Chapter 6
Momentum equation

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Significant learning outcomes

Conceptual Knowledge

- Explain the steps in deriving the momentum equation.
- Define an inertial reference frame.
- Identify the accumulation and momentum flux terms in the momentum equation.

Procedural Knowledge

- Apply the component form of the momentum equation to stationary and moving control volumes.

Typical Applications

- For jets, vanes, nozzles and pipe sections, calculate forces and moments.

6.1 Momentum equation: Derivation

- **Newton's Second Law** : The net force acting on a body is equal to the product of mass and acceleration of the body or
- The net force acting on a body is equal to the rate of change of momentum of the body

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})_{sys}}{dt}$$

6.1 Momentum equation: Derivation

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho d\forall + \int_{cs} b\rho \vec{V}.d\vec{A}$$

$$B = m\vec{V}, \quad b = \vec{V}$$

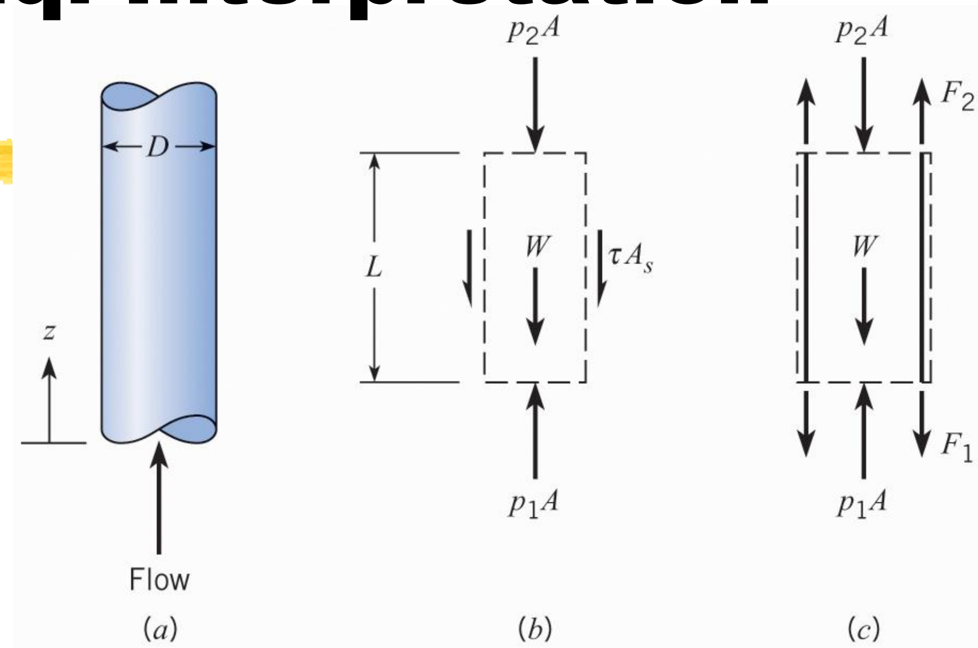
$$\frac{d(m\vec{V})}{dt} = \frac{d}{dt}\int_{cv} \vec{V}\rho d\forall + \int_{cs} \vec{V}\rho \vec{V}.d\vec{A}$$

$$\boxed{\sum \vec{F} = \frac{d}{dt}\int_{cv} \vec{V}\rho d\forall + \int_{cs} \vec{V}\rho \vec{V}.d\vec{A}}$$

6.2 Momentum Eq: Interpretation

- **Force diagrams**
  - Body forces
  - Surface forces

Considering the fluid in order to calculate shear stress (Fig. b)



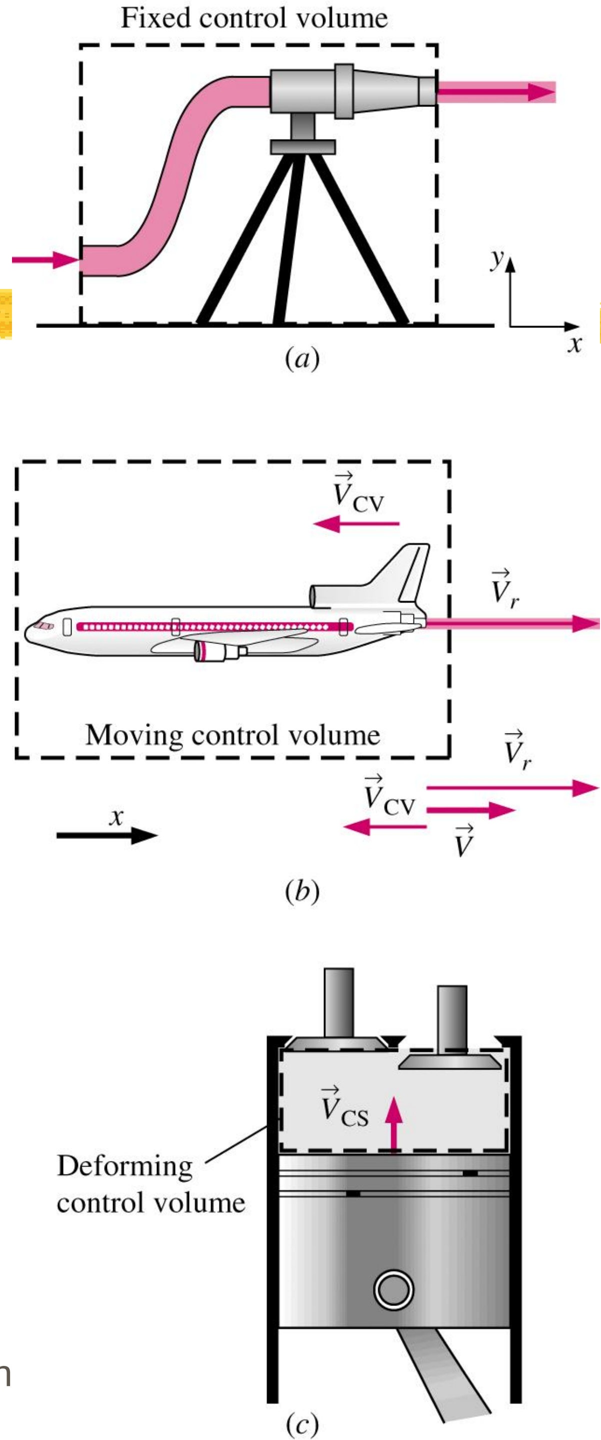
$$\sum F = (p_1 - p_2)\frac{\pi}{4}D^2 - \tau\pi DL - \gamma(\pi D^2/4)L$$

In order to calculate tensile forces in the pipe wall (Fig. c)

$$\sum F = (p_1 - p_2)\frac{\pi}{4}D^2 - -F_1 - F_2 - (W_p + \gamma(\pi D^2/4)L)$$

Choosing a control volume

- Fixed and moving control volumes
- Choice of control surfaces



(a) (b) (c)

Systematic approach

Problem Setup

Select an appropriate control volume. Sketch the control volume and coordinate axes. Select an inertial reference frame.

Identify governing equations.

Force Analysis and Diagram

Sketch body force(s) (gravity, buoyancy) on the force diagram.

Sketch surface forces on the force diagram (forces due to pressure distribution, shear stress distribution, and supports and structures).

Momentum Analysis and Diagram

Evaluate the momentum accumulation term. If the flow is steady and other materials in the control volume are stationary, the momentum accumulation is zero. Otherwise, the momentum accumulation term is evaluated by integration, and an appropriate vector is added to the momentum diagram.

Sketch momentum flow vectors on the momentum diagram.

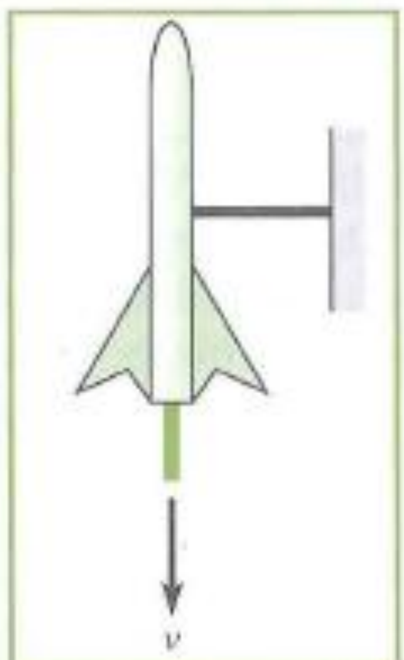
6.3 Applications

- Fluid jets
- Nozzles
- Vanes
- Pipe bends
- Drag force on wind tunnel models
- Force on a sluice gate

EXAMPLE 6.1 THRUST OF ROCKET

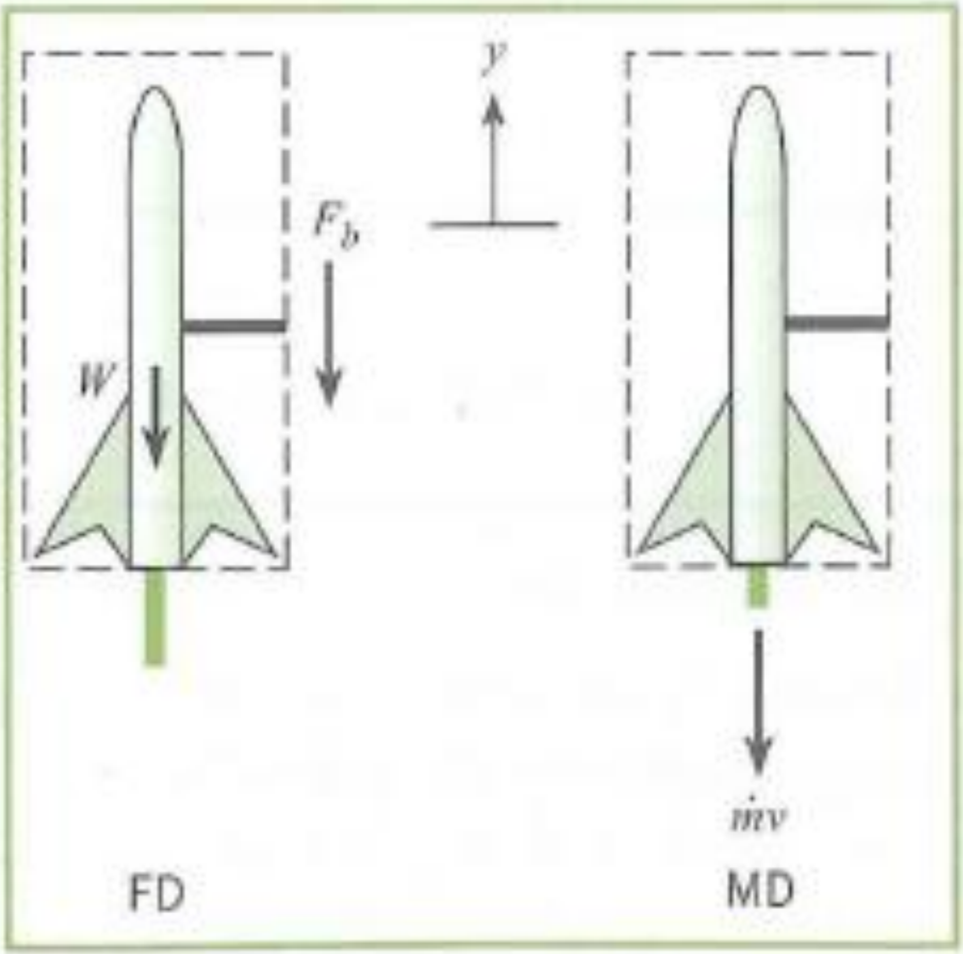
The sketch below shows a 40 g rocket, of the type used for model rocketry, being fired on a test stand in order to evaluate thrust. The exhaust jet from the rocket motor has a diameter of $d = 1$ cm, a speed of $v = 450$ m/s, and a density of $\rho = 0.5$ kg/m^3. Assume the pressure in the exhaust jet equals ambient pressure, and neglect any momentum changes inside the rocket motor. Find the force F_b acting on the beam that supports the rocket.

Sketch:



Solution

1. The control volume chosen is shown below. The control volume is stationary.



2. From the force diagram, the force on the control surface exerted by the beam is chosen as downward (negative z-direction) with magnitude F_b. (The corresponding force exerted by the rocket on the beam is upward.) The weight also acts downward. Also there is no pressure force at the nozzle exit plane because exit pressure is atmospheric.
3. The momentum diagram shows only one momentum outflow and no inflow.
4. Momentum equation in z-direction:

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho \, d\mathcal{V} + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz}$$

5. Sum of forces.

$$\sum F_z = (-F_b - W)$$

$$= -F_b - mg$$

6. Evaluation of momentum terms.

- Accumulation term: No changes in control volume,

$$\frac{d}{dt}\int_{cv} v_z \rho \, d\forall = 0.$$

- Momentum inflow: No inflow, $\sum_{cs} \dot{m}_i v_{iz} = 0.$

- Momentum outflow: $\sum_{cs} \dot{m}_o v_{oz} = \dot{m}(-v) = -\rho A v^2$

7. Force on beam:

$$-F_b - mg = -\rho A v^2$$

$$F_b = \rho A v^2 - mg$$

$$= (0.5 \text{ kg/m}^3)(\pi \times 0.01^2 \text{ m}^2/4)(450^2 \text{ m}^2/\text{s}^2)$$

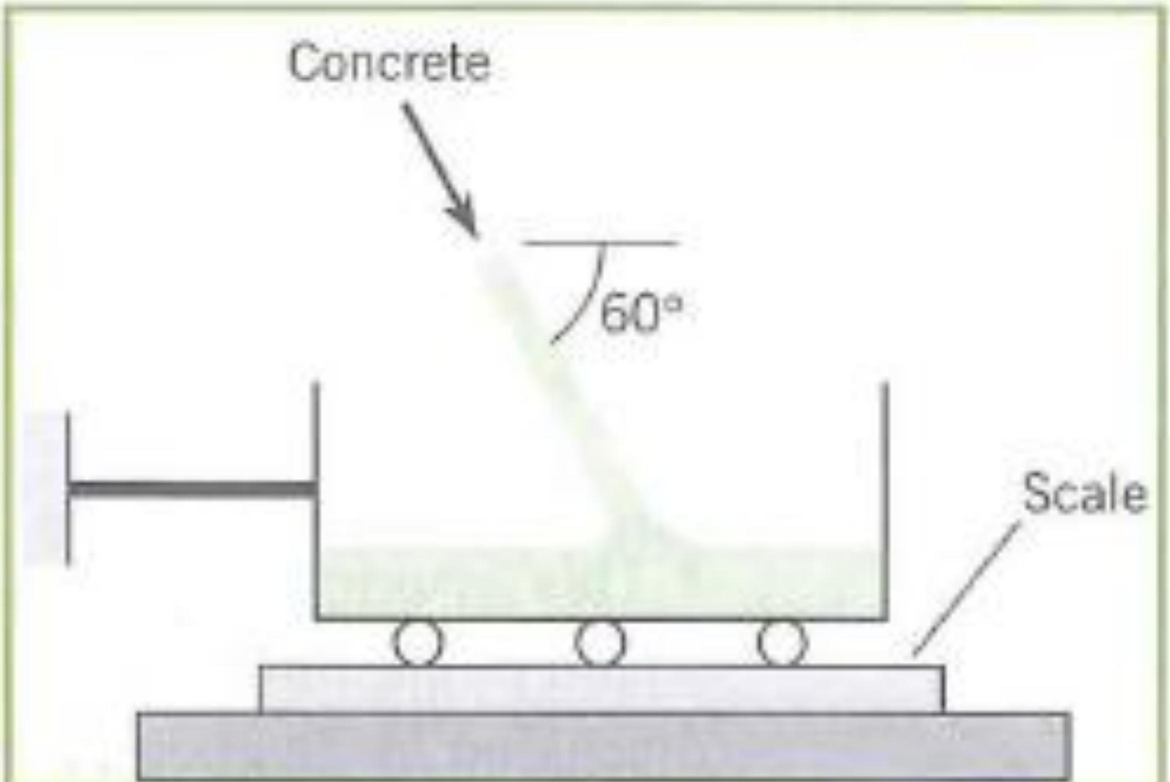
$$- (0.04 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_b = \boxed{7.56 \text{ N}}$$

The direction of F_b (on the beam) is upward.

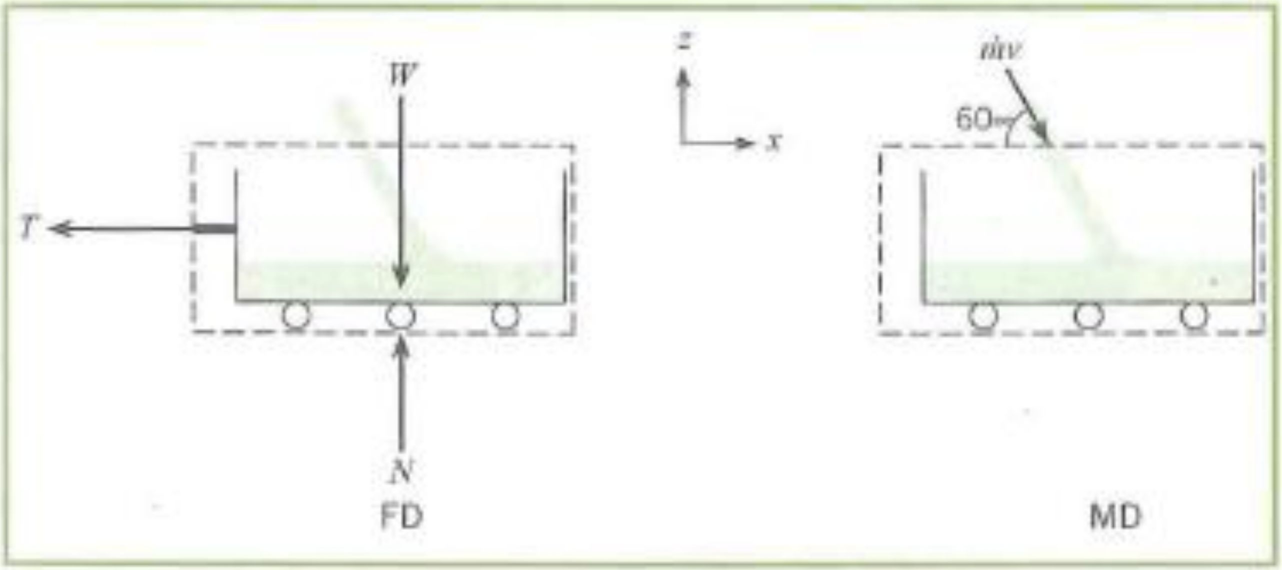
EXAMPLE 6.2 CONCRETE FLOWING INTO CART

As shown in the sketch, concrete flows into a cart sitting on a scale. The stream of concrete has a density of $\rho = 2400\ \mathrm{kg/m^3}$, an area of $A = 0.1\ \mathrm{m^2}$, and a speed of $v = 3\ \mathrm{m/s}$. At the instant shown, the weight of the cart plus the concrete is 3600 N. Determine the tension in the cable and the weight recorded by the scale. Assume steady flow.



Solution

1. Control volume selected is shown on diagram. Control volume is stationary.



2. Force diagram shows the tension in the cable and the weight on the scale.
3. Momentum diagram shows only an inflow of momentum. Velocity of the concrete in the tank is neglected.
4. Component momentum equations
   - Momentum equation in x-direction

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \qquad \text{(a)}$$

- Momentum equation in z-direction

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho \, d\forall + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz} \qquad \text{(b)}$$

5. Forces from the force diagram

$$\sum F_x = -T$$

$$\sum F_z = N - W$$

6. Evaluation of momentum terms

- Momentum accumulation: $v_x = 0$, $v_z = 0$, so

$$\frac{d}{dt}\int_{cv} v_x \rho \, d\forall = 0, \ \frac{d}{dt}\int_{cv} v_z \rho \, d\forall = 0.$$

- Momentum inflow

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v \cos 60° = \rho A v^2 \cos 60°$$

$$\sum_{cs} \dot{m}_i v_{iz} = m(-v \sin 60°) = -\rho A v^2 \sin 60°$$

- Momentum outflow: No outflow, so,

$$\sum_{cs} \dot{m}_o v_{ox} = 0, \text{ and } \sum_{cs} \dot{m}_o v_{oz} = 0.$$

7. Evaluate tension in cable using (a).

$$-T = -\rho A v^2 \cos 60°$$

$$T = (2400\ \text{kg/m}^3)(0.1\ \text{m}^2)(3\ \text{m/s})^2 \cos 60°$$

$$= \boxed{1080\ \text{N}}$$

Evaluate force on scale using (b).

$$N - W = -(-\rho A v^2 \sin 60°)$$

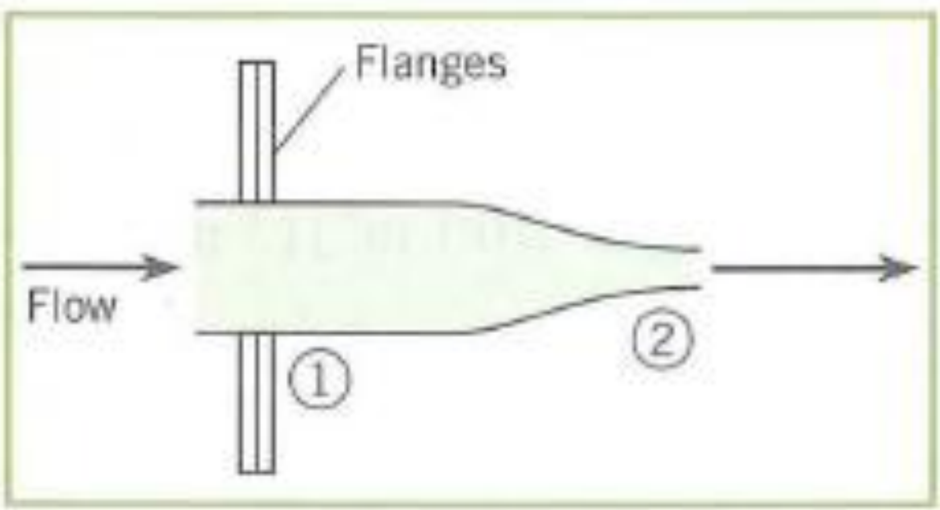
$$N = W + \rho A v^2 \sin 60°$$

$$= 3600\ \text{N} + 1871\ \text{N} = \boxed{5471\ \text{N}}$$

EXAMPLE 6.3 FORCE ON A NOZZLE

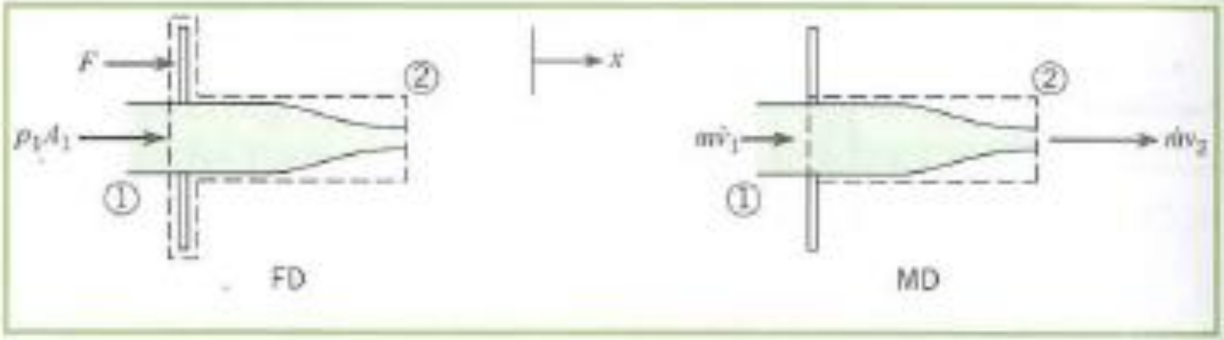
The sketch shows air flowing through a nozzle. The inlet pressure is $p_1 = 105$ kPa abs, and the air exhausts into the atmosphere, where the pressure is 101.3 kPa abs. The nozzle has an inlet diameter of 60 mm and an exit diameter of 10 mm, and the nozzle is connected to the supply pipe by flanges. Find the air speed at the exit of the nozzle and the force required to hold the nozzle stationary. Assume the air has a constant density of 1.22 kg/m^3. Neglect the weight of the nozzle.

Sketch:



Solution

1. Select control volume (and control surface). Control volume is stationary.



2. Force diagram shows force due to pressure and force from flange.
3. Momentum diagram shows a momentum inflow and outflow.
4. Application of the Bernoulli equation between sections 1 and 2

$$p_1 + \gamma z_1 + \frac{1}{2}\rho v_1^{\,2} = p_2 + \gamma z_2 + \frac{1}{2}\rho v_2^{\,2}$$

- Set $z_1 = z_2$.
- Set $p_2 = 0$ kPa gage, and now

$$p_1 = 105\text{ kPa} - 101.3\text{ kPa} = 3.7\text{ kPa gage.}$$

The Bernoulli equation simplifies to

$$p_1 + \rho v_1^2/2 = \rho v_2^2/2$$

From the continuity equation,

$$v_1 A_1 = v_2 A_2$$

$$v_1 d_1^2 = v_2 d_2^2$$

Substitute into the Bernoulli equation and solve for v_2:

$$v_2 = \sqrt{\frac{2p_1}{\rho(1-(d_2/d_1)^4)}}$$

Evaluate exit velocity:

$$v_2 = \sqrt{\frac{2 \times 3.7 \times 1000 \text{ Pa}}{(1.22 \text{ kg/m}^3)(1-(10/60)^4)}} = 77.9 \text{ m/s}$$

Inlet velocity is

$$v_1 = v_2\left(\frac{d_2}{d_1}\right)^2$$

$$= 77.9 \text{ m/s} \times \left(\frac{1}{6}\right)^2 = 2.16 \text{ m/s}$$

5. Momentum equation

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\mathcal{V} + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

6. Sum of forces in x-direction

$$\sum F_x = F + p_1 A_1$$

7. Term-by-term evaluation of momentum terms
   - Accumulation term: Flow is steady,

$$\frac{d}{dt}\int_{cv} v_x \rho \, d\mathcal{V} = 0.$$

   - Momentum outflux with one outflow at section 2,

$$\sum_{cs} \dot{m}_o v_{ox} = \dot{m} v_2.$$

   - Momentum influx with one inlet at section 1,

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_1.$$

8. Force on flange

$$F + p_1A_1 = \dot{m}(v_2 - v_1)$$

$$F = \rho A_1 v_1(v_2 - v_1) - p_1A_1$$

$$= (1.22\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.06\ \text{m})^2(2.16\ \text{m/s})$$

$$\times (77.9 - 2.16)(\text{m/s})$$

$$-3.7 \times 1000\ \text{N/m}^2 \times \left(\frac{\pi}{4}\right)(0.06\ \text{m})^2$$

$$= 0.564\ \text{N} - 10.46\ \text{N} = \boxed{-9.90\ \text{N}}$$

Because F is negative, the direction is opposite to the direction assumed on the force diagram. Hence, the force on the control surface acts in the negative x-direction, but the force on the flange will be in the positive direction.

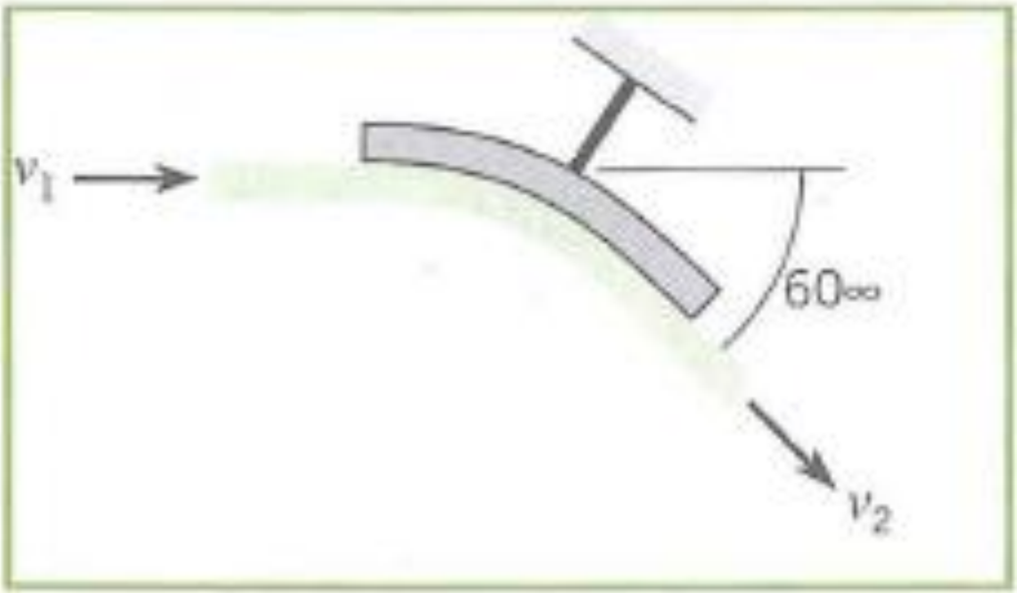
$$\text{Force on flange} = \boxed{9.90\ \text{N}}$$

The tension in the bolts holding the flange will be increased.

EXAMPLE 6.4 WATER DEFLECTED BY A VANE

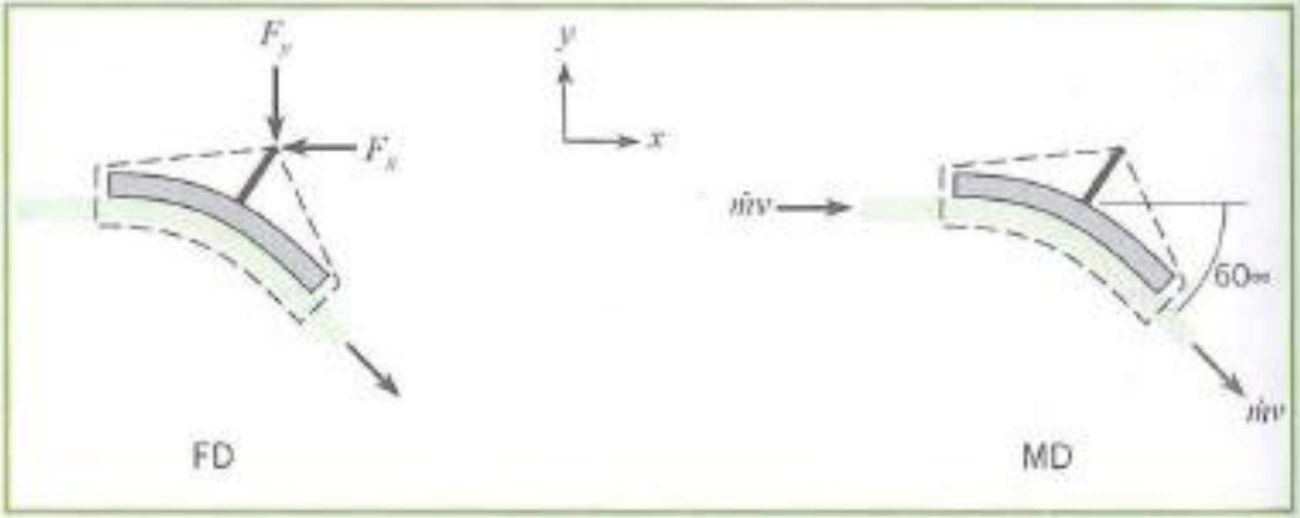
A water jet is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 30 m/s and a diameter of 3 cm. Find the force exerted by the jet on the vane. Neglect the influence of gravity.

Sketch:



Solution

1. The control volume selected is shown in the sketch. The control volume is stationary.



2. The force diagram shows only the reaction force.
3. The momentum diagram shows an inflow and outflow.
4. Vector form of momentum equation.

$$\sum \boldsymbol{F} = \frac{d}{dt}\int_{cv} \rho \boldsymbol{v}\, dV + \sum_{cs} \dot{m}_o \boldsymbol{v}_o - \sum_{cs} \dot{m}_i \boldsymbol{v}_i$$

5. Force vector is

$$\sum \boldsymbol{F} = -F_x \mathbf{i} - F_y \mathbf{j}$$

6. Evaluation of momentum terms

- Control volume is stationary, $\dfrac{d}{dt}\int_{cv} \rho \mathbf{v}\, d\mathcal{V} = 0$
- Momentum outflow vector,

$$\sum_{cs} \dot{m}_o \mathbf{v}_o = [(\dot{m}v \cos 60°)\mathbf{i} - (\dot{m}v \sin 60°)\mathbf{j}].$$

- Momentum inflow vector, $\sum_{cs} \dot{m}_i \mathbf{v}_i = \dot{m}v\mathbf{i}.$

7. Mass flow rate

$$\begin{aligned}\dot{m} &= \rho A v \\ &= 1000\ \text{kg/m}^3(\pi \times 0.015^2\ \text{m}^2)(30\ \text{m/s}) \\ &= 21.21\ \text{kg/s}\end{aligned}$$

8. Force

$$-F_x\mathbf{i} - F_y\mathbf{j} = (\dot{m}v \cos 60° - \dot{m}v)\mathbf{i} - (\dot{m}v \sin 60°)\mathbf{j}$$

For each component,

$$-F_x = \dot{m}v\cos 60^\circ - \dot{m}v$$

$$-F_y = -\dot{m}v\sin 60^\circ$$

Force in x-direction

$$F_x = \dot{m}v(1-\cos 60^\circ)$$
$$= (21.21\ \text{kg/s})(30\ \text{m/s})(1-\cos 60^\circ)$$
$$F_x = \boxed{318.15\ \text{N}}$$

Force in y-direction

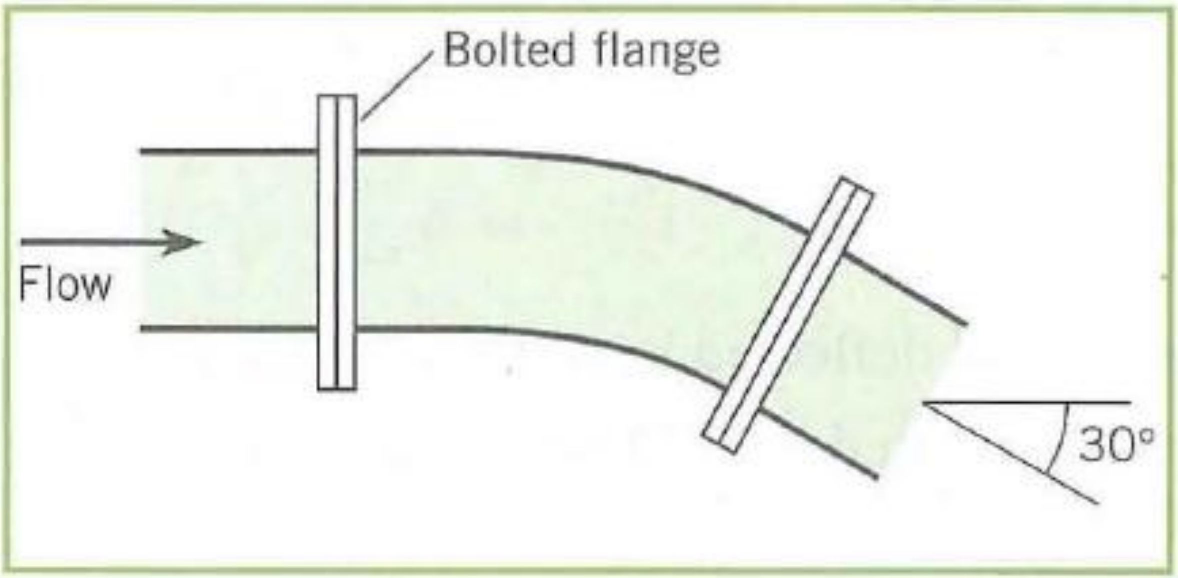
$$F_y = \dot{m}v\sin 60^\circ$$
$$= (21.21\ \text{kg/s})(30\ \text{m/s})\sin 60^\circ$$
$$F_y = \boxed{551.05\ \text{N}}$$

The force of the jet on the vane ($\mathbf{F}_{jet}$) is opposite in direction to the force required to hold the vane stationary ($\mathbf{F}$). Therefore,

$$\mathbf{F}_{jet} = (318.15\ \text{N})\mathbf{i} + (551.05\ \text{N})\mathbf{j}$$

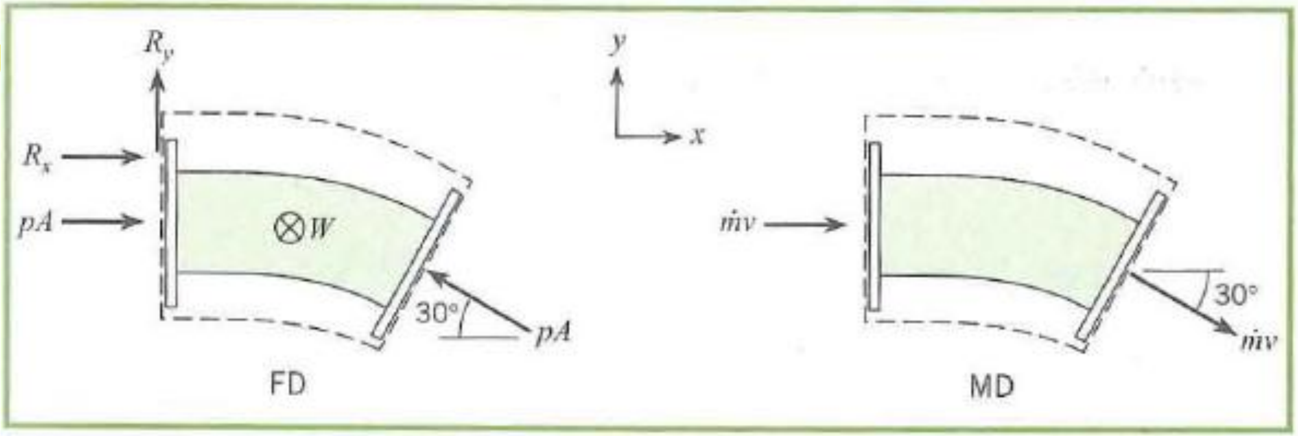
EXAMPLE 6.6 FORCES ACTING ON A PIPE BEND

A 1 m–diameter pipe bend shown in the diagram is carrying crude oil ($S = 0.94$) with a steady flow rate of 2 m^3/s. The bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is 1.2 m^3, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the net force required to hold the bend in place.



Solution

1. The control volume selected is shown. The control volume is stationary. The z-direction is outward from the page.



2. The force diagram shows pressure forces and the component reaction forces.
3. Vector form of momentum equation

$$\sum \boldsymbol{F} = \frac{d}{dt}\int_{cv} \rho \boldsymbol{v} \, dV + \sum_{cs} \dot{m}_o \boldsymbol{v}_o - \sum_{cs} \dot{m}_i \boldsymbol{v}_i$$

4. Sum of the forces: The weight of the pipe and fluid therein is W and acts in the negative z-direction.

$$\sum \boldsymbol{F} = (R_x + pA - pA\cos 30°)\mathbf{i} + (R_y + pA\sin 30°)\mathbf{j}$$
$$+ (R_z - W)\mathbf{k}$$

5. Momentum terms

- Accumulation term for stationary control volume is

$$\frac{d}{dt}\int_{cv} \rho \mathbf{v}\, d\mathcal{V} = 0.$$

- Momentum outflow is

$$\sum_{cs} \dot{m}_o \mathbf{v}_o = (\dot{m} v \cos 30°)\mathbf{i} - (\dot{m} v \sin 30°)\mathbf{j}.$$

- Momentum inflow is $\sum_{cs} \dot{m}_i \mathbf{v}_i = (\dot{m} v)\mathbf{i}$.

6. Reaction force

$$(R_x + pA - pA\cos 30°)\mathbf{i} + (R_y + pA\sin 30°)\mathbf{j} + (R_z - W)\mathbf{k}$$
$$= [mv(\cos 30 - 1)]\mathbf{i} - (\dot{m} v \sin 30°)\mathbf{j}$$

- Equating components

$$R_x + pA - pA\cos 30° = \dot{m} v \cos 30° - \dot{m} v$$
$$R_y + pA\sin 30° = -\dot{m} v \sin 30°$$
$$R_z - W = 0$$

- Pressure force

$$pA = (75\ \text{kN/m}^2)(\pi \times 0.5^2\ \text{m}^2) = 58.9\ \text{kN}$$

- Fluid speed

$$v = Q/A = \frac{(2\ \text{m}^3/\text{s})}{(\pi \times 0.5^2\ \text{m}^2)} = 2.55\ \text{m/s}$$

- Momentum flux

$$\dot{m}v = \rho Q v = (0.94 \times 1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(2.55\ \text{m/s})$$
$$= 4.79\ \text{kN}$$

Reaction force in x-direction

$$R_x = -(pA + \dot{m}v)(1 - \cos 30°)$$
$$= -(58.9 + 4.79)(\text{kN})(1 - \cos 30°) = \boxed{-8.53\ \text{kN}}$$

Reaction force in *y*-direction

$$R_y = -(pA + \dot{m}v)\sin 30°$$

$$= -(58.9 + 4.79)(\text{kN})(\sin 30°) = \boxed{-31.8\text{ kN}}$$

Reaction force in *z*-direction. (The bend weight includes the oil plus the empty pipe).

$$W = \gamma \mathcal{V} + 4\text{ kN}$$

$$= (0.94 \times 9.81\text{ kN/m}^3)(1.2\text{ m}^3) + 4\text{ kN} = \boxed{15.1\text{ kN}}$$

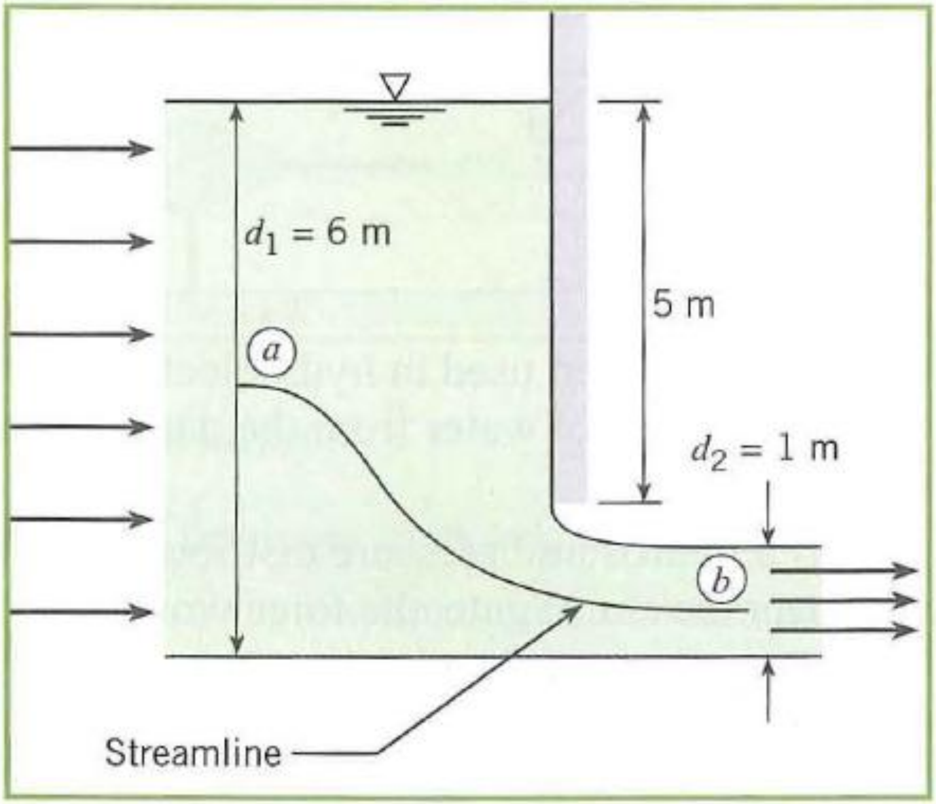
Reaction force vector

$$\mathbf{R} = (-8.53\text{ kN})\mathbf{i} + (-31.8\text{ kN})\mathbf{j} + (15.1\text{ kN})\mathbf{k}$$

EXAMPLE 6.9 FORCE ON A SLUICE GATE

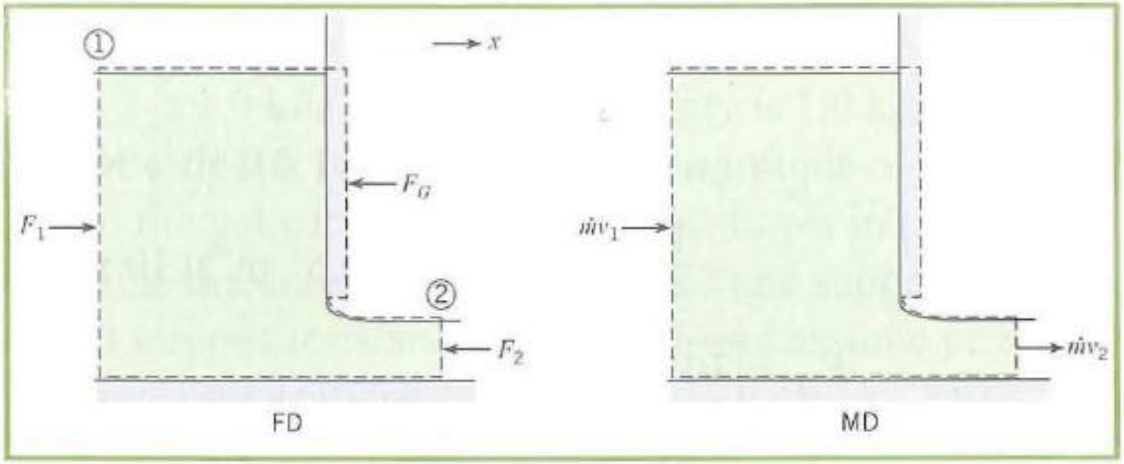
A sluice gate is used to control the water flow rate over a dam. The gate is 6 m wide, and the depth of the water above the bottom of the sluice gate is 5 m. The depth of the water upstream of the gate is 6 m, and the depth downstream is 1 m. Estimate the flow rate under the gate and the force on the gate. The water density is 1000 kg/m^3.

Sketch:



Solution

1. The control volume selected is shown. The control volume is stationary.



2. The force diagram shows forces due to pressure and the force on the gate.
3. The momentum diagram shows an influx and outflux of momentum.
4. Component form of momentum equation in x-direction.

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

5. The Bernoulli equation between points a and b along the streamline.

$$\frac{p_a}{\gamma} + z_a + \frac{v_1^2}{2g} = \frac{p_b}{\gamma} + z_b + \frac{v_2^2}{2g}$$

The piezometric pressure is constant across sections 1 and 2, so $\frac{p_a}{\gamma}+z_a=d_1$ and $\frac{p_b}{\gamma}+z_b=d_2$. From continuity equation, Eq. (5.27), $v_1 d_1 w = v_2 d_2 w$ where w is the flow width. Combine the Bernoulli and continuity equations.

$$2g(d_1-d_2)=v_2^2-v_1^2=v_2^2\left(1-\frac{d_2^2}{d_1^2}\right)$$

$$v_2=\frac{1}{\sqrt{1-\frac{d_2^2}{d_1^2}}}\sqrt{2g(d_1-d_2)}$$

Velocities and discharge

$$v_2=\frac{1}{\sqrt{1-\left(\frac{1}{6}\right)^2}}\sqrt{2\times 9.8\ \text{m/s}^2\ (6-1)\ \text{m}}=10.045\ \text{m/s}$$

$$v_1=\frac{d_2}{d_1}v_2=\frac{1\ \text{m}}{6\ \text{m}}\times 10.045\ \text{m/s}=1.674\ \text{m/s}$$

$$Q=v_2 d_2 w=10.045\ \text{m/s}\times 1\ \text{m}\times 6\ \text{m}=\boxed{60.3\ \text{m}^3/\text{s}}$$

6. Sum of the forces from force diagram

$$\sum F_x = F_1 - F_2 - F_G$$

From equation for force on planar surface, Eq. (3.23), $F = \bar{p}A$,

$$F_1 = \frac{\gamma d_1}{2} d_1 w$$

$$F_2 = \frac{\gamma d_2}{2} d_2 w$$

7. Evaluation of momentum terms

- Accumulation term for steady flow is $\frac{d}{dt}\int_{cv} v_x \rho \, dV = 0$.
- Momentum inflow with one inlet is $\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_1$.
- Momentum outflow with one outlet is $\sum_{cs} \dot{m}_o v_{ox} = \dot{m} v_2$.

8. Force on sluice gate

$$w - \frac{\gamma}{2} d_2^2 w - F_G = \dot{m}(v_2 - v_1)$$

$$F_G = \frac{\gamma}{2} w(d_1^2 - d_2^2) + \rho Q(v_1 - v_2)$$

$$F_G = \frac{9810 \text{ N/m}^3}{2} \times 6 \text{ m}(6^2 - 1^2) \text{ m}^2 + 1000 \text{ kg/m}^3$$

$$\times 60.27 \text{ m}^3/\text{s} \times (1.674 - 10.045)$$

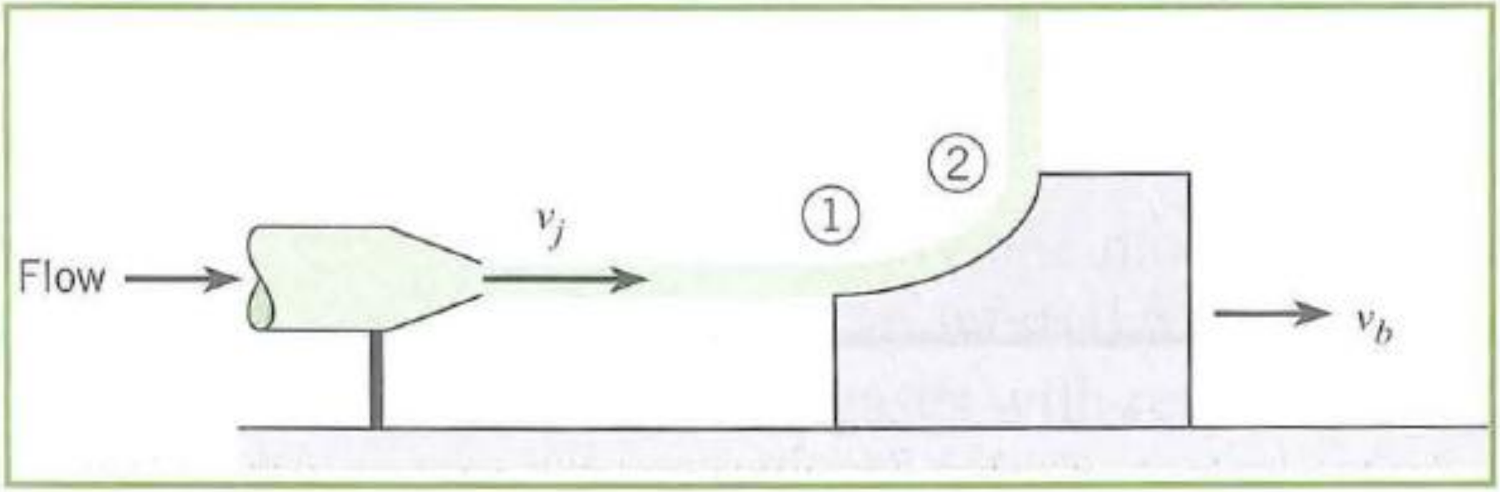
$$= \boxed{526 \text{ kN}}$$

EXAMPLE 6.10 JET IMPINGING ON MOVING BLOCK

A stationary nozzle produces a water jet with a speed of 50 m/s and a cross-sectional area of 5 cm^2. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed of 25 m/s on a surface with friction. The density of the water is 1000 kg/m^3. Find the frictional force F acting on the block.

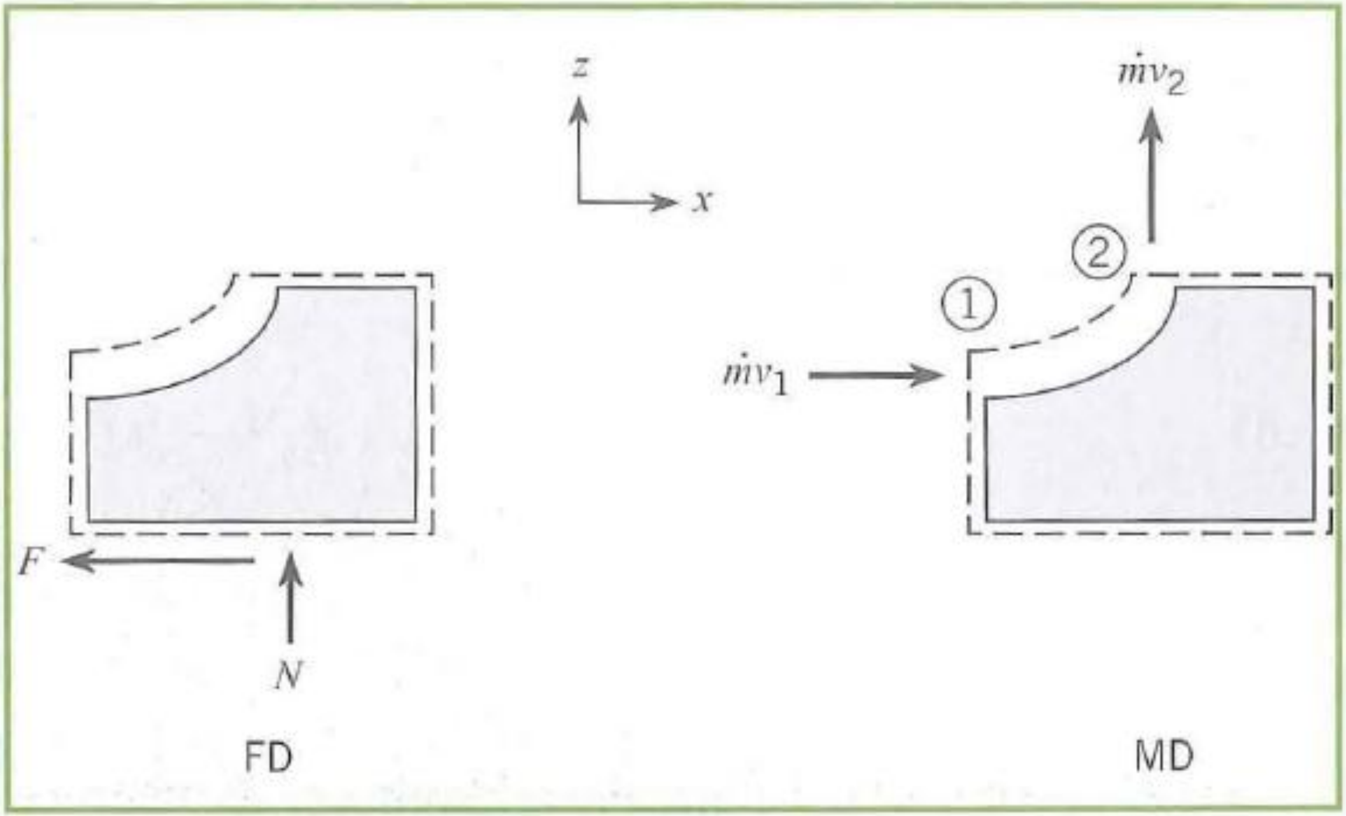
Solve the problem using two different inertial reference frames: (a) the moving block and (b) the stationary nozzle.

Sketch:



Solution

1. The control volume selected is shown in the sketch. The control volume is not stationary.



2. The force diagram shows one force in the horizontal direction.
3. The momentum diagram shows an influx and outflux of momentum.

4. Momentum equation

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\text{V\!\!\!-} + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

5. The sum of the forces

$$\sum F_x = -F$$

6. Evaluation of terms in momentum equation
   (a) Inertial reference frame on cart

- Accumulation term with $v_x = 0$ is $\frac{d}{dt}\int_{cv} v_x \rho \, d\text{V\!\!\!-} = 0$.
- Momentum inflow for x-component of velocity at station 1, $v_{ix} = v_j - v_b$, is

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m}(v_j - v_b)$$

- Momentum outflow for x-component of velocity at station 2, $v_{ox} = 0$, is

$$\sum_{cs} \dot{m}_o v_{ox} = 0$$

(b) Inertial reference frame at nozzle

- Accumulation term with $v_x = v_b =$ constant, is

$$\frac{d}{dt}\int_{cv} v_x \rho \, d\forall = 0.$$

- Momentum inflow for x-component of velocity at station 1, $v_{xi} = v_j$, is

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_j$$

- Momentum outflow for x-component of velocity at station 2, $v_{xo} = v_b$, is

$$\sum_{cs} \dot{m}_o v_{ox} = \dot{m} v_b$$

7. Mass flow rate. Since flow is steady with respect to the block, $\dot{m}_i = \dot{m}_o = \dot{m}$.

$$\dot{m} = \rho A (v_j - v_b)$$

8. Evaluate force.

(a) Moving block as inertial reference frame

$$-F = -\rho A(v_j - v_b)^2$$

$$F = \rho A(v_j - v_b)^2$$

(b) Stationary nozzle as inertial reference frame

$$-F = \dot{m}v_b - \dot{m}v_j = -\dot{m}(v_j - v_b)$$

$$F = \rho A(v_j - v_b)^2$$

Force on cart

$$F_x = (1000\ \text{kg/m}^3)(5 \times 10^{-4}\ \text{m}^2)(50 - 25)^2(\text{m/s})^2$$

$$F_x = \boxed{312\ \text{N}}$$

Review

Note that the same answer for force is obtained independent of the inertial reference frame chosen.