Solution to Assignment No.3

4-9 a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

$$P(X > 2.75) = \int_{2.75}^{2.8} 2dx = 2(0.05) = 0.10.$$

b) If the probability density function is centered at 2.55 meters, then $f_X(x) = 2$ for 2.3 < x < 2.8 and all rods will meet specifications.

4-21. $F(x) = \int_0^x 0.5x dx = \frac{0.5x^2}{2}\Big|_0^x = 0.25x^2$ for 0 < x < 2. Then,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25x^2, & 0 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

4-29. a) $E(X) = \int_5^\infty x10e^{-10(x-5)}dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_5^\infty + \int_5^\infty e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_5^\infty = 5.1$$

Now, $V(X) = \int_5^\infty (x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_5^\infty + 2\int_5^\infty (x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X) the integral above is recognized to equal 0.

Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^\infty 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^\infty = e^{-10(5.1-5)} = 0.3679$

4-38. a) $P(X > 35) = \int_{35}^{40} 0.1dx = 0.1x\Big|_{35}^{40} = 0.5$

b) P(X > x)=0.90 and $P(X > x) = \int_x^{40} 0.1dt = 0.1(40 - x)$.

Now, 0.1(40-x) = 0.90 and x = 31

c) E(X) = (30 + 40)/2 = 35 and V(X) = $\frac{(40-30)^2}{12} = 8.33$

4-45. a) P(X < 11) = $P\left(Z < \frac{11-5}{4}\right)$

= P(Z < 1.5)

= 0.93319

b) P(X > 0) = $P\left(Z > \frac{0-5}{4}\right)$ = P(Z > –1.25) = 1 – P(Z < –1.25)

= 0.89435

c) P(3 < X < 7) = $P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$ = P(–0.5 < Z < 0.5) = P(Z < 0.5) – P(Z < –0.5)

= 0.38292

d) P(–2 < X < 9) = $P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$ = P(–1.75 < Z < 1) = P(Z < 1) – P(Z < –1.75)]

= 0.80128

e) P(2 < X < 8) = $P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$ =P(–0.75 < Z < 0.75) = P(Z < 0.75) – P(Z < –0.75)

= 0.54674

4-49. a) P(X > 0.62) = $P\left(Z > \frac{0.62-0.5}{0.05}\right)$

= P(Z > 2.4)
= 1 – P(Z <2.4)
= 0.0082

b) P(0.47 < X < 0.63) = $P\left(\frac{0.47-0.5}{0.05} < Z < \frac{0.63-0.5}{0.05}\right)$

= P(–0.6 < Z < 2.6)
= P(Z < 2.6) – P(Z < –0.6)
= 0.99534 – 0.27425
= 0.72109

c) P(X < x) = $P\left(Z < \frac{x-0.5}{0.05}\right)$ = 0.90.

Therefore, $\frac{x-0.5}{0.05}$ = 1.28 and x = 0.564.

4-59. a) P(X > 0.0026) = $P\left(Z > \frac{0.0026-0.002}{0.0004}\right)$

= P(Z > 1.5)
= 1-P(Z < 1.5)
= 0.06681.

b) P(0.0014 < X < 0.0026) = $P\left(\frac{0.0014-0.002}{0.0004} < Z < \frac{0.0026-0.002}{0.0004}\right)$

= P(–1.5 < Z < 1.5)
= 0.86638.

c) P(0.0014 < X < 0.0026) = $P\left(\frac{0.0014-0.002}{\sigma} < Z < \frac{0.0026-0.002}{\sigma}\right)$

$= P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right)$.

Therefore, $P\left(Z < \frac{0.0006}{\sigma}\right)$ = 0.9975. Therefore, $\frac{0.0006}{\sigma}$ = 2.81 and σ = 0.000214.