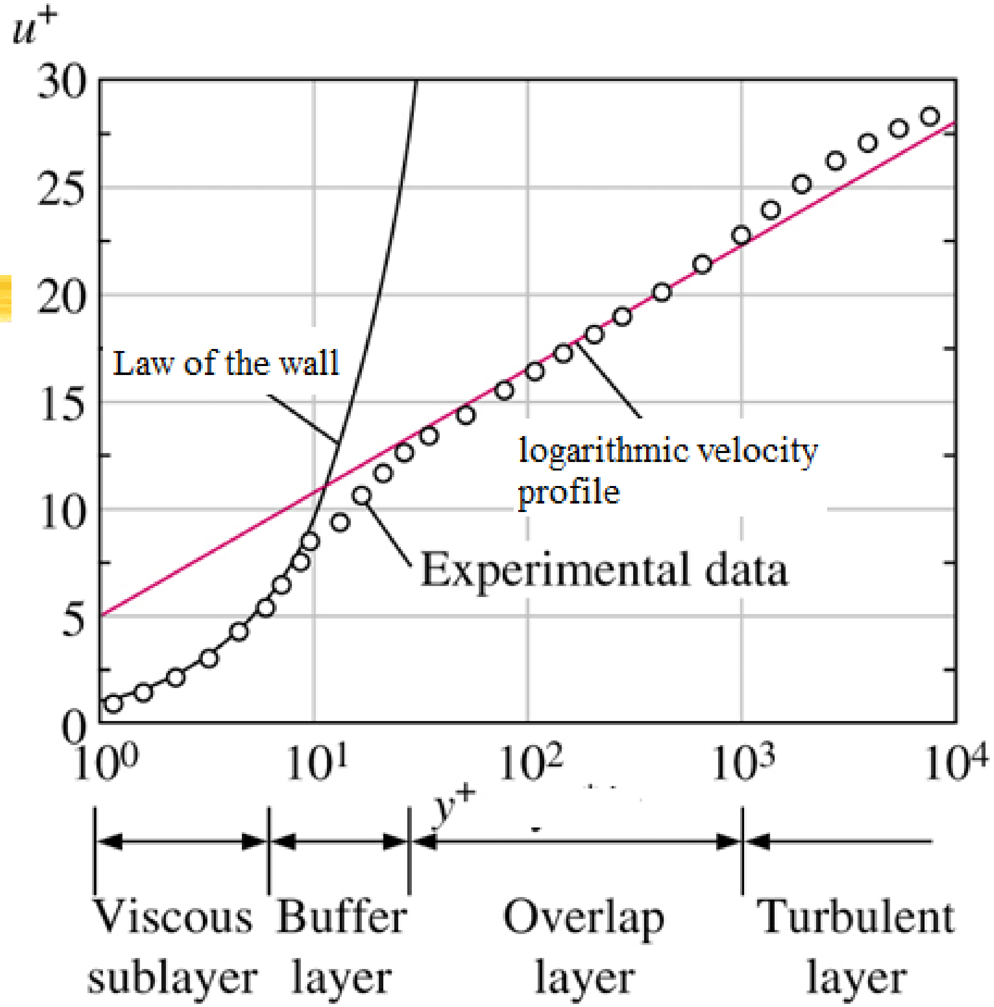
Turbulent flow

$$\frac{V}{V_*} = \frac{yV_*}{\nu}$$

$$\left(V_* = \sqrt{\tau_0 / \rho}\right)$$

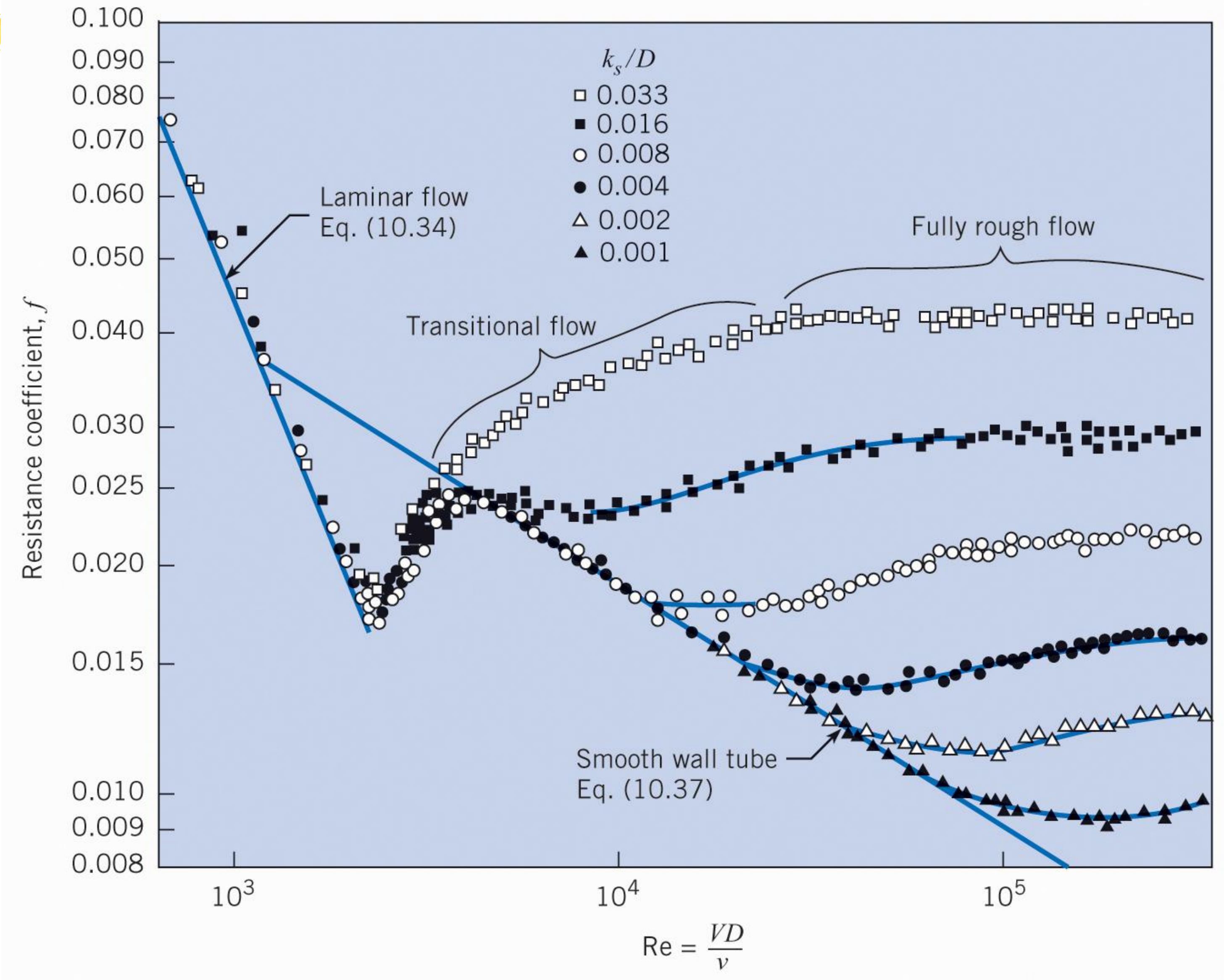
$$\frac{V}{V_*} = 2.5 \ln\left(\frac{yV_*}{\nu}\right) + 5.0$$

$$y^+ = \frac{yV_*}{\nu} \qquad u^+ = \frac{V}{V_*}$$



Colebrook Equation $$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Moody diagram



Boundary Material	k_s, Millimeters	k_s, Inches
Glass, plastic	Smooth	Smooth
Copper or brass tubing	0.0015	6×10^{-5}
Wrought iron, steel	0.046	0.002
Asphalted cast iron	0.12	0.005
Galvanized iron	0.15	0.006
Cast iron	0.26	0.010
Concrete	0.3 to 3.0	0.012–0.12
Riveted steel	0.9–9	0.035–0.35
Rubber pipe (straight)	0.025	0.001

Swamee and Jain formula

$$f = \frac{0.25}{\left[\log\left(\frac{k_s / D}{3.7} + \frac{5.74}{(\text{Re})^{0.9}}\right)\right]^2}$$

10.7 Solving turbulent flow problems

Types of fluid flow problems

Case	Given	Find
1	L, D, Q	h_f
2	L, D, h_f	Q
3	L, Q, h_f	D

EXAMPLE 10.3 HEAD LOSS IN A PIPE (CASE 1)

Water ($T = 20°C$) flows at a rate of $0.05\ m^3/s$ in a 20 cm asphalted cast-iron pipe. What is the head loss per kilometer of pipe?

Problem Definition

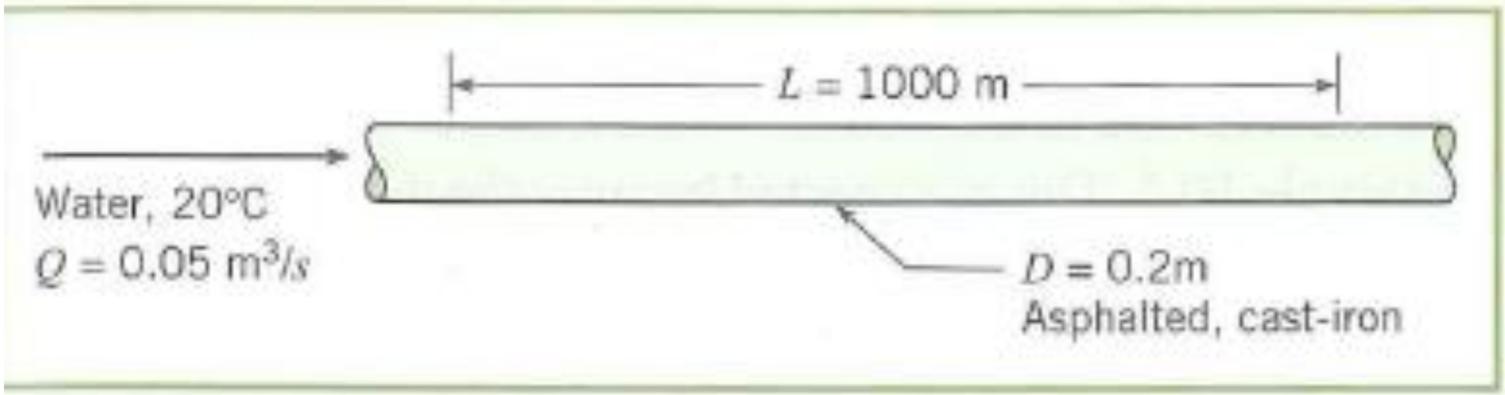
Situation: Water is flowing in a pipe.

Find: Head loss (in meters) for $L = 1000$ m.

Assumptions: Fully developed flow.

Properties: Water (20°C), Table A.5: $\nu = 1 \times 10^{-6}\ m^2/s$.

Sketch:



Solution

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.05\ \text{m}^3/\text{s}}{(\pi/4)(0.02\ \text{m})^2} = 1.59\ \text{m/s}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.59\ \text{m/s})(0.20\ \text{m})}{10^{-6}\ \text{m}^2/\text{s}} = 3.18 \times 10^5$$

3. Resistance coefficient

- Equivalent sand roughness (Table 10.4): $k_s = 0.12$ mm
- Relative roughness:
  $k_s/D = (0.00012\ \text{m})/(0.2\ \text{m}) = 0.0006$
- Look up f on the Moody diagram for $\text{Re} = 3.18 \times 10^5$ and $k_s/D = 0.0006$:

$$f = 0.019$$

4. Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.019\left(\frac{1000\ \text{m}}{0.20\ \text{m}}\right)\left(\frac{1.59^2\ \text{m}^2/\text{s}^2}{2(9.81\ \text{m/s}^2)}\right)$$

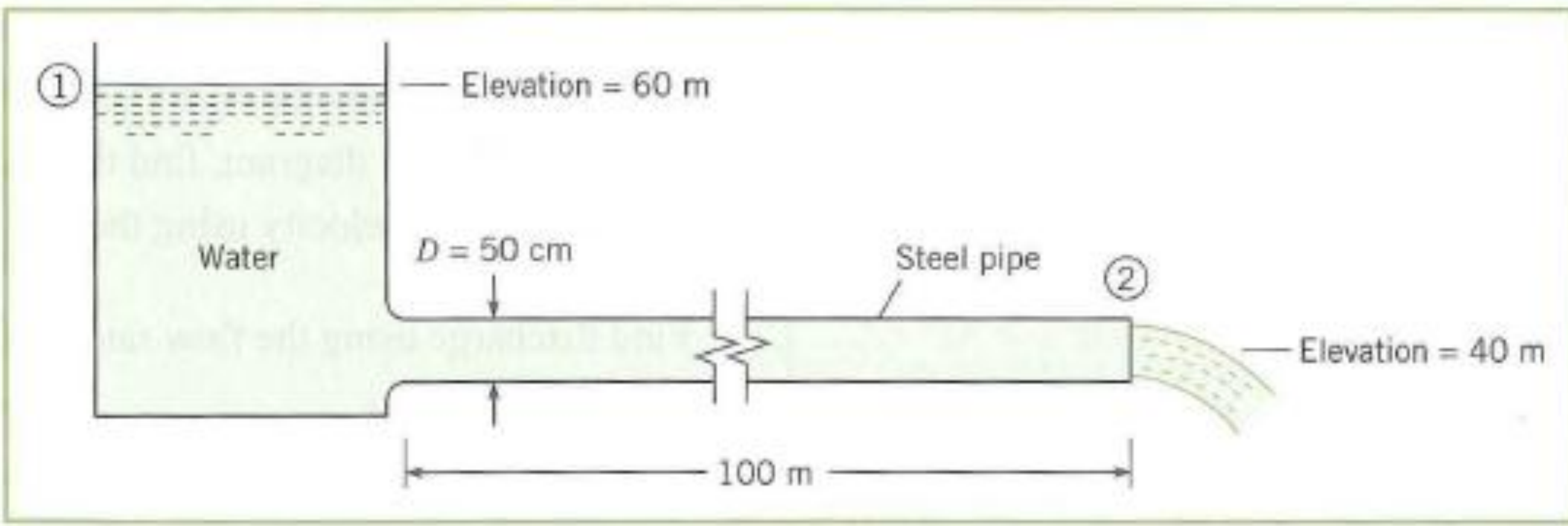
$$= \boxed{12.2\ \text{m}}$$

EXAMPLE 10.5 FLOW RATE IN A PIPE (CASE 2)

Water ($T = 20°C$) flows from a tank through a 50 cm diameter steel pipe. Determine the discharge of water.

Properties:

1. Water (20°C), Table A.5: $\nu = 1 \times 10^{-6}\ m^2/s$.
2. Steel pipe, Table 10.4, equivalent sand roughness: $k_s = 0.046$ mm. Relative roughness (k_s/D) is 9.2×10^{-5}.



Solution

1. Energy equation (reservoir surface to outlet)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 60 = 0 + \frac{V_2^2}{2g} + 40 + f\frac{L}{D}\frac{V_2^2}{2g}$$

or

$$V = \left(\frac{2g \times 20}{1 + 200f}\right)^{1/2} \qquad (1)$$

2. First trial (iteration 1)
   - Guess a value of $f = 0.020$.
   - Use eq. (1) to calculate $V = 8.86$ m/s.
   - Use $V = 8.86$ m/s to calculate $\mathrm{Re} = 4.43 \times 10^6$.
   - Use $\mathrm{Re} = 4.43 \times 10^6$ and $k_s/D = 9.2 \times 10^{-5}$ on the Moody diagram to find that $f = 0.012$.
   - Use eq. (1) with $f = 0.012$ to calculate $V = 10.7$ m/s.

3. Second trial (iteration 2)

- Use $V = 10.7$ m/s to calculate Re $= 5.35 \times 10^6$.
- Use Re $= 5.35 \times 10^6$ and $k_s/D = 9.2 \times 10^{-5}$ on the Moody diagram to find that $f = 0.012$.

4. Convergence. The value of $f = 0.012$ is unchanged between the first and second trials. Therefore, there is no need for more iterations.
5. Flow rate

$$Q = VA = (10.7\ \text{m/s}) \times (\pi/4) \times (0.50)^2\ \text{m}^2 = \boxed{2.10\ \text{m}^3/\text{s}}$$

EXAMPLE 10.6 FINDING PIPE DIAMETER (CASE 3)

What size of asphalted cast-iron pipe is required to carry water (15°C) at a discharge of 0.08 m^3/s and with a head loss of 1 m per 300 m of pipe?

Problem Definition

Situation: Water is flowing in a asphalted cast-iron pipe.

$Q = 0.08\ m^3/s$.

Find: Pipe diameter (in m) so that head loss is 1 m per 300 m of pipe length.

Assumptions: Fully developed flow.

Solution

1. Develop an equation to use for iteration.
   - Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = f\left(\frac{L}{D}\right)\left(\frac{Q^2/A^2}{2g}\right) = \frac{fLQ^2}{2g(\pi/4)^2D^5}$$

   - Solve for pipe diameter

$$D^5 = \frac{fLQ^2}{0.785^2(2gh_f)} \qquad (1)$$

2. Iteration 1

- Guess $f = 0.015$.
- Solve for diameter using eq. (1):

$$D^5 = \frac{0.015(300\text{ m})(0.08\text{ m}^3/\text{s})^2}{0.785^2(19.62\text{ m/s}^2)(1\text{ m})} = 2.38 \times 10^{-3}\text{ m}^5$$

$$D = 0.299\text{ m}$$

- Find parameters needed for calculating f:

$$V = \frac{Q}{A} = \frac{0.08\text{ m}^3/\text{s}}{(\pi/4)(0.299^2\text{ m}^2)} = 1.14\text{ m/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.14\text{ m/s})(0.299\text{ m})}{(1.14 \times 10^{-6}\text{ m}^2/\text{s})} = 3 \times 10^5$$

$$k_s/D = 0.12/(0.299 \times 10^3) = 0.0004$$

• Calculate f using Eq. (10.39): $f = 0.0177$.

3. In the table below, the first row contains the values from iteration 1. The value of $f = 0.0178$ from iteration 1 is used for the initial value for iteration 2. Notice how the solution has converged by iteration 3.

Iteration #	Initial f	D	V	Re	k_s/D	New f
		(m)	(m/s)			
1	0.0150	0.299	1.14	2.99E+05	4.00E–04	0.0177
2	0.0177	0.309	1.07	2.90E+05	3.88E–04	0.0176
3	0.0176	0.309	1.07	2.90E+05	3.88E–04	0.0176
4	0.0176	0.309	1.07	2.90E+05	3.88E–04	0.0176

Specify a pipe with a 31 cm inside diameter.

10.8 Combined head loss

Major loss (due to friction) $h_{L,\text{major}} = f\,\frac{L}{D}\frac{\overline{V}^2}{2g}$

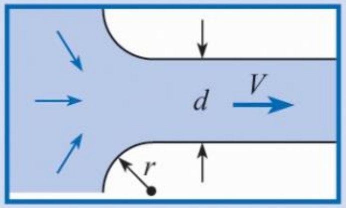
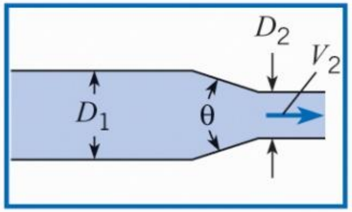
Minor losses (entrance, contraction, expansion, exit, valves and bends)

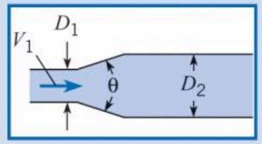
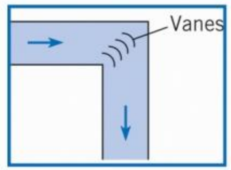
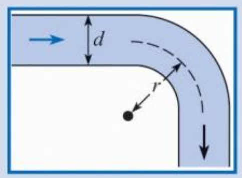
$$h_{L,\text{minor}} = K\,\frac{\overline{V}^2}{2g}$$

$$h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}}$$

$$h_{L,\text{total}} = \left(\sum f\,\frac{L}{D} + \sum K\right)\frac{\overline{V}^2}{2g}$$

Table 10.5 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

Description	Sketch	Additional Data		K	Source
Pipe entrance $h_L = K_e V^2/2g$		r/d		K_e	(10)[†]
		0.0		0.50	
		0.1		0.12	
		>0.2		0.03	
Contraction $h_L = K_C V_2^2/2g$		D_2/D_1	K_C $\theta = 60°$	K_C $\theta = 180°$	(10)
		0.00	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
		0.90	0.06	0.10	

Description	Sketch	Additional Data	K	Source
Expansion $h_L = K_E V_1^2/2g$		D_1/D_2 ; K_E $\theta = 20°$	K_E $\theta = 180°$	(9)
		0.00 ;	1.00	
		0.20 ; 0.30	0.87	
		0.40 ; 0.25	0.70	
		0.60 ; 0.15	0.41	
		0.80 ; 0.10	0.15	
90° miter bend		Without vanes	$K_b = 1.1$	(15)
		With vanes	$K_b = 0.2$	(15)
90° smooth bend		r/d		(16) and (9)
		1	$K_b = 0.35$	
		2	0.19	
		4	0.16	
		6	0.21	
		8	0.28	
		10	0.32	
Threaded pipe fittings	Globe valve—wide open		$K_v = 10.0$	(15)
	Angle valve—wide open		$K_v = 5.0$	
	Gate valve—wide open		$K_v = 0.2$	
	Gate valve—half open		$K_v = 5.6$	
	Return bend		$K_b = 2.2$	
	Tee			
	Straight-through flow		$K_t = 0.4$	
	Side-outlet flow		$K_t = 1.8$	
	90° elbow		$K_b = 0.9$	
	45° elbow		$K_b = 0.4$	

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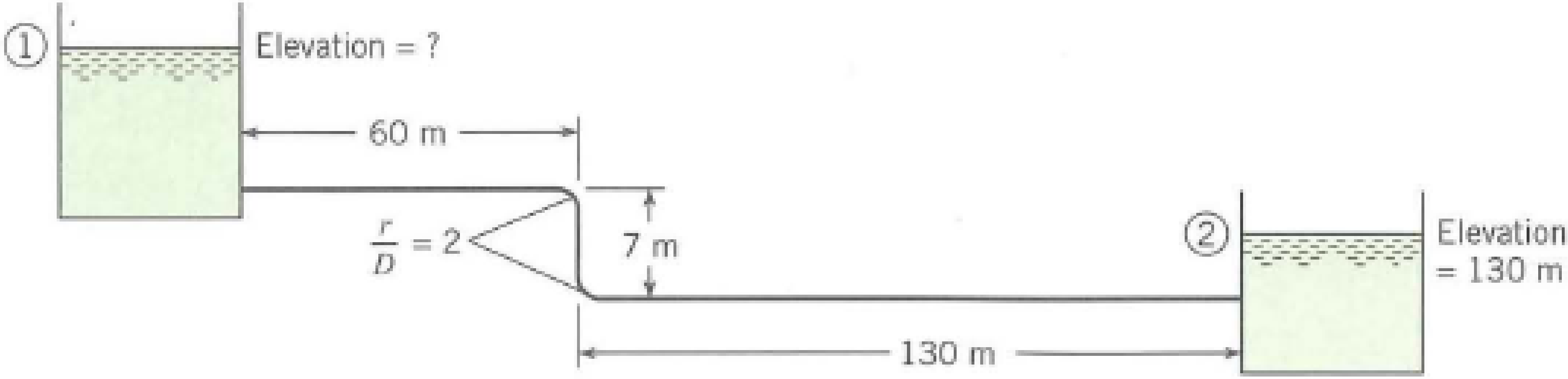
EXAMPLE 10.7 PIPE SYSTEM WITH COMBINED HEAD LOSS

If oil ($\nu = 4 \times 10^{-5}\ \text{m}^2/\text{s}$; $S = 0.9$) flows from the upper to the lower reservoir at a rate of $0.028\ \text{m}^3/\text{s}$ in the 15 cm smooth pipe, what is the elevation of the oil surface in the upper reservoir?

Properties:

1. Oil: $\nu = 4 \times 10^{-5}\ \text{m}^2/\text{s}$, $S = 0.9$.
2. Minor head loss coefficients, Table 10.5;
   entrance $= K_e = 0.5$; bend $= K_b = 0.19$;
   outlet $= K_E = 1.0$.



Solution

1. Energy equation and term-by-term analysis

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\overline{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\overline{V}_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + z_1 + 0 = 0 + 0 + z_2 + 0 + h_L$$

$$z_1 = z_2 + h_L$$

Interpretation: Change in elevation head is balanced by the total head loss.

2. Combined head loss equation

$$h_L = \sum_{\text{pipes}} f\frac{L}{D}\frac{V^2}{2g} + \sum_{\text{components}} K\frac{V^2}{2g}$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + \left(2K_b\frac{V^2}{2g} + K_e\frac{V^2}{2g} + K_E\frac{V^2}{2g}\right)$$

$$= \frac{V^2}{2g}\left(f\frac{L}{D} + 2K_b + K_e + K_E\right)$$

3. Combine eqs. (1) and (2).

$$z_1 = z_2 + \frac{V^2}{2g}\left(f\frac{L}{D} + 2K_b + K_e + K_E\right)$$

4. Resistance coefficient

- Flow rate equation (5.8)

$$V = \frac{Q}{A} = \frac{(0.028\ \text{m}^3/\text{s})}{(\pi/4)(0.15\ \text{m})^2} = 1.58\ \text{m/s}$$

- Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{1.58\ \text{m/s}(0.15\ \text{m})}{4 \times 10^{-5}\ \text{m}^2/\text{s}} = 5.93 \times 10^3$$

Thus, flow is turbulent.

- Swamee-Jain equation (10.39)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{5930^{0.9}}\right)\right]^2} = 0.036$$

5. Calculate z_1 using (3):

$$z_1 = (130 \text{ m}) + \frac{(1.58 \text{ m/s})^2}{2(9.81)\text{m/s}^2}$$

$$\left(0.036\frac{(197 \text{ m})}{(0.15 \text{ m})} + 2(0.19) + 0.5 + 1.0\right)$$

$$\boxed{z_1 = 136 \text{ m}}$$