# Chapter 4
# Flowing fluids and pressure variation

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# Significant learning outcomes

## Conceptual Knowledge

- Distinguish between steady, unsteady, uniform, and nonuniform flows.
- Distinguish between convective and local acceleration.
- Describe the steps to derive the Bernoulli equation from Euler's equation.

## Procedural Knowledge

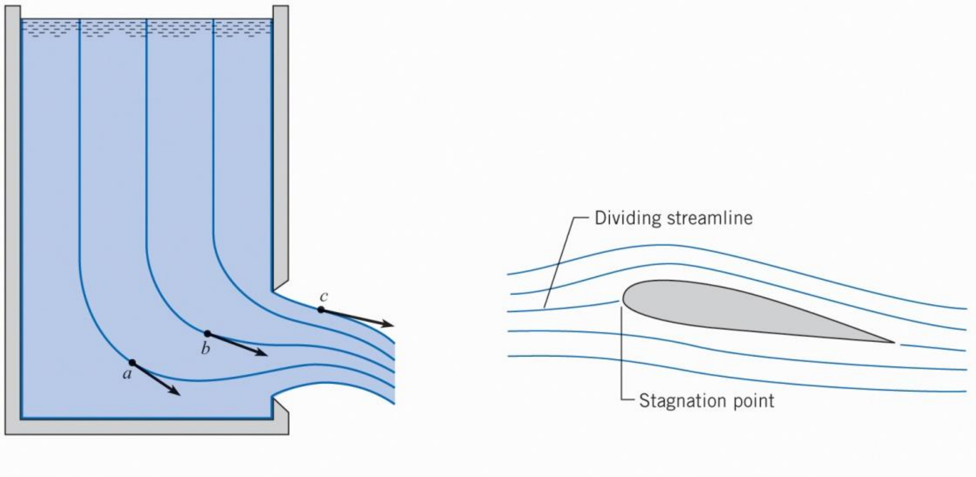
- Apply Euler's equation to predict pressure.
- Apply the Bernoulli equation to pressure and velocity variations.

## Applications (typical)

- In variable area ducts, relate pressure and velocity distributions.
- Measurement of velocity with stagnation tube or a Pitot-static tube.

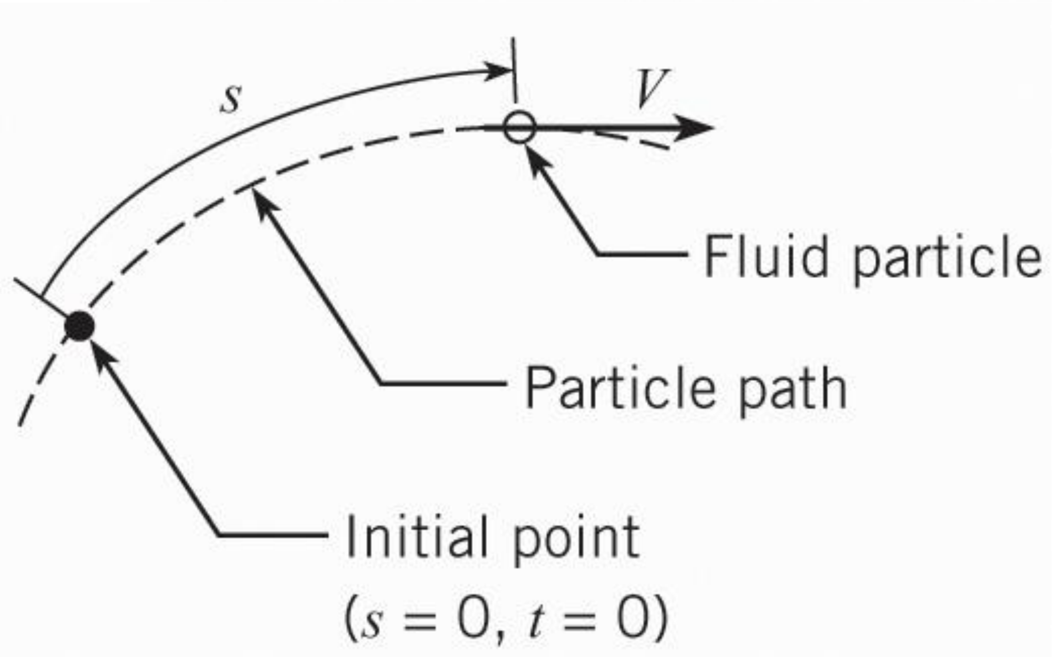
# 4.1 Descriptions of fluid motion

Streamline: *A curve tangent to the instantaneous local velocity everywhere.*

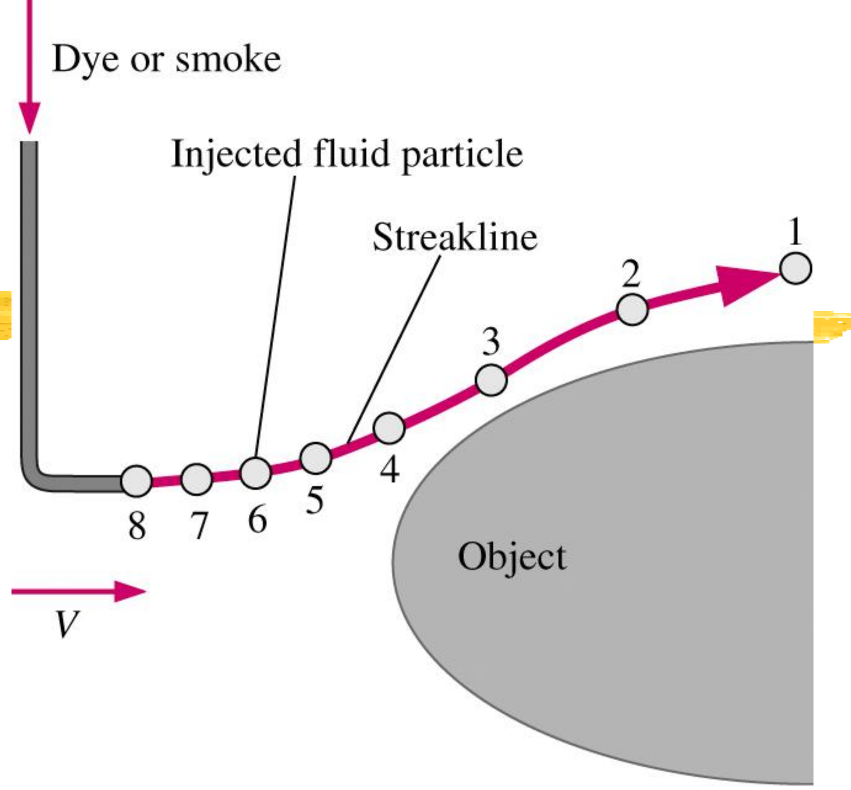



(a) (b)

Pathline: *Actual path traveled by an individual fluid particle*

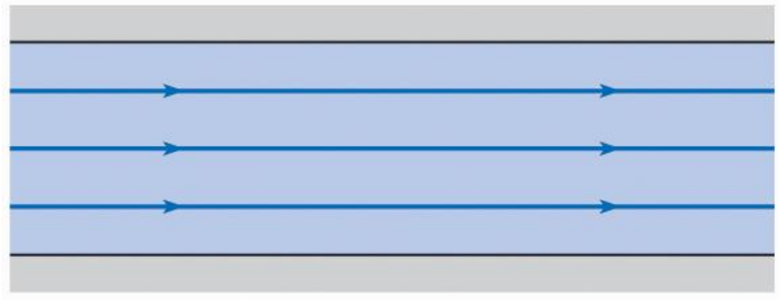


Streakline: *locus of fluid particles that have passed sequentially through a certain point in the flow*



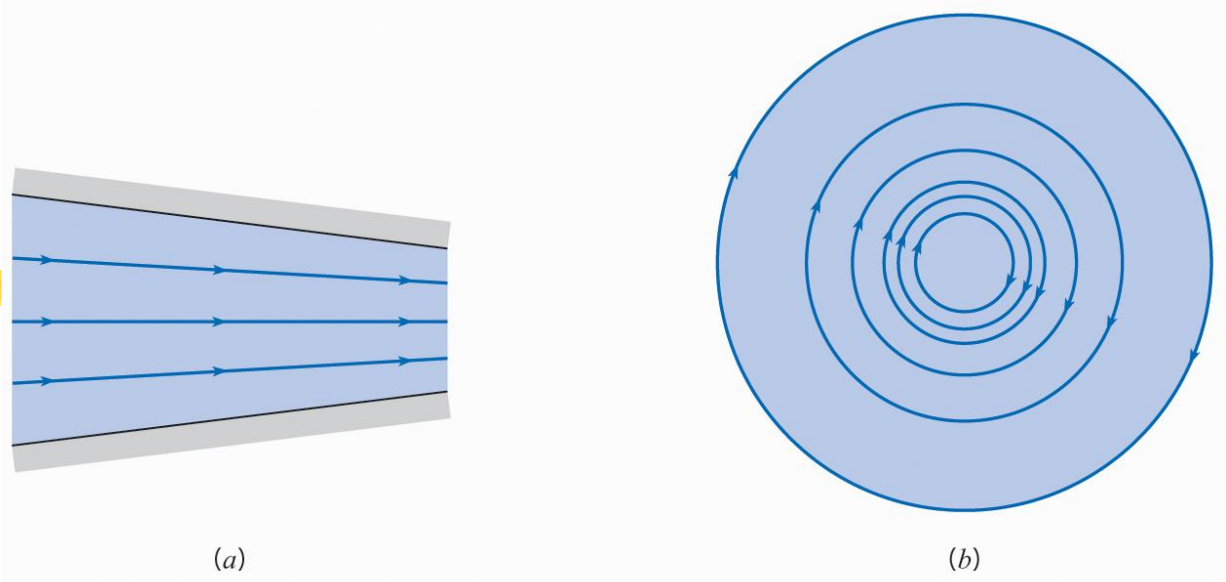

$$\vec{V} = \vec{V}(s,t)$$

Uniform flow $$\frac{\partial \vec{V}}{\partial s} = 0$$



# Nonuniform flow

$$\frac{\partial \vec{V}}{\partial s} \neq 0$$

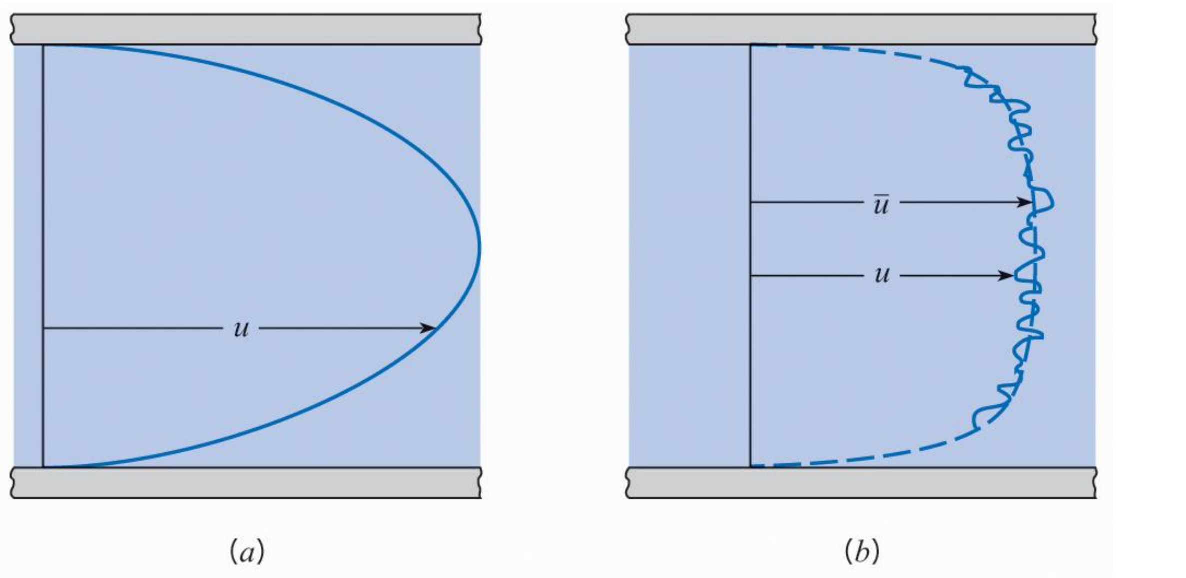


Steady flow $\frac{\partial \vec{V}}{\partial t} = 0$

Unsteady flow $\frac{\partial \vec{V}}{\partial t} \neq 0$

(a) Laminar flow

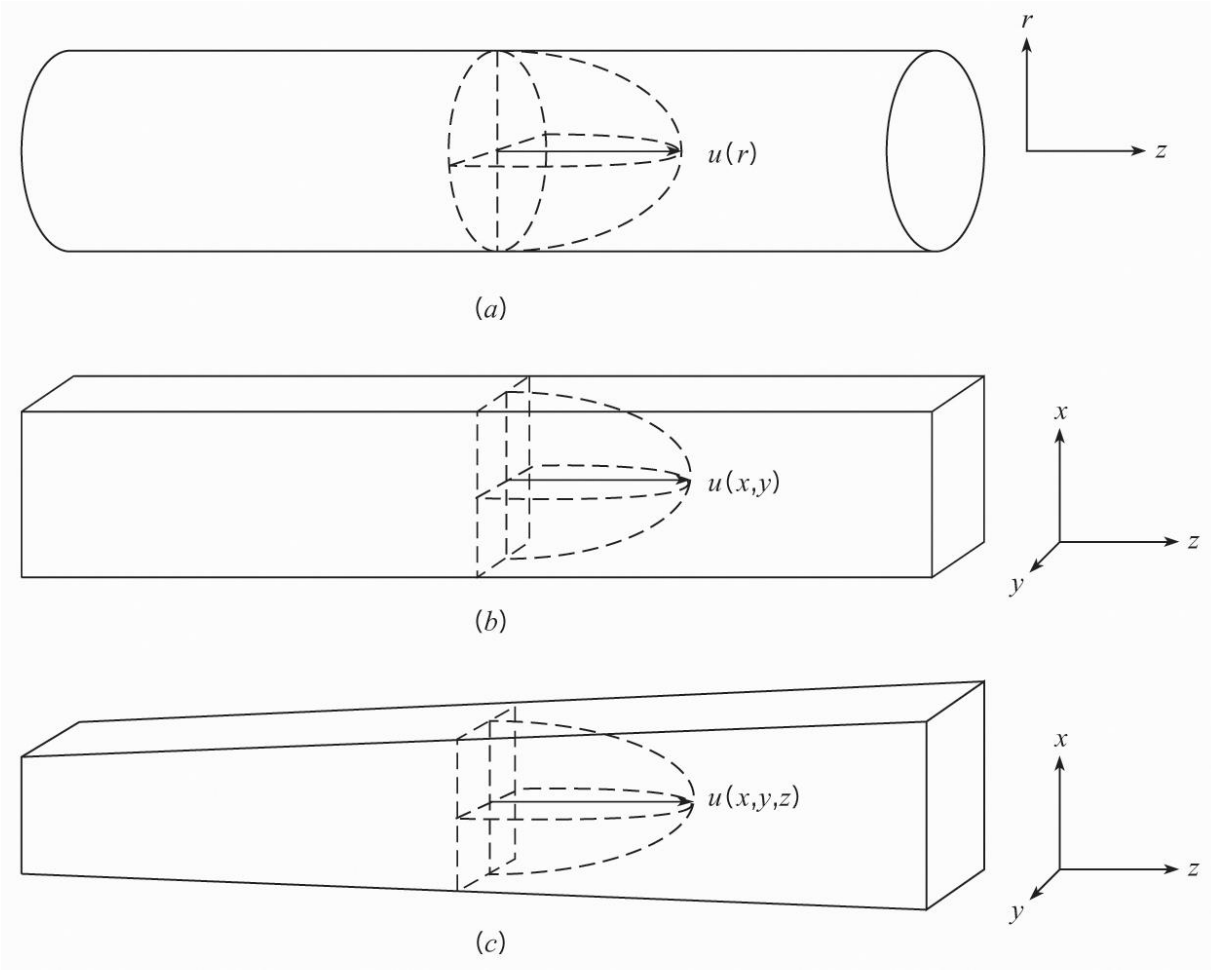
(b) Turbulent flow



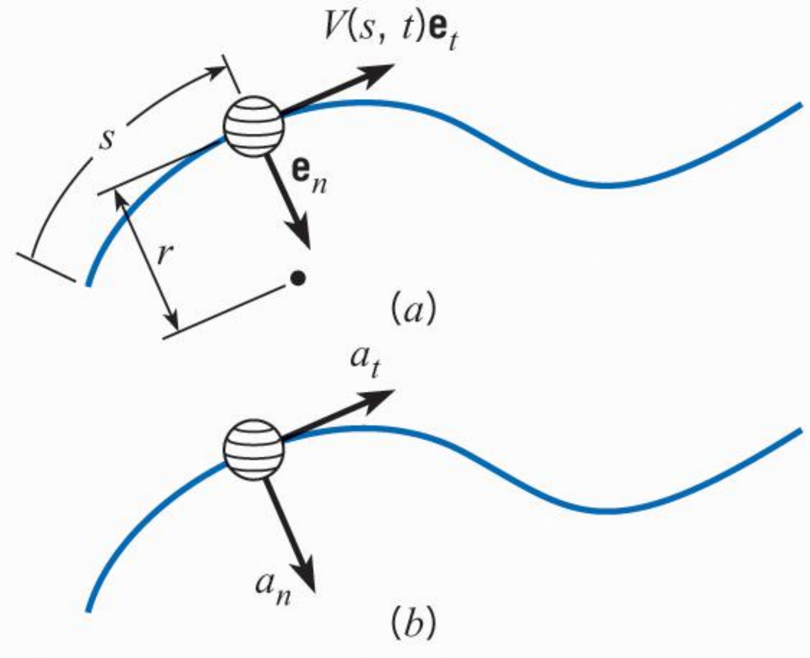
(a) One dimensional flow

(b) Two dimensional flow

(c) Three dimensional flow



# 4.2 Acceleration



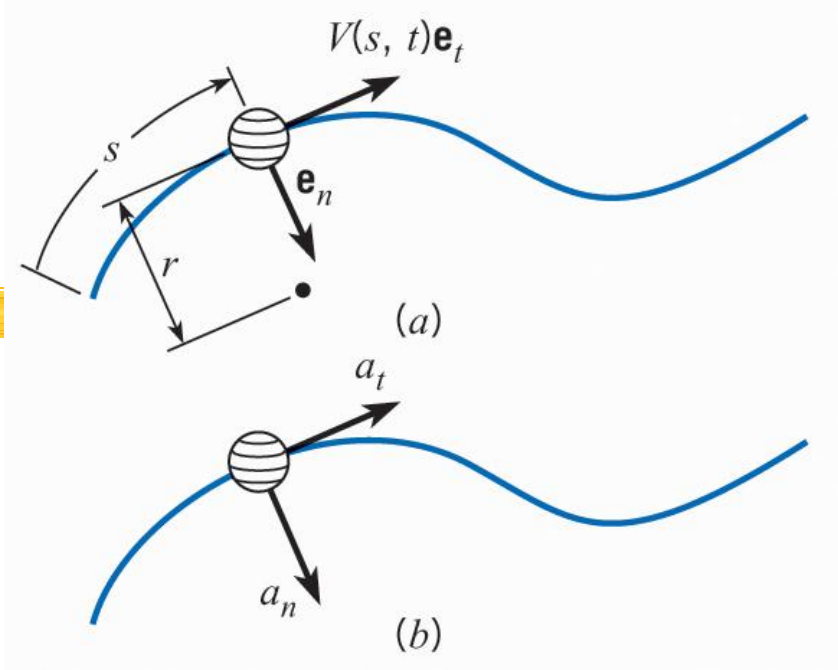

$$\vec{V} = V(s,t)\vec{e}_t$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \left(\frac{dV}{dt}\right)\vec{e}_t + V\left(\frac{d\vec{e}_t}{dt}\right)$$

$$\frac{dV(s,t)}{dt} = \left(\frac{\partial V}{\partial s}\right)\left(\frac{ds}{dt}\right) + \left(\frac{\partial V}{\partial t}\right)$$

$$\frac{dV}{dt} = V\left(\frac{\partial V}{\partial s}\right) + \left(\frac{\partial V}{\partial t}\right)$$

# 4.2 Acceleration




$$\frac{d\vec{e}_t}{dt} = \frac{V}{r}\vec{e}_n$$

$$\vec{a} = \left( \underbrace{V\frac{\partial V}{\partial s}}_{convective} + \underbrace{\frac{\partial V}{\partial t}}_{local} \right)\vec{e}_t + \underbrace{\left( \frac{V^2}{r} \right)}_{centripetal}\vec{e}_n$$

## EXAMPLE 4.1 EVALUATING ACCELERATION IN A NOZZLE

A nozzle is designed such that the velocity in the nozzle varies as

$$u(x) = \frac{u_0}{1.0 - 0.5x/L}$$

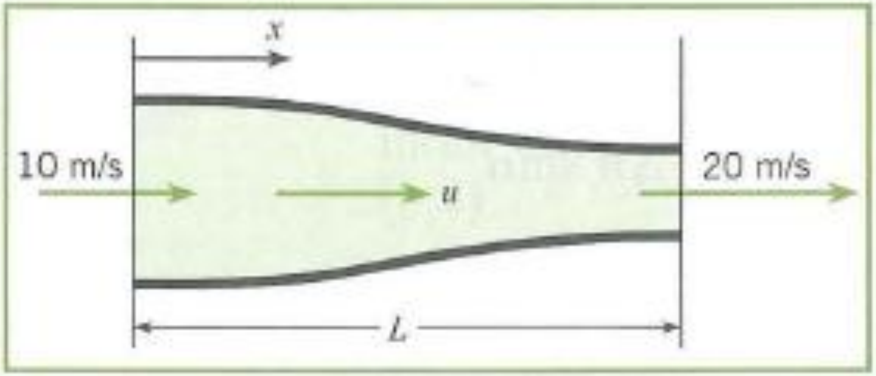
where the velocity $u_0$ is the entrance velocity and $L$ is the nozzle length. The entrance velocity is 10 m/s, and the length is 0.5 m. The velocity is uniform across each section. Find the acceleration at the station halfway through the nozzle $(x/L = 0.5)$.

### *Problem Definition*

**Situation:** Given velocity distribution in a nozzle.

**Find:** Acceleration at nozzle midpoint.

**Sketch:**



**Assumptions:** Flow field is quasi–one-dimensional (negligible velocity normal to nozzle centerline).

*Plan*

1. Select the pathline along the centerline of the nozzle.
2. Evaluate the convective, local, and centripetal accelerations in Eq. (4.5).
3. Calculate the acceleration.

*Solution*

The distance along the pathline is $x$, so $s$ in Eq. 4.5 becomes $x$ and $V$ becomes $u$. The pathline is straight, so $r \longrightarrow \infty$.

1. Evaluation of terms:

- Convective acceleration

$$\frac{\partial u}{\partial x} = -\frac{u_0}{(1-0.5x/L)^2} \times \left(-\frac{0.5}{L}\right)$$

$$= \frac{1}{L}\frac{0.5u_0}{(1-0.5x/L)^2}$$

$$u\frac{\partial u}{\partial x} = 0.5\frac{u_0^2}{L}\frac{1}{(1-0.5x/L)^3}$$

Evaluation at $x/L = 0.5$:

$$u\frac{\partial u}{\partial x} = 0.5 \times \frac{10^2}{0.5} \times \frac{1}{0.75^3}$$

$$= 237\ \text{m/s}^2$$

- Local acceleration

$$\frac{\partial u}{\partial t} = 0$$

- Centripetal acceleration

$$\frac{u^2}{r} = 0$$

2. Acceleration

$$a_x = 237\ \text{m/s}^2 + 0$$

$$= \boxed{237\ \text{m/s}^2}$$

$$a_n\ (\text{normal to pathline}) = \boxed{0}$$

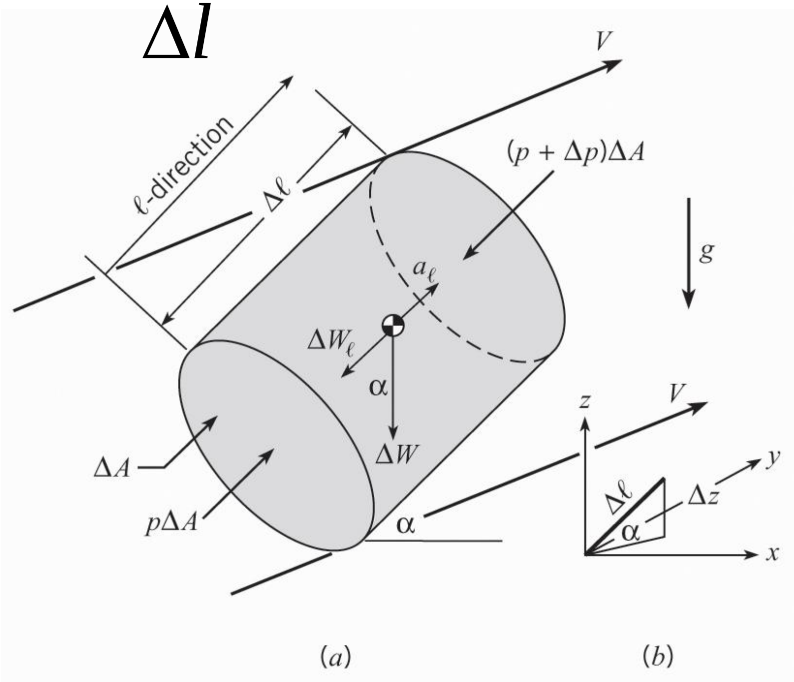
# 4.3 Euler's equation

$$\sum F_l = ma_l$$

$$F_{pressure} + F_{gravity} = ma_l \qquad (m = \rho \Delta A \Delta l)$$

$$F_{pressure} = p\Delta A - (p + \Delta p)\Delta A = -\Delta p \Delta A$$

$$F_{gravity} = -\Delta W_l = -\Delta W \sin\alpha = -\Delta W \frac{\Delta z}{\Delta l}$$




$$-\Delta p \Delta A - \gamma \Delta l \Delta A \frac{\Delta z}{\Delta l} = \rho \Delta A \Delta l a_l$$

$$-\frac{\Delta p}{\Delta l} - \gamma \frac{\Delta z}{\Delta l} = \rho a_l$$

$$-\frac{\partial}{\partial l}(p + \gamma z) = \rho a_l$$

## EXAMPLE 4.2 APPLICATION OF EULER'S EQUATION TO ACCELERATION OF A FLUID

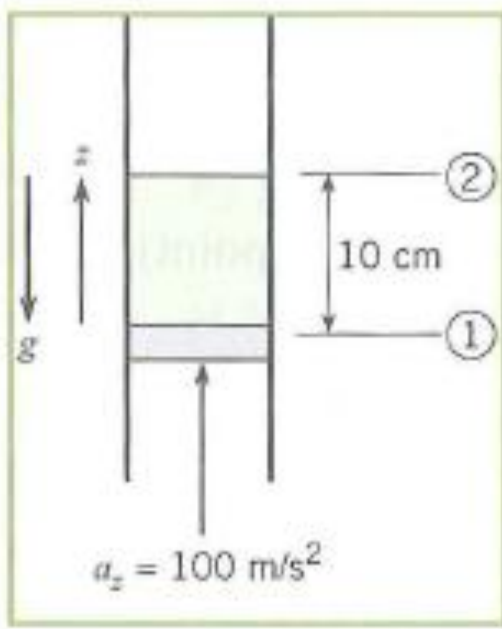
A column water in a vertical tube is being accelerated by a piston in the vertical direction at 100 $m/s^2$. The depth of the water column is 10 cm. Find the gage pressure on the piston. The water density is $10^3$ $kg/m^3$.

*Problem Definition*

**Situation:** A column of water is being accelerated by a piston.

**Find:** The gage pressure on the piston.

**Sketch:**



### Solution

1. Because the acceleration is constant there is no dependence on time so the partial derivative in Euler's equation can be replaced by an ordinary derivative. Euler's equation in $z$-direction:

$$\frac{d}{dz}(p + \gamma z) = -\rho a_z$$

2. Integration between sections 1 and 2:

$$\int_1^2 d(p + \gamma z) = \int_1^2 (-\rho a_z)dz$$

$$(p_2 + \gamma z_2) - (p_1 + \gamma z_1) = -\rho a_z(z_2 - z_1)$$

3. Substitution of limits:

$$p_1 = (\gamma + \rho a_z)\Delta z = \rho(g + a_z)\Delta z$$

4. Evaluation of pressure:

$$p_1 = 10^3 \text{ kg/m}^3 \times (9.81 + 100) \text{ m/s}^2 \times 0.1 \text{ m}$$

$$p_1 = \boxed{10.9 \times 10^3 \text{ Pa} = 10.9 \text{ kPa, gage}}$$

# 4.5 The Bernoulli equation along a streamline

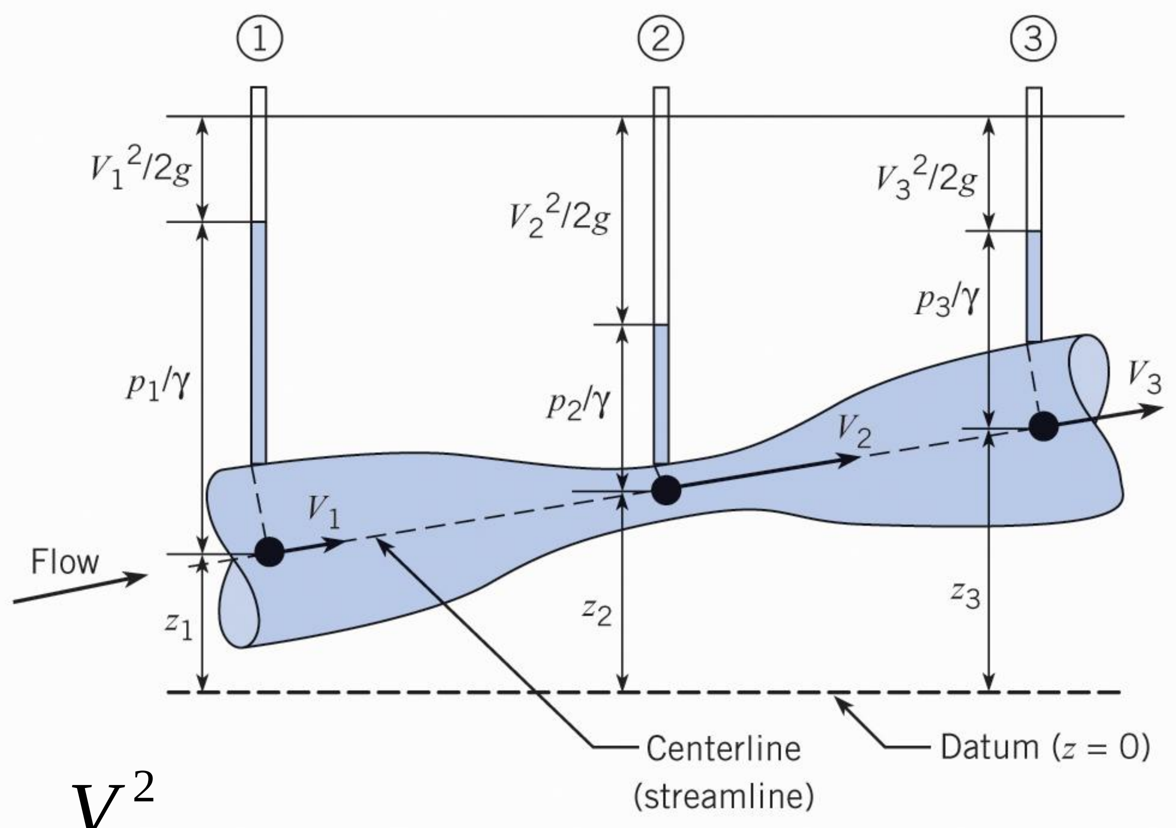
$$-\frac{\partial}{\partial s}\left(p+\gamma z\right)=\rho a_t$$

$$a_t = V\frac{\partial V}{\partial s}+\frac{\partial V}{\partial t}$$

$$-\frac{d}{ds}\left(p+\gamma z\right)=\rho V\frac{\partial V}{\partial s}\quad\left(\frac{\partial V}{\partial t}=0,\ steady\ \ flow\right)$$

$$\frac{d}{ds}\left(p+\gamma z+\rho\frac{V^2}{2}\right)=0$$

$$p+\gamma z+\rho\frac{V^2}{2}=C$$



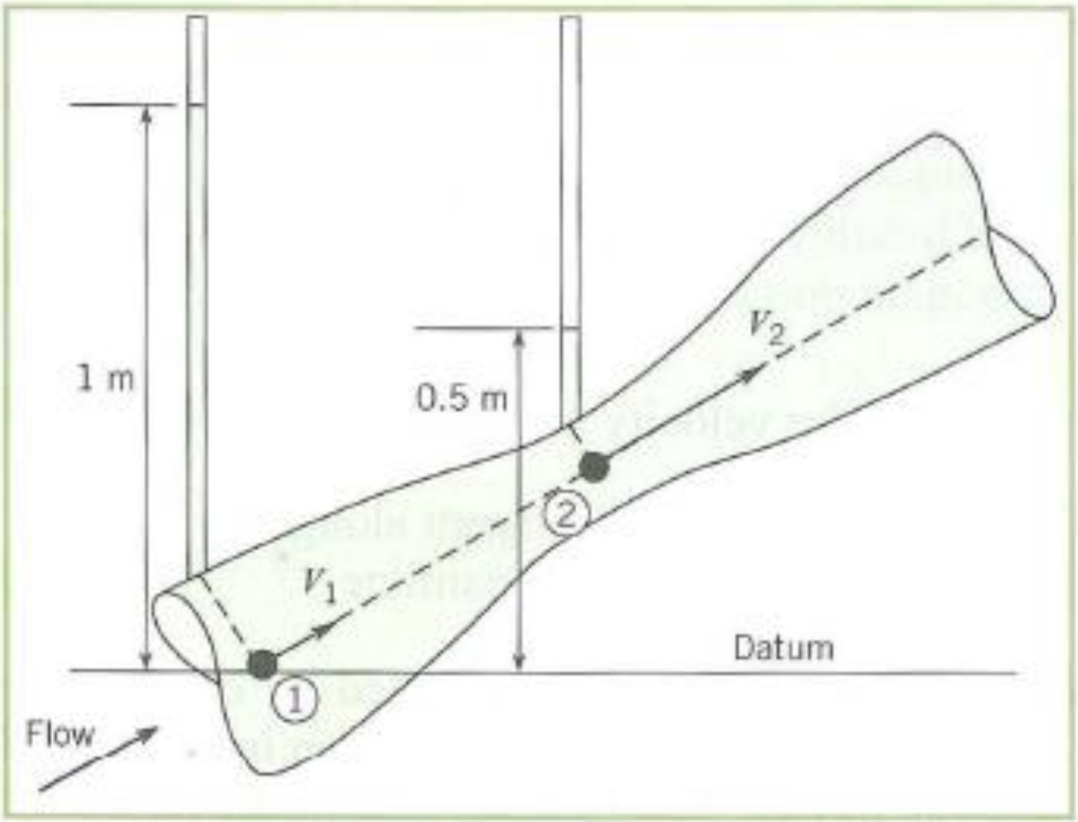

$$\underbrace{\frac{p}{\gamma}}_{\text{pressure head}} + \underbrace{z}_{\text{elevation head}} + \underbrace{\frac{V^2}{2g}}_{\text{velocity head}} = \underbrace{C}_{\text{constant along streanline}}$$

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^{\,2}}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^{\,2}}{2g} = \frac{p_3}{\gamma} + z_3 + \frac{V_3^{\,2}}{2g}$$

## EXAMPLE 4.6 VELOCITY IN A VENTURI SECTION

Piezometric tubes are tapped into a venturi section as shown in the figure. The liquid is incompressible. The upstream piezometric head is 1 m, and the piezometric head at the throat is 0.5 m. The velocity in the throat section is twice as large as in the approach section. Find the velocity in the throat section.

**Sketch:**



*Solution*

1. The Bernoulli equation with $V_2 = 2V_1$ gives

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2g} = \frac{3V_1^2}{2g}$$

$$V_1^2 = \frac{2g}{3}(h_1 - h_2)$$

$$V_2 = 2\sqrt{\frac{2g}{3}(h_1 - h_2)}$$

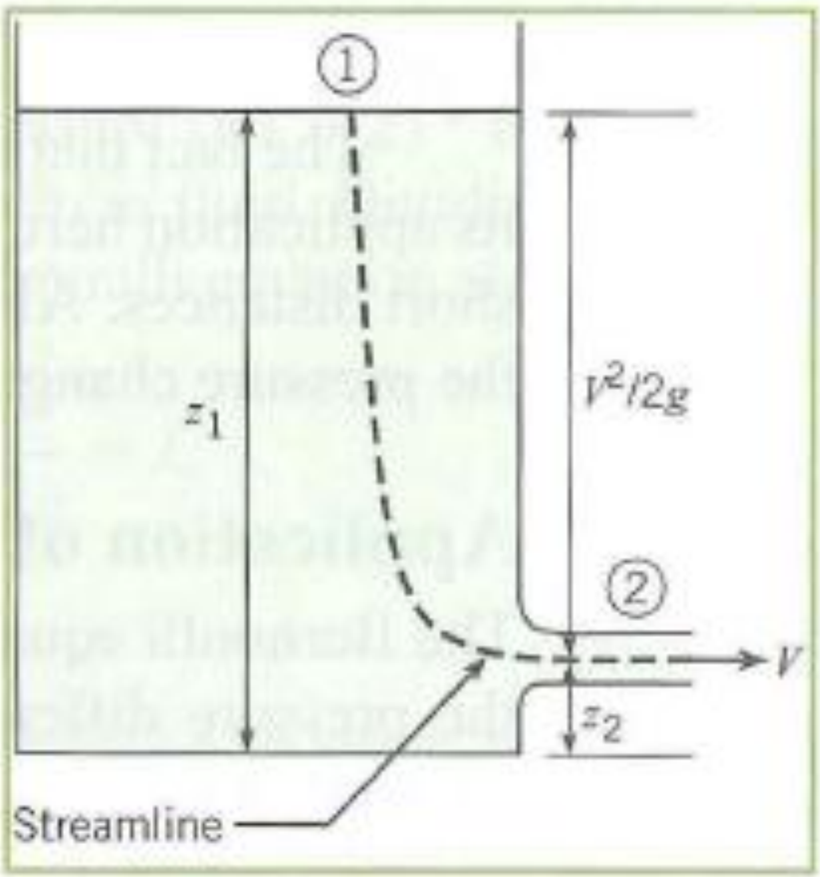
2. Substitution of values and velocity calculation:

$$V_2 = 2\sqrt{\frac{2 \times 9.81 \text{ m/s}^2 (1 - 0.5) \text{ m}}{3}}$$

$$= \boxed{3.62 \text{ m/s}}$$

## EXAMPLE 4.7 OUTLET VELOCITY FROM DRAINING TANK

An open tank is filled with water and drains through a port at the bottom of the tank. The elevation of the water in the tank is 10 m above the drain. The drain port is at atmospheric pressure. Find the velocity of the liquid in the drain port.



*Solution*

1. The Bernoulli equation between points 1 and 2 on streamline:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

2. The pressure at the outlet and the tank surface are the same (atmospheric), so $p_1 = p_2$. The velocity at the tank surface is much less than in the drain port so $V_2^2 \gg V_1^2$. Solution for $V_2$:

$$z_1 - z_2 = \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2g(z_1 - z_2)}$$

3. Velocity calculation:

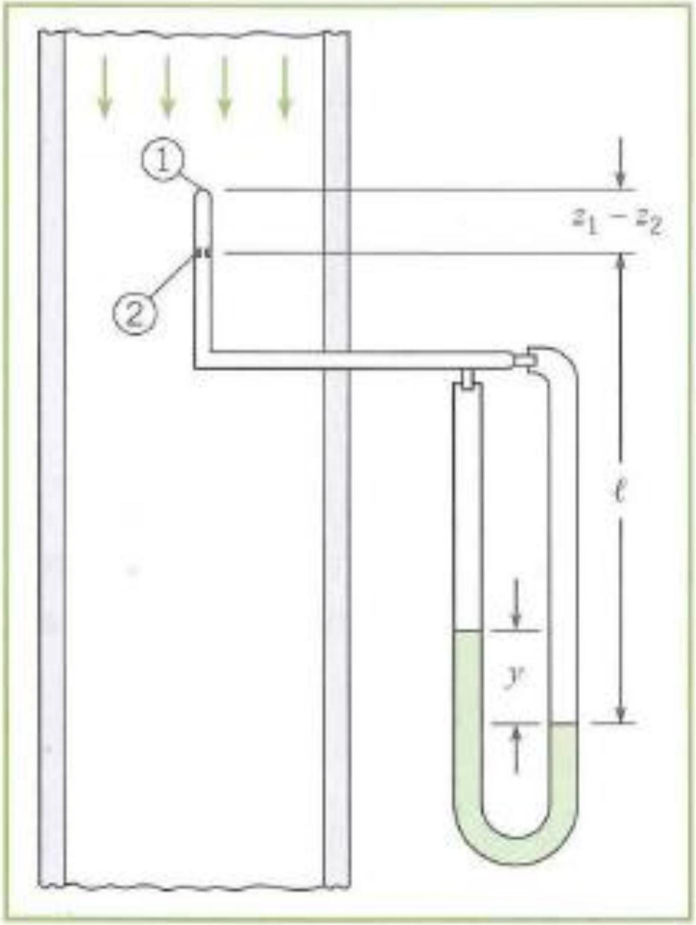
$$V_2 = \sqrt{2 \times 9.81\ \text{m/s}^2 \times 10\ \text{m}}$$

$$= \boxed{14\ \text{m/s}}$$

## EXAMPLE 4.8 APPLICATION OF PITOT EQUATION WITH MANOMETER

A mercury manometer is connected to the Pitot-static tube in a pipe transporting kerosene as shown. If the deflection on the manometer is 18 cm, what is the kerosene velocity in the pipe? Assume that the specific gravity of the kerosene is 0.81.

**Sketch:**



## *Solution*

1. Manometer equation between points 1 and 2 on Pitot-static tube:

$$p_1 + (z_1 - z_2)\gamma_{\text{kero}} + \ell\gamma_{\text{kero}} - y\gamma_{\text{Hg}} - (\ell - y)\gamma_{\text{kero}} = p_2$$

or

$$p_1 + \gamma_{\text{kero}} z_1 - (p_2 + \gamma_{\text{kero}} z_2) = y(\gamma_{\text{Hg}} - \gamma_{\text{kero}})$$

$$p_{z,1} - p_{z,2} = y(\gamma_{\text{Hg}} - \gamma_{\text{kero}})$$

2. Substitution into the Pitot-static tube equation:

$$V = \left[\frac{2}{\rho_{\text{kero}}} y(\gamma_{\text{Hg}} - \gamma_{\text{kero}})\right]^{1/2}$$

$$= \left[2gy\left(\frac{\gamma_{\text{Hg}}}{\gamma_{\text{kero}}} - 1\right)\right]^{1/2}$$

3. Velocity evaluation:

$$V = \left[2 \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.18\ \text{m}\left(\frac{13.55}{0.81} - 1\right)\right]^{1/2}$$

$$= \left[2 \times 9.81 \times 0.18(16.7 - 1)\text{m}^2/\text{s}^2\right]^{1/2}$$

$$= \boxed{7.45\ \text{m/s}}$$